Lecture - Spectral theory of operators on discrete spaces
SoSe 2024

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Topics: In this lecture we study the basic spectral theory of self-adjoint Schrödinger operators on (infinite) weighted graphs. More precisely, we investigate the bottom of the spectrum and the bottom of the essential spectrum (respectively absence of the essential spectrum) in terms of the geometry of the underlying graphs by establishing the theorems of Allegretto-Piepenbrink, Buser-Cheeger-Dodzuik, Persson and Brooks-Sturm. The discrete setting has the advantage that many technical details related to local regularity of functions can be ignored. The plan for the lecture is as follows:

1. Basics on self-adjoint operators induced by quadratic forms.

2. Self-adjoint Schrödinger operators on discrete spaces and the Beurling-Deny criteria.

3. Basics on the essential spectrum.

4. The essential spectrum for Schrödinger operators on graphs and the Theorem's of Allegretto-Piepenbrink and Persson.

5. Intrinsic metrics and the Buser-Cheeger-Dodzuik inequality.

6. The theorem of Brooks-Sturm.

Prerequisites: Basic knowledge on topics covered in the course Funktionalanalysis I (basics on Hilbert spaces, bounded and closed operators).

Times: Wednesdays 11:15 - 12:45 in Room P801 Paulinum.

Exercises: There are no exercise classes for this lecture. However, I include many exercises in the notes and I encourage everyone to try to solve them.

Literature: Matthias Keller, Daniel Lenz und Radosław Wojciechowski. *Graphs and Discrete Dirichlet spaces.* Grundlagen der mathematischen Wissenschaften 358, Springer 2021.