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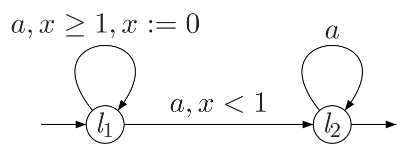
Institut für Informatik

„Real-Timed Automata“
Exercise 2

The following exercises must be submitted 19.05.2014 *before* the lecture.

1. Construct the region automaton for the following timed automaton:



2. Prove the remaining open case for Lemma 3.7:
   Assume $\langle (l,\nu),(l,r)\rangle \in R$ *and* $(l,r) \xrightarrow{a}_{\mathcal{R}} (l',r')$. *Then there exists some* (l',ν') *such that* $(l,\nu) \xrightarrow{a}_D (l',\nu')$ *and* $\langle (l',\nu'),(l',r')\rangle \in R$.

3. For $a, b \in \mathbb{Z}$ with $a \leq b$, we use $[a,b]$ to denote the set $\{x \in \mathbb{Z} \mid a \leq x \leq b\}$.

   A *bounded one-counter automaton* (BOCA, for short) is a tuple $\mathcal{B} = (Q, b, \Delta, q_\iota)$, where Q is a finite set of control states, $b \in \mathbb{N}$ is a global counter bound, $\Delta \subseteq (Q \times [-b,+b] \times Q)$ is a finite set of transitions, and $q_\iota \in Q$ is the initial control state. A *configuration* is a pair (q,v), where $q \in Q$ is the current control state, and $v \in [0,b]$ is the current value of the counter. We define the transition relation $\rightarrow$ on the set of configurations as follows: $\langle (q,v),(q',v')\rangle \in \rightarrow$ iff there is a transition $(q,d,q') \in \Delta$ such that $v' = v + d$. A *computation* is a finite sequence $\prod_{1 \leq i \leq n} \langle (q_{i-1}, v_{i-1}),(q_i,v_i)\rangle$ such that $q_0 = q_\iota$, $v_0 = 0$, and $\langle (q_{i-1}, v_{i-1}),(q_i,v_i)\rangle \in \rightarrow$ for every $i \in \{1,\ldots,n\}$. The reachability problem is: given a bounded one-counter automaton $\mathcal{B} = (Q, b, \Delta, q_\iota)$ and $q \in Q$, is there a computation ending in q?

   A *bounded one-counter automaton with inequality tests* is a tuple $\mathcal{I} = (Q, b, \Delta, q_\iota)$, where Q, b, q_ι are as above, and $\Delta \subseteq (Q \times [-b,+b] \times [0,b] \times [0,b] \times Q)$ is a finite set of transitions. In a transition of the form $(q,d,g_1,g_2,q') \in \Delta$, g_1 and g_2 determine the lower and upper bound on the value of the counter. Accordingly, the transition relation $\Rightarrow$ on the set of configurations is defined by $\langle (q,v),(q',v')\rangle \in \Rightarrow$ iff there is a transition $(q,d,g_1,g_2,q') \in \Delta$ such that $g_1 \leq v \leq g_2$ and $v' = v + d$. Computations and the reachability problem are defined as for BOCA.

   Prove that the reachability problem for bounded one-counter automata with inequality tests is log-space reducible to the reachability problem for BOCA.
   Hint: Give a translation from bounded one-counter automata with inequality tests to BOCA. Use the global counter bound to test inequalities against the counter.

4. An instance of the SUBSET-SUM problem consists of a pair (A,t), where $A \subseteq \mathbb{N}$ is a finite set of the natural numbers, and $t \in \mathbb{N}$ is a natural number. The SUBSET-SUM problem is to decide, whether there exists a subset B of A, such that $\sum_{a \in B} a = t$. This problem is NP-complete.

   Show by a reduction of the SUBSET-SUM problem that the reachability problem for timed automata with two clocks is NP-hard.