Trees Abound
Part I: Tree Automata

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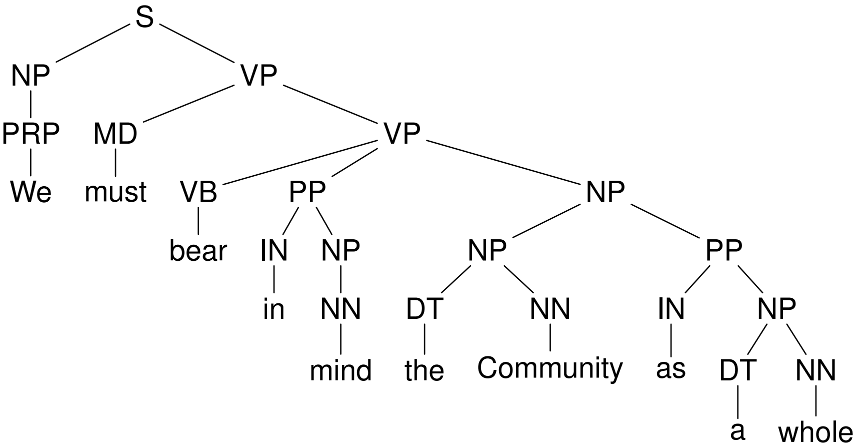
Trees?


S
NP
VP
PRP
MD
VP
We
must
VB
PP
NP
bear
IN
NP
NP
PP
in
NN
DT
NN
IN
NP
mind
the
Community
as
DT
NN
a
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Tree Language

How to represent a set of trees?

- enumerate them
- enumerate them cleverly (e.g., add sharing)
- parse forest of a CFG

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Parse Forest of a CFG

Example

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\begin{aligned}
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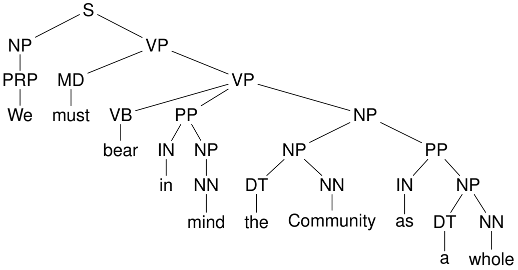

Parse Forest of a CFG

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$$S \to NP\ VP \qquad VP \to MD\ VP$$
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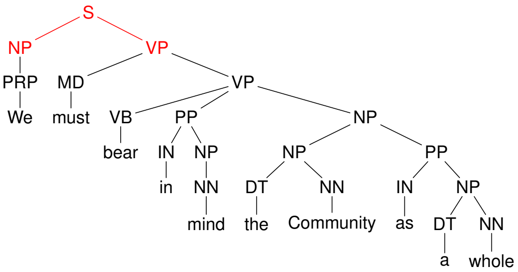

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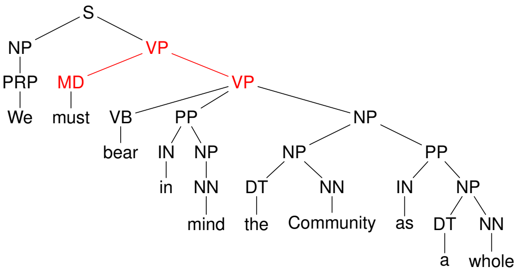

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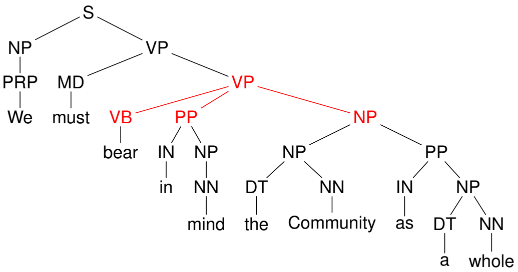

Parse Forest of a CFG

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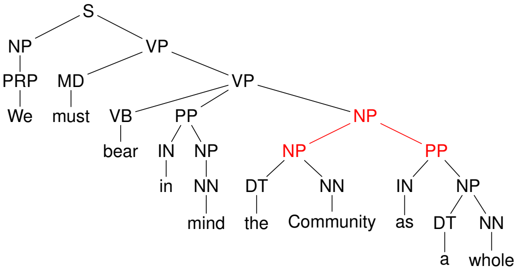

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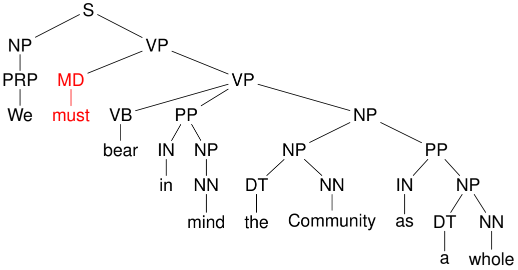

Local Tree Grammar

Definition (Gécseg, Steinby 1984)

A local tree grammar G is a finite set of CFG productions (together with a start nonterminal S)

Definition (Generated tree language)

$L(G)$ contains exactly the trees in which

- the root is labeled S
- "label $\rightarrow$ child labels" is a production of G for each internal node

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Local Tree Grammar

Theorem

Local tree grammars recognize exactly the parse forests of CFG

Properties

- ✓ simple
- ✓ no ambiguity (unique explanation for each recognized tree)
- ✗ not closed under BOOLEAN operations (union/intersection/complement: ✗/✓/✗)
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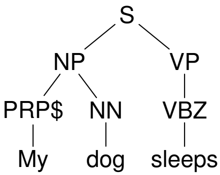
Local Tree Grammar

No ambiguity



is in $L(G)$ if and only if all the productions in it are in G

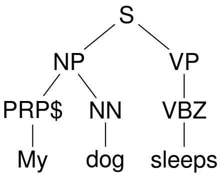
Local Tree Grammar

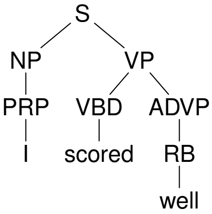
Theorem

Local tree languages are not closed under union

Proof.

The following single-element tree languages are local:





But their union is not local as it must also recognize:

My dog scored well **I sleeps* □

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- tree substitution grammar

Local Tree Grammar

Generalization

- CFG production $\mathrm{L} \rightarrow \mathrm{R}_1\ \mathrm{R}_2\ \mathrm{R}_3$ represented by tree

$$\begin{array}{ccc} & \mathrm{L} & \\ \diagup & | & \diagdown \\ \mathrm{R}_1 & \mathrm{R}_2 & \mathrm{R}_3 \end{array}$$

- “Glue” fragments together to obtain larger trees:

$$\mathrm{S}$$

- But why only small tree fragments?

Local Tree Grammar

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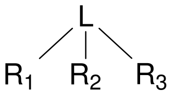

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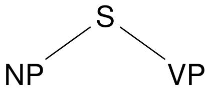

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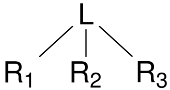

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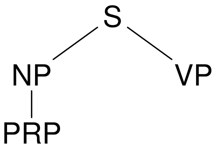

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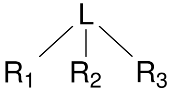

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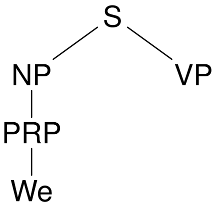

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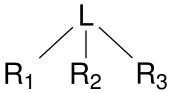

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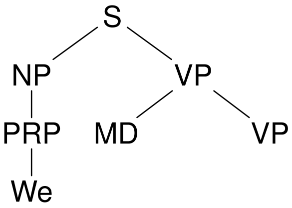

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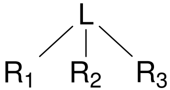

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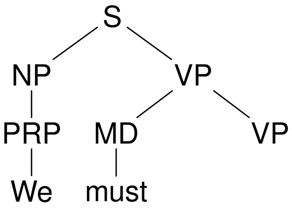

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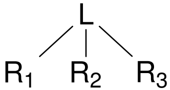

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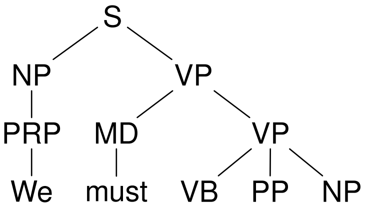

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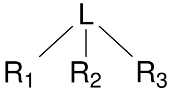

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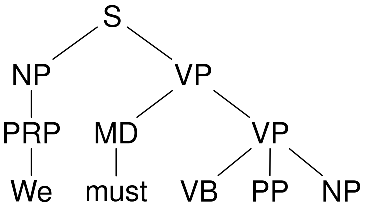

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Tree Substitution Grammar

Definition

A tree substitution grammar is a finite set of tree fragments (together with a start nonterminal S)

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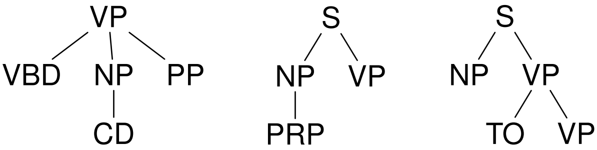
Example (Typical fragments [Post, ACL 2011])



Tree Substitution Grammar

Theorem

local tree languages $\subsetneq$ *tree substitution languages*

Proof.

Can trivially express all finite tree languages □

Remarks

- can express many finite-distance dependencies
- *extended domain of locality*

Tree Substitution Grammar

Properties

- ✓ simple
- ✓ more expressive than local tree grammars
- ✗ ambiguity (several explanations for a recognized tree)
- ✗ not closed under BOOLEAN operations
  (union/intersection/complement: ✗/✗/✗)
- ✗ not closed under (non-injective) relabelings
- ✗ …

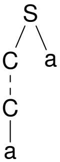
Tree Substitution Grammar

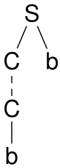
Theorem

Tree substitution languages are not closed under union

Proof.

Counterexample must be infinite $\rightsquigarrow$ artificial example





$L_1 = \{\mathrm{S}(\mathrm{C}^n(a), a) \mid n \in \mathbb{N}\}$ $\qquad$ $L_2 = \{\mathrm{S}(\mathrm{C}^n(b), b) \mid n \in \mathbb{N}\}$

Their union is not a tree substitution language $\square$

Tree Substitution Grammar

Theorem

Tree substitution languages are not closed under intersection

Proof.

Ideas? □

Tree Substitution Grammar

Experiment [Post, Gildea, ACL 2009]

grammar	size	Prec.	Recall	F1
CFG	46k	75.37	70.05	72.61
“spinal” TSG	190k	80.30	78.10	79.18
“minimal subset” TSG	2,560k	76.40	78.29	77.33

(on WSJ Sect. 23)

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- tree substitution grammar with latent variables

Tree Substitution Grammar with Latent Variables

Definition (Shindo et al., ACL 2012 best paper)

A tree substitution grammar with latent variables is
a tree substitution grammar together with a functional relabeling

Remark

Typically symbols that are relabeled to X are written as X-*n*

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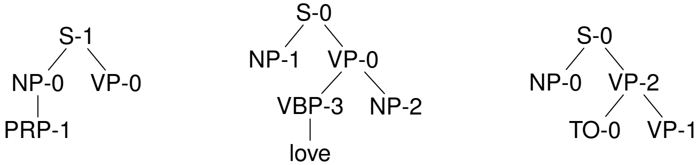
Example (Typical fragments)



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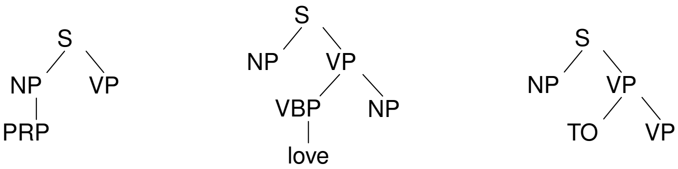

Tree Substitution Grammar with Latent Variables

Experiment [Shindo et al., ACL 2012 best paper]

grammar	F1 score $\|w\| \leq 40$	F1 score full
TSG [Post, Gildea, 2009]	82.6	
TSG [Cohn et al., 2010]	85.4	84.7
CFGlv [Collins, 1999]	88.6	88.2
CFGlv [Petrov, Klein, 2007]	90.6	90.1
CFGlv [Petrov, 2010]		91.8
TSGlv (single)	91.6	91.1
TSGlv (multiple)	92.9	92.4
Discriminative Parsers		
Carreras et al., 2008		91.1
Charniak, Johnson, 2005	92.0	91.4
Huang, 2008	92.3	91.7

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Let us look at a really old model

Overview

Regular Tree Grammar

Definition (Brainerd, 1969)

A regular tree grammar is a tuple $G = (Q, \Sigma, I, P)$ with

- alphabet of nonterminals Q
- alphabet of terminals Σ
- initial nonterminals $I \subseteq Q$
- finite set of productions $P \subseteq Q \times T_\Sigma(Q)$

Remark

Instead of (q, r) we write $q \to r$

Regular Tree Grammar

Example

- $Q = \{q_0, q_1, q_2, q_3, q_4, q_5, q_6\}$
- $\Sigma = \{\mathrm{VP}, \mathrm{NP}, \mathrm{S}, \dots\}$
- $I = \{q_0\}$
- and the following productions:

$$q_4 \to \mathrm{VP}(q_5, \mathrm{NP}(q_2), q_3) \qquad q_0 \to \mathrm{S}(\mathrm{NP}(q_1), q_4) \qquad q_0 \to \mathrm{S}(q_6, \mathrm{VP}(q_2, q_4))$$

Regular Tree Grammar

Definition (Derivation Semantics)

Sentential forms: $t, u \in T_\Sigma(Q)$

$$\boxed{t \Rightarrow_G u}$$

if there exist position $w \in \mathrm{pos}(t)$ and production $q \to r \in P$

- $t = t[q]_w$
- $u = t[r]_w$

Definition (Recognized tree language)

$$L(G) = \{t \in T_\Sigma \mid \exists q \in I \colon q \Rightarrow_G^* t\}$$

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Regular Tree Grammar

Example (Productions)

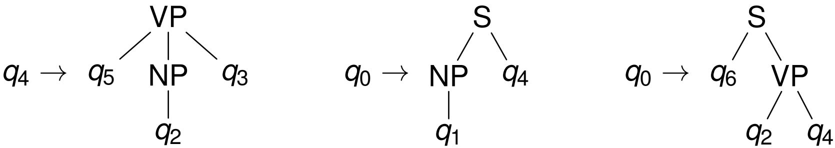
$$q_4 \to \mathrm{VP}(q_5, \mathrm{NP}(q_2), q_3) \qquad q_0 \to \mathrm{S}(\mathrm{NP}(q_1), q_4) \qquad q_0 \to \mathrm{S}(q_6, \mathrm{VP}(q_2, q_4))$$

Example (Derivation)

$$q_0$$

Regular Tree Grammar

Example (Productions)



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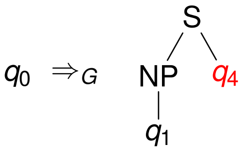

Regular Tree Grammar

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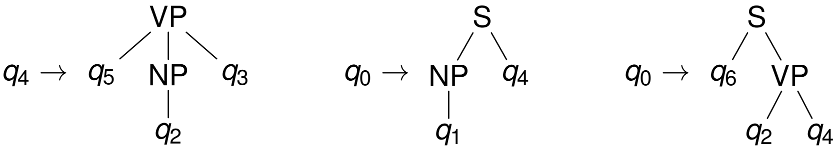

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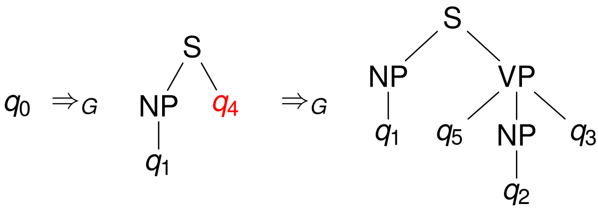

Regular Tree Grammar

Theorem

tree substitution languages $\subsetneq$ *regular tree languages*

Proof.

We can express the union counterexample easily $\square$

Remarks

- can organize finite information transport
  (even over unbounded distance)

Regular Tree Grammar

Properties

- ✓ simple
- ✓ more expressive than tree substitution grammars
- ✗ ambiguity (several explanations for a recognized tree)
- ✓ closed under all BOOLEAN operations
  (union/intersection/complement: ✓/✓/✓)
- ✓ closed under (non-injective) relabelings
- ✓ …

Regular Tree Grammar

Definition (Brainerd, 1969)

G is in normal form if $r = \sigma(q_1, \ldots, q_k)$ with $\sigma \in \Sigma$ and $q_1, \ldots, q_k \in Q$ for all $q \to r \in P$

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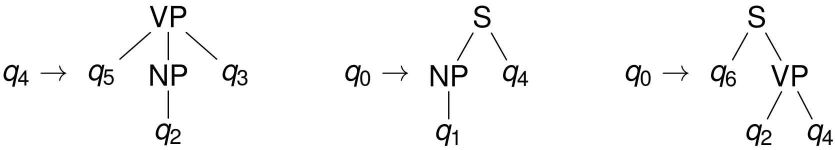


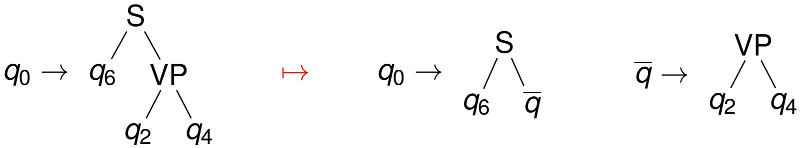
Regular Tree Grammar

Theorem (Brainerd, 1969)

Any G is equivalent to a regular tree grammar in normal form

Proof.

Simply cut large rules introducing new states

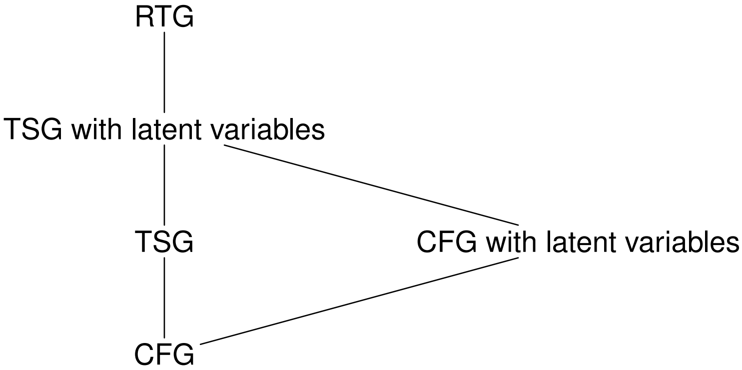


$\square$

Regular Tree Grammar

Theorem (Folk, Lore, 1972)

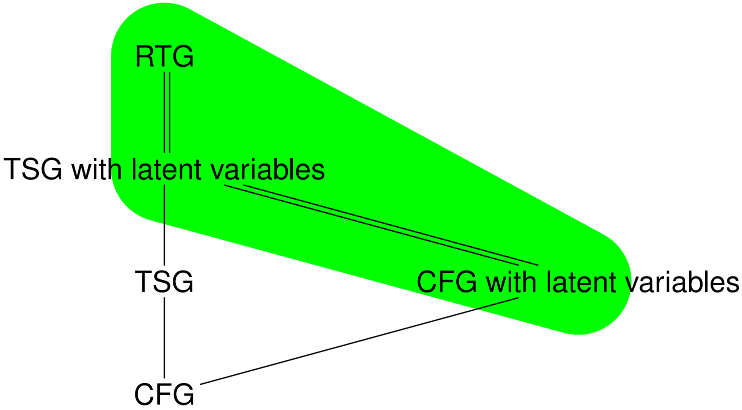
regular tree languages = *relabeled local tree languages*



Regular Tree Grammar

Theorem (Folk, Lore, 1972)

regular tree languages $=$ *relabeled local tree languages*



Berkeley Parser

Example (Berkeley parser — English grammar)

S-1 $\to$ ADJP-2 S-1	$0.0035453455987323125 \cdot 10^{0}$
S-1 $\to$ ADJP-1 S-1	$2.108608433271444 \cdot 10^{-6}$
S-1 $\to$ VP-5 VP-3	$1.6367163259885093 \cdot 10^{-4}$
S-2 $\to$ VP-5 VP-3	$9.724998692152419 \cdot 10^{-8}$
S-1 $\to$ PP-7 VP-0	$1.0686659961009547 \cdot 10^{-5}$
S-9 $\to$ “ NP-3	$0.012551243773149695 \cdot 10^{0}$

$\rightsquigarrow$ Regular tree grammar

Recent NLP Result

Corollary

The grammar of [SHINDO et al., ACL 2012 best paper] can be implemented in the BERKELEY *parser*

Remark

- the main contribution of SHINDO et al. is not the TSGlv
- it is probably the intricate 3-layer back-off model

Recent NLP Result

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Overview

Tree Automaton

Definition (Thatcher, 1970; Rounds, 1970)

tree automaton is a regular tree grammar in normal form

Remarks

- bottom-up: rules written as $X(q_1, \ldots, q_k) \to q$
- top-down: rules written as $q \to X(q_1, \ldots, q_k)$

Tree Automaton

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Remarks

- bottom-up: rules written as $\mathrm{X}(q_1, \ldots, q_k) \to q$
- top-down: rules written as $q \to \mathrm{X}(q_1, \ldots, q_k)$

Determinism

Definition

- top-down deterministic if $\forall q \in Q, k \in \mathbb{N}, \mathrm{X} \in \Sigma$
  $\exists$ at most one $q_1, \ldots, q_k \in Q$: $q \to \mathrm{X}(q_1, \ldots, q_k) \in P$
- bottom-up deterministic if $\forall k \in \mathbb{N}, \mathrm{X} \in \Sigma, q_1, \ldots, q_k \in Q$
  $\exists$ at most one $q \in Q$: $\mathrm{X}(q_1, \ldots, q_k) \to q \in P$

(red determines blue)

Theorem (Thatcher, Wright, 1968; Doner, 1970)

top-down deterministic $\subsetneq$ *bottom-up deterministic* $=$ *RTL*

Proof.

By a standard subset construction and a simple counterexample $\square$

Determinism

tree automata

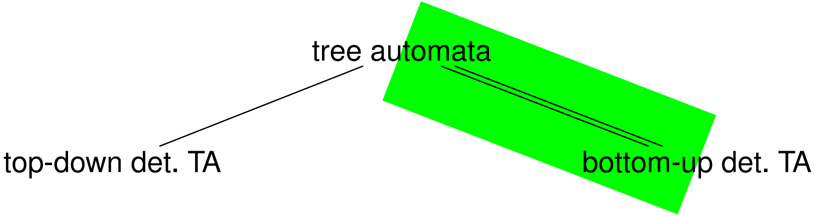
top-down det. TA

bottom-up det. TA

Remark

finite tree languages $\not\subseteq$ top-down deterministic

Determinism



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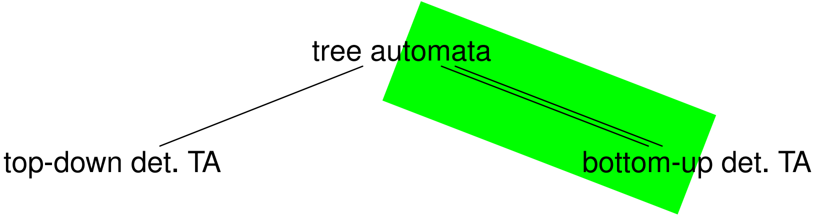

Remark

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Operations on Regular Tree Languages

Theorem

Regular tree languages are closed under

- *all* BOOLEAN *operations*
- *substitution (quotients) and iteration*
- *(non-deterministic) relabelings*
- *linear homomorphisms*
- *inverse homomorphisms*

Operations on Regular Tree Languages

Theorem

Regular tree languages are closed under substitution

Definition

$L, L' \subseteq T_\Sigma$ tree languages and $\mathrm{X} \in \Sigma$

$$\boxed{L[\mathrm{X} \leftarrow L']}$$

contains all trees obtained from a tree of L
by replacing each leaf labeled X by a tree of L'

Operations on Regular Tree Languages

Theorem

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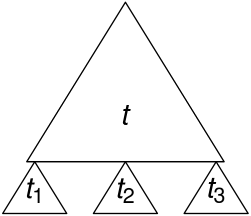
$$\boxed{L[\mathrm{X} \leftarrow L']}$$



$t \in L$

$t_1, t_2, t_3 \in L'$

Efficient Representation

Definition

A tree automaton is minimal in $\mathcal{C}$
if all equivalent tree automata of $\mathcal{C}$ are at least as large

Theorem

Complexity of minimization problems:

outp. \ inp. model	DTA	NTA
$\mathcal{C} =$ DTA	NL	(EXPTIME)
$\mathcal{C} =$ NTA	PSPACE	PSPACE

Overview

Weighted Tree Automaton

Definition (BERSTEL, REUTENAUER, 1982)

A weighted tree automaton is a tree automaton together with a map $c\colon P \to S$

Semantics

- S forms a semiring $(S, +, \cdot, 0, 1)$
- production weights are multiplied $(\cdot)$ in a derivation
- weights of multiple (left-most) derivations for the same tree are summed $(+)$

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Weighted Tree Automaton

Remarks

- BERKELEY parser uses weighted tree automata
- but has a best-derivation semantics

Theoretical research

- Minimization wrt. best-derivation semantics
- Minimization wrt. n-best-derivation semantics
- Foundational investigation of those semantics

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Bisimulation Minimization

(needs additive cancellation)

Experiment with BERKELEY parser

	states		productions	
English grammar	1,133	100%	1,842,218	100%
backward minimal	548	48%	626,600	34%
forward minimal	791	70%	767,153	42%
backward/forward minimal	366	32%	272,675	15%
forward/backward minimal	381	34%	309,845	17%
f/b/f/b minimal	375	33%	295,836	16%

These might be buggy

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Full Minimization

Theorem (Berstel, Reutenauer, 1982)

Weighted tree automata over fields can effectively be minimized

Remarks

- even smaller than bisimulation-minimal WTA
- implementations for weighted string automata are efficient
- no implementation for WTA yet

Summary

How to represent a set of trees?

- ~~enumerate them~~
- ~~enumerate them cleverly (e.g., add sharing)~~
- ~~parse forest of a CFG~~
- ~~tree substitution grammar~~
- ~~tree substitution grammar with latent variables~~
- ~~parse forest of a CFG with latent variables~~
- ~~...~~

regular tree grammar

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regular tree grammar

Many theoretical results still to be tried in practice!

Trees Abound
Part II: Tree Transducers

Andreas Maletti

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Universität Stuttgart, Germany

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Paris — September 26, 2012

From Automata to Transducers

Idea

Synchronous grammars have synchronous (linked) non-terminals that develop at the same time

Example

- join two productions $q_1 \to r_1$ and $q_2 \to r_2$ to $(q_1, q_2) \to (r_1, r_2)$
- demand $q_1 = q = q_2$ for simplicity and write $r_1 \xrightarrow{q} r_2$
- productions develop input and output trees at the same time

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From Automata to Transducers

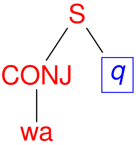
$\boxed{q}$ $\boxed{q}$

Used rule:

Next rule:

$$\boxed{q} \xrightarrow{q} \mathrm{S}(\mathrm{CONJ}(\mathrm{wa}), \boxed{q})$$

From Automata to Transducers



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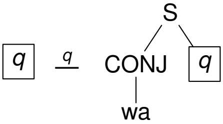

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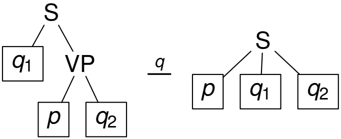

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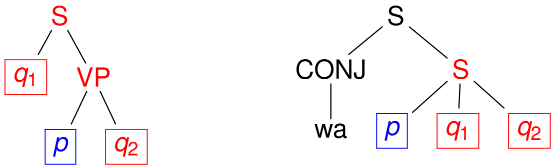

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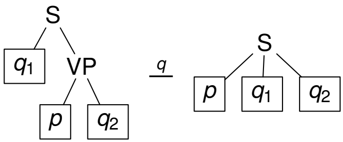

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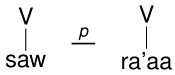

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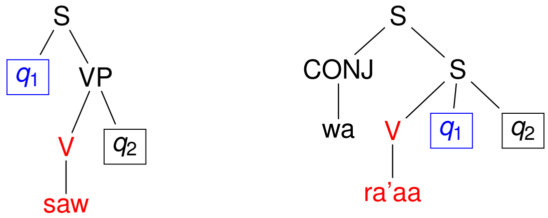

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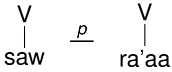

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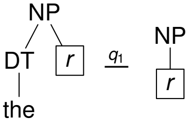

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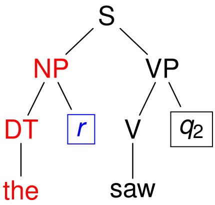

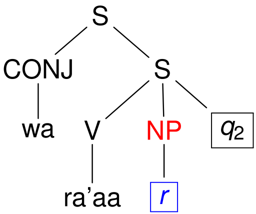

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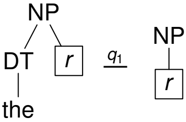

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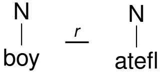

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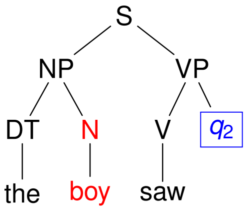

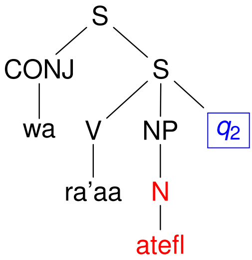

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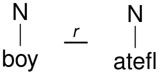

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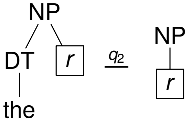

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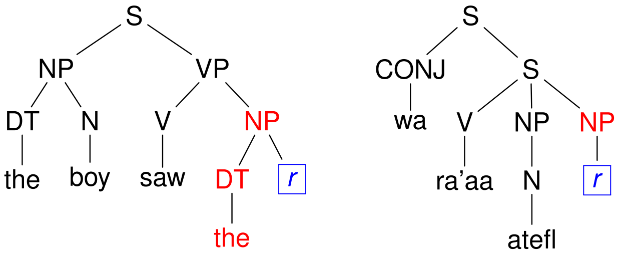

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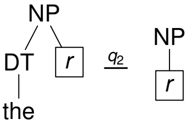

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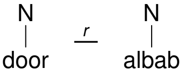

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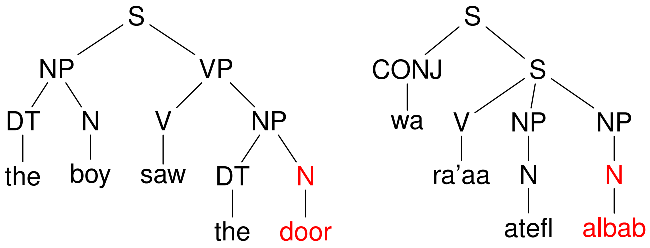

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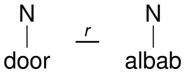


Next rule:

From Automata to Transducers

Remarks

- synchronization breaks the normalization proof
- the grammar/automaton model makes a difference

Output model: RTG and input model:

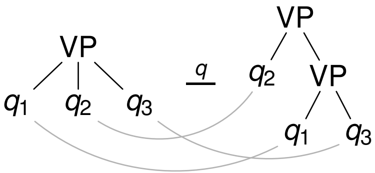
- NTA $\rightsquigarrow$ linear top-down tree transducer
- RTG $\rightsquigarrow$ linear extended top-down tree transducer

Overview

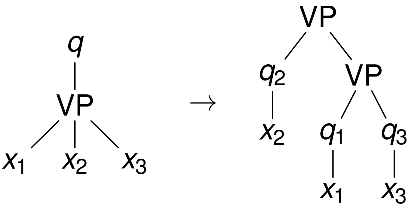
1. Quick Recall
2. Top-down Tree Transducers
3. Extended Top-down Tree Transducers
4. Extended Multi Bottom-up Tree Transducers

Rule Transformation

Synchronous grammar rule:



Top-down tree transducer rule:



Top-down Tree Transducer

Definition (THATCHER, 1970)

A top-down tree transducer is a system $M = (Q, \Sigma, \Delta, I, R)$ with

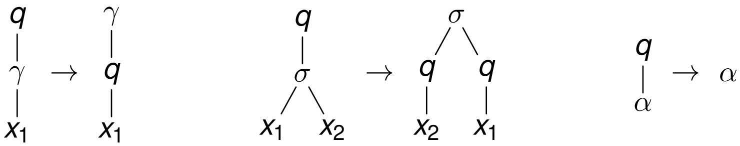
- alphabet of states Q
- input alphabet Σ; output alphabet Δ
- initial states $I \subseteq Q$
- finite set of rules $R \subseteq Q(\Sigma(X)) \times T_\Delta(Q(X))$ such that $\mathrm{var}(r) \subseteq \mathrm{var}(\ell)$ and ℓ is linear for all $(\ell, r) \in R$

Top-down Tree Transducer

Example

Mirror-image top-down tree transducer $(Q, \Sigma, \Sigma, Q, R)$ with

- $Q = \{q\}$
- $\Sigma = \{\sigma^{(2)}, \gamma^{(1)}, \alpha^{(0)}\}$
- the following rules in R



Top-down Tree Transducer

Definition

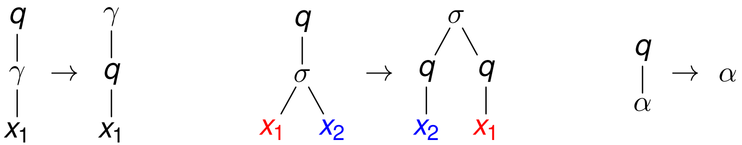
Sentential forms $\xi, \zeta \in T_\Delta(Q(T_\Sigma))$

$$\boxed{\xi \Rightarrow_M \zeta}$$

if there exist $\ell \to r \in R$, position $w \in \mathrm{pos}(\xi)$, substitution $\theta \colon X \to T_\Sigma$

- $\xi = \xi[\ell\theta]_w$
- $\zeta = \xi[r\theta]_w$

Derivation Example



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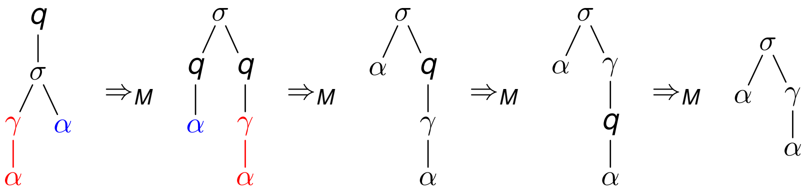

Derivation Semantics

Definition

$$M = \{\langle t, u\rangle \in T_\Sigma \times T_\Delta \mid \exists q \in I \colon q(t) \Rightarrow_M^* u\}$$

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Example

Top-down tree transducer N with

$$\{\langle \sigma(t, u), \sigma(u, t)\rangle \mid t, u \in T_{\{\gamma, \alpha\}}\} \subseteq N$$

$$q(\gamma(x_1)) \to \gamma(q(x_1)) \qquad q(\sigma(x_1, x_2)) \to \sigma(q(x_2), q(x_1)) \qquad q(\alpha) \to \alpha$$

Syntactic Restrictions

Definition

Transducer $M = (Q, \Sigma, \Delta, I, R)$ is

- linear if r is linear for every $\ell \to r \in R$
- nondeleting if $\text{var}(r) = \text{var}(\ell)$ for every $\ell \to r \in R$
- strict if $r \notin Q(X)$ for every $\ell \to r \in R$

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Example

Mirror-image transducer is linear, nondeleting, and strict (lns-TOP)

$$q(\gamma(x_1)) \to \gamma(q(x_1)) \qquad q(\sigma(x_1, x_2)) \to \sigma(q(x_2), q(x_1)) \qquad q(\alpha) \to \alpha$$

Expressive Power

Properties [ENGELFRIET, 1975]

- T1 "Copying of an input tree and processing the copies differently"
- T2 Cannot inspect deleted input tree

Remark

T2 has been addressed
$\rightsquigarrow$ top-down tree transducers with regular look-ahead [ENGELFRIET, 1977]

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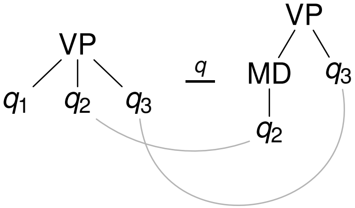
Regular Look-Ahead

Can be simulated by allowing un-linked nonterminals on the input side



- these develop without effect on the output
- can generate any regular tree language

Composition

Definition (Comp)

$\tau \subseteq T_\Sigma \times T_\Delta$ and $\tau' \subseteq T_\Delta \times T_\Gamma$

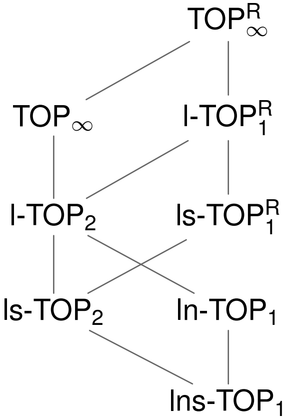
$$\boxed{\tau \,;\, \tau' = \{(s,u) \mid \exists t \in T_\Delta \colon (s,t) \in \tau, (t,u) \in \tau'\}}$$

Example (Double mirror-image)

$$N \,;\, N = \mathrm{id}$$

$$q(\gamma(x_1)) \to \gamma(q(x_1)) \qquad q(\sigma(x_1, x_2)) \to \sigma(q(x_2), q(x_1)) \qquad q(\alpha) \to \alpha$$

Expressive Power

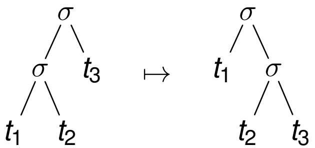


composition closure indicated in subscript

Desirable Properties

Rotations

$$\textsc{Rot} = \{\langle \sigma(\sigma(t_1, t_2), t_3),\ \sigma(t_1, \sigma(t_2, t_3))\rangle \mid t_1, t_2, t_3 \in T_\Sigma\}$$



Preservation of regularity (Pres)

Given $\tau \subseteq T_\Sigma \times T_\Delta$ and $L \subseteq T_\Sigma$ regular, is $\tau(L)$ regular?

$$\boxed{\tau(L) = \{u \mid \exists t \in L\colon (t, u) \in \tau\}}$$

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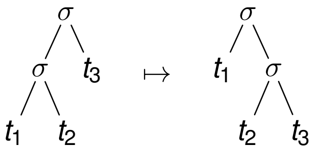
Rotations

$$\text{ROT} = \{\langle \sigma(\sigma(t_1, t_2), t_3),\ \sigma(t_1, \sigma(t_2, t_3)) \rangle \mid t_1, t_2, t_3 \in T_\Sigma\}$$



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Summary

Model \ Criterion	ROT	SYM	PRES	PRES^{-1}	COMP
Ins-TOP	✗	✗	✓	✓	✓
In-TOP	✗	✗	✓	✓	✓
Is-TOP	✗	✗	✓	✓	✗$_2$
I-TOP	✗	✗	✓	✓	✗$_2$
Is-TOP^R	✗	✗	✓	✓	✓
I-TOP^R	✗	✗	✓	✓	✓
TOP	✓	✗	✗	✓	✗$_\infty$
TOP^R	✓	✗	✗	✓	✗$_\infty$

(SYM = symmetric)

Overview

Extended Top-down Tree Transducer

Definition (GRAEHL et al., 2009)

A top-down tree transducer is a system $M = (Q, \Sigma, \Delta, I, R)$

- finite set of states Q
- input alphabet Σ; output alphabet Δ
- initial states $I \subseteq Q$
- finite set of rules $R \subseteq Q(\Sigma(X)) \times T_\Delta(Q(X))$ such that $\mathrm{var}(r) \subseteq \mathrm{var}(\ell)$ and ℓ is linear for all $(\ell, r) \in R$

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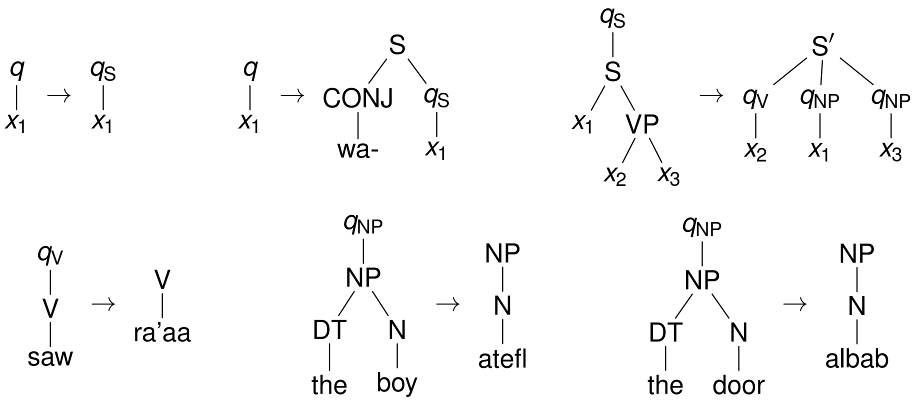
Extended Top-down Tree Transducer

Example



Extended Top-down Tree Transducer

Definition

Sentential forms $\xi, \zeta \in T_\Delta(Q(T_\Sigma))$

$$\boxed{\xi \Rightarrow_M \zeta}$$

if there exist $\ell \to r \in R$, position $w \in \mathrm{pos}(\xi)$, substitution $\theta \colon X \to T_\Sigma$

- $\xi = \xi[\ell\theta]_w$
- $\zeta = \xi[r\theta]_w$

Definition

$$M = \{\langle t, u\rangle \in T_\Sigma \times T_\Delta \mid \exists q \in I \colon q(t) \Rightarrow_M^* u\}$$

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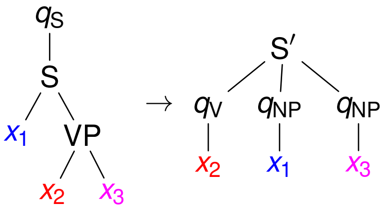

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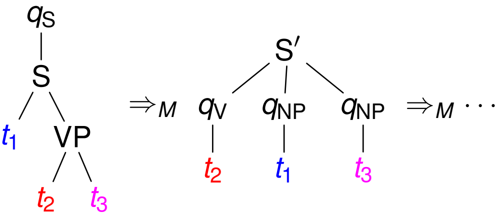
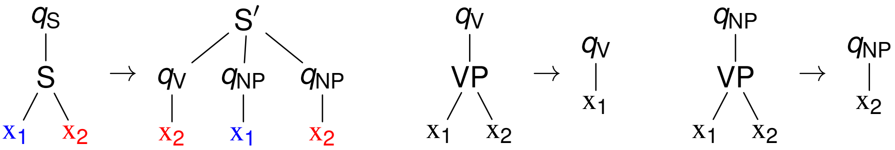


Simulation by Copying and Deletion



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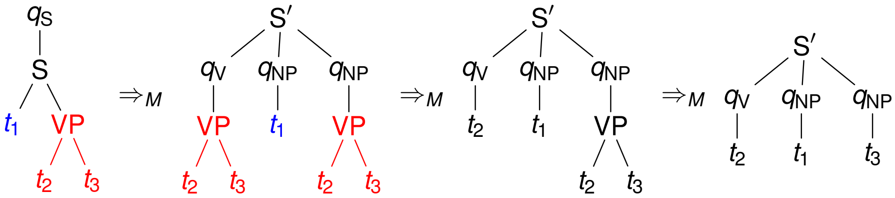

Syntactic Restrictions

Definition

Extended top-down tree transducer $M = (Q, \Sigma, \Delta, I, R)$ is

- linear, nondeleting, strict as before
- ε-free if $\ell \notin Q(X)$ for every $\ell \to r \in R$

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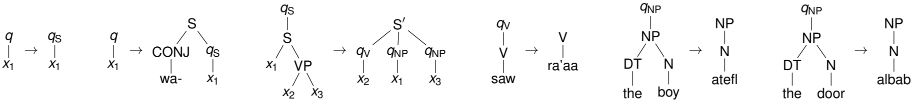
Example

Our example transducer is linear, nondeleting, ~~strict~~, and ~~ε-free~~



Expressive Power

Properties [Graehl et al., 2009]

- X1 Finite look-ahead
- X2 Deep attachment of variables
- X3 Infinitely many outputs for one input

Remark

T1 and T2 still apply

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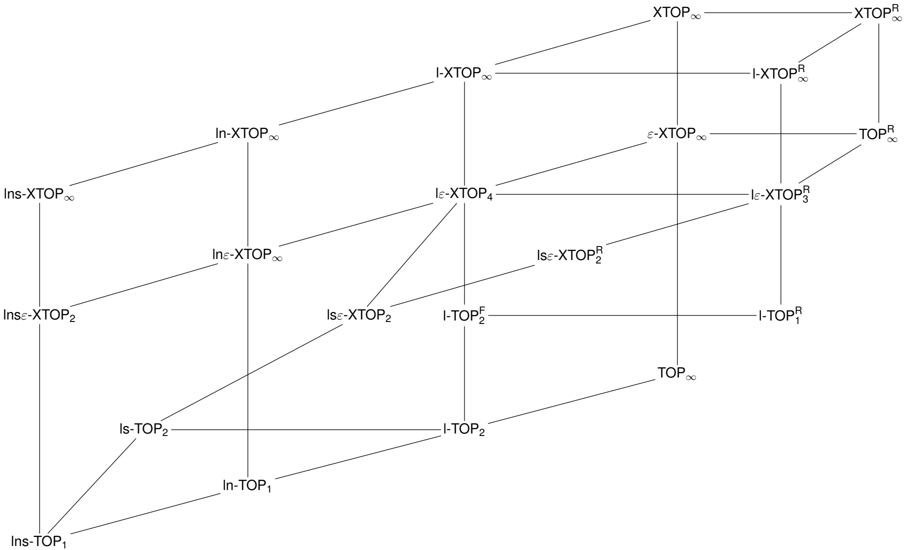

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lns-XTOP	✓	✗	✓	✓	✗$_\infty$
lsε-XTOP$^{(R)}$	✓	✗	✓	✓	✗$_2$
lε-XTOP	✓	✗	✓	✓	✗$_4$
lε-XTOP^R	✓	✗	✓	✓	✗$_3$
(s)l-XTOP$^{(R)}$	✓	✗	✓	✓	✗$_\infty$
XTOP	✓	✗	✗	✓	✗$_\infty$
XTOP^R	✓	✗	✗	✓	✗$_\infty$

Overview

Extended Multi Bottom-up Tree Transducer

Definition

An extended multi bottom-up tree transducer $M = (Q, \Sigma, \Delta, F, R)$ with

- ranked alphabet of states Q
- input alphabet Σ; output alphabet Δ
- final states $F \subseteq Q_1$ (all unary)
- finite set of rules $R \subseteq T_\Sigma(Q(X)) \times Q(T_\Delta(X))$ such that $\text{var}(r) \subseteq \text{var}(\ell)$ and ℓ is linear for all $(\ell, r) \in R$

Properties

linear, nondeleting, strict, ε-free as before

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Extended Multi Bottom-up Tree Transducer

Example (Duplication)

Extended multi bottom-up tree transducer $(Q, \Sigma, \Sigma, \{f\}, R)$

- $Q = \{q^{(2)}, f^{(1)}\}$
- $\Sigma = \{\sigma, a, b, e\}$
- R contains:

$$e \to q(e, e) \qquad a(q(x_1, x_2)) \to q(a(x_1), a(x_2)) \qquad b(q(x_1, x_2)) \to q(b(x_1), b(x_2)) \qquad q(x_1, x_2) \to f(\sigma(x_1, x_2))$$

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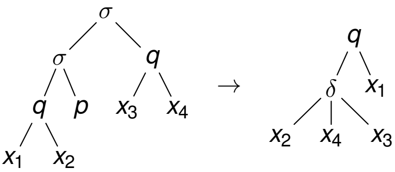
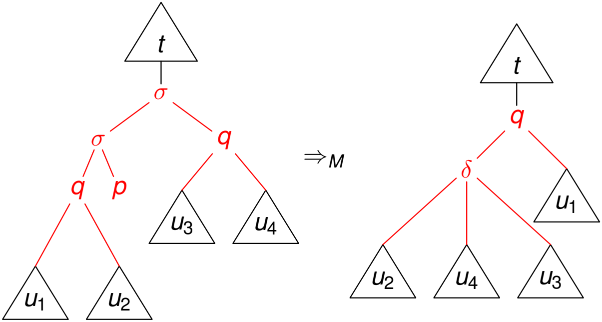
Extended Multi Bottom-up Tree Transducer

Rule:



Derivation:



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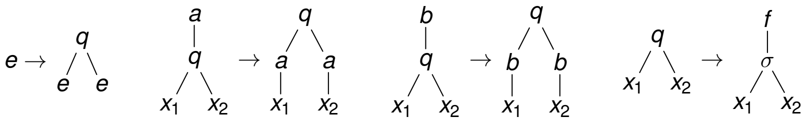
Example (Derivation)

$$a(b(b(e)))$$

Extended Multi Bottom-up Tree Transducer



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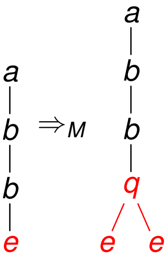

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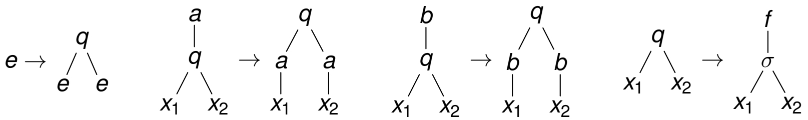

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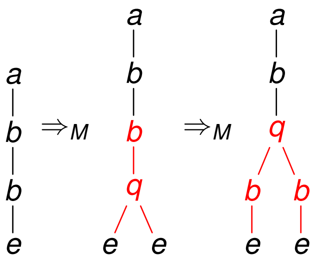

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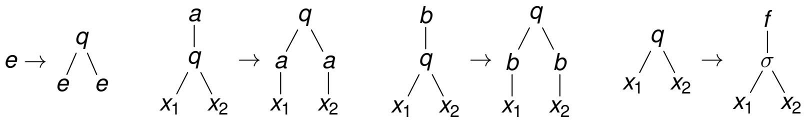

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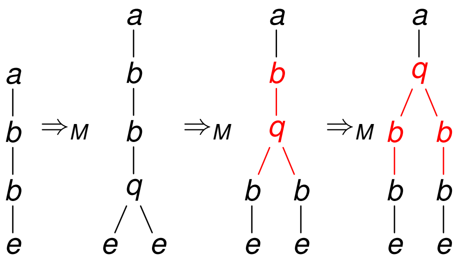

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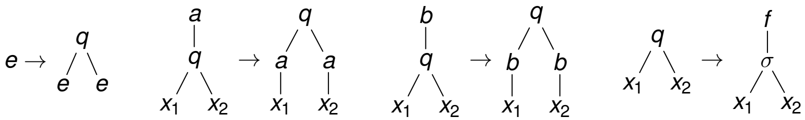

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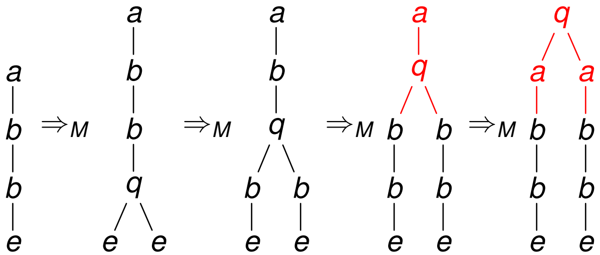

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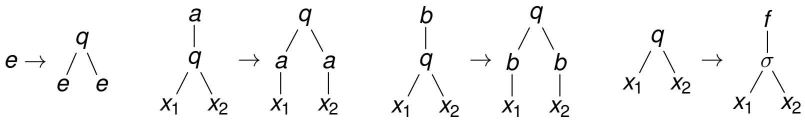

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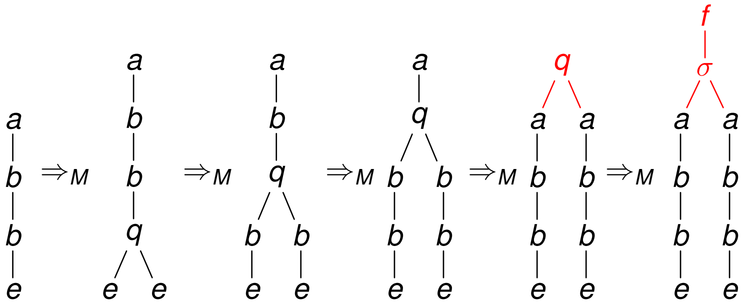

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Example (Duplication)

It computes $\{(t, \sigma(t, t)) \mid t \in T_\Sigma\}$

Its image is not a regular tree language

Subclasses

Definition

Extended multi bottom-up tree transducer $(Q, \Sigma, \Delta, F, R)$ is

- extended bottom-up tree transducer if $Q = Q_1$
- multi bottom-up tree transducer if $\ell \in \Sigma(Q(X))$ for all $\ell \to r \in R$
- bottom-up tree transducer if both previous conditions hold

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Expressive Power

Theorem (Engelfriet et al. '09)

$$\text{l-XTOP}^{\text{R}} = \text{l-XBOT}$$

Proof.

Standard construction trading input-deletion for output-deletion
see l-TOP $\subseteq$ l-BOT by [Engelfriet '75] □

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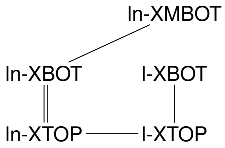

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$$\begin{tikzcd}
 & \text{ln-XMBOT} \\
\text{ln-XBOT} \arrow[ur, dash] \arrow[d, equal] & \text{l-XBOT} \arrow[d, dash] \\
\text{ln-XTOP} \arrow[r, dash] & \text{l-XTOP}
\end{tikzcd}$$

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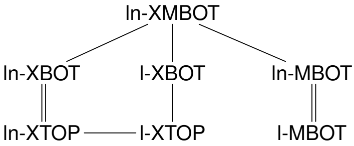

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Theorem (ENGELFRIET et al. ’09)

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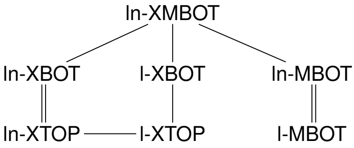

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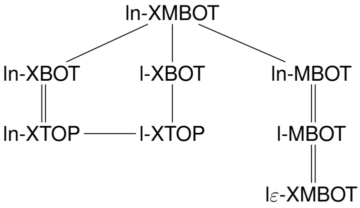

Expressive Power

Definition

XTOP M sensible if $|u| \in \mathcal{O}(|t|)$ for all $(t, u) \in M$

Theorem (Maletti '12)

$$\text{sensible XTOP} \subseteq \text{ln-MBOT}$$

Proof.

- use (essentially) construction of [Engelfriet, Maneth '03]
- obtain finitely copying ε-XTOP
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Expressive Power

Corollary

All relevant extended top-down tree transducers can be simulated by linear and nondeleting extended multi bottom-up tree transducers

Further Properties

Theorem

$$\text{ln-MBOT} \not\subseteq \text{XTOP}^{\text{R}}$$

Theorem (GILDEA '12)

$$\underset{\text{out}}{\mathit{yield}}(\text{ln-MBOT}) = \text{LCFRS}$$

Further Properties

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lsε-XTOP$^{(\mathrm{R})}$	✓	✗	✓	✓	$✗_2$
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lε-XTOP$^{\mathrm{R}}$	✓	✗	✓	✓	$✗_3$
(s)l-XTOP$^{(\mathrm{R})}$	✓	✗	✓	✓	$✗_\infty$
XTOP$^{(\mathrm{R})}$	✓	✗	✗	✓	$✗_\infty$
l(n)-XMBOT	✓	✗	✗	✓	✓
XMBOT	✓	✗	✗	✓	$✗_\infty$
reg.-preserving l-XMBOT	✓	✗	✓	✓	✓
invertable l-XMBOT	✓	✓	✓	✓	✓