Hyper-minimization for deterministic tree automata

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Contents

Hyper-Minimization

Intuition

Minimize automaton allowing a finite number of errors

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Minimize automaton allowing a finite number of errors

model/process	hyper-minimization	hyper-optimization
DFA	$\mathcal{O}(m \log n)$	$\mathcal{O}(mn)$
DBA	$\mathcal{O}(mn)$	???
DCA	$\mathcal{O}(mn)$	???
DTA	$\mathcal{O}(\ell mn)$	???

DTA = deterministic tree automaton

DBA / DCA = deterministic BÜCHI / Co-BÜCHI automaton

Why Tree Automata?

Applications

- Parsing (tree substitution grammars with latent variables)
- XML processing
- . . .

Parsing

- requires huge tree automata
- typically non-deterministic (but can be determinized)
- obtained from corpora (linguistic resources)

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Parsing of Natural Languages

Standard approach

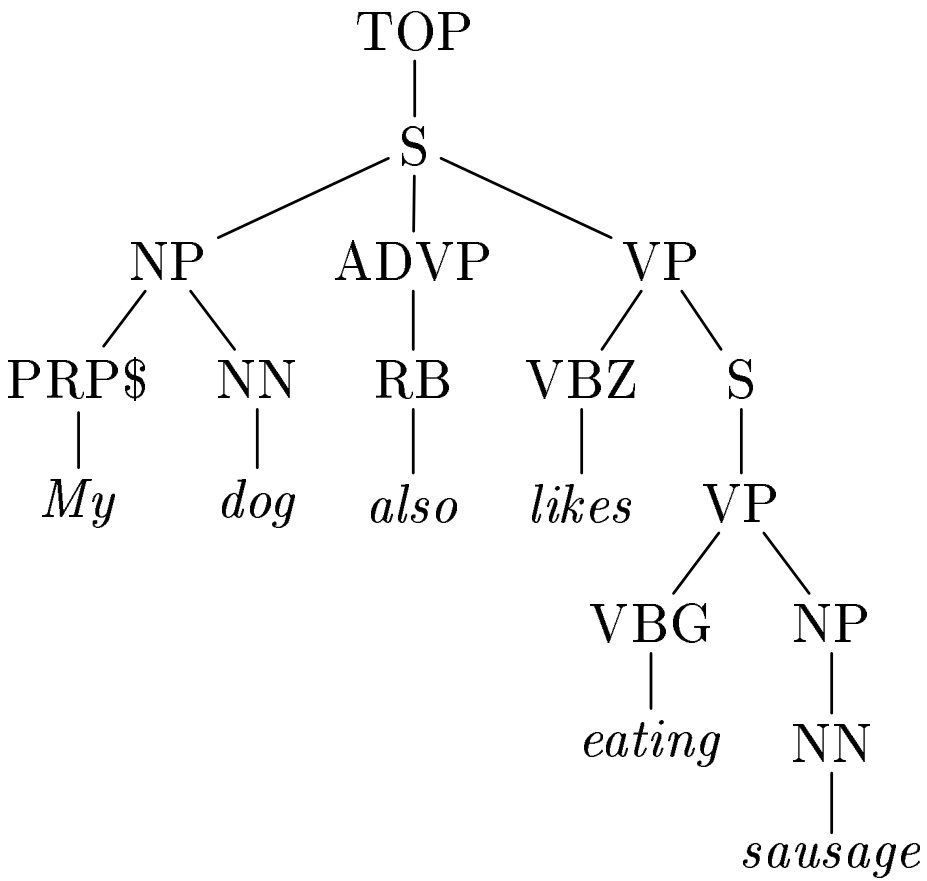
- supervised training (i.e., from annotated examples)
- but annotated samples are small, expensive
- can contain errors or inconsistencies

Intuition

- annotated sample not complete (positives only)
- annotation errors and inconsistencies (errors in annotation)

$\rightarrow$ reasonable training must generalize from finite observations to infinite language

Parse Tree


TOP
S
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ADVP
VP
PRP$
NN
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VBZ
S
My
dog
also
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VP
VBG
NP
eating
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sausage

Why Hyper-Minimization?

Advantages

- makes the DTA smaller $\rightarrow$ efficiency gain
- reduces spurious, artificial effects
- allows inspection of the “core” (the recursive structure)

Disadvantages

- gains sometimes rather small (in particular for DTA)
- no discrimination between errors
- no non-trivial limit on the number of errors

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Deterministic Tree Automaton

Definition (GÉCSEG, STEINBY 1984)

(Q, Σ, δ, F) deterministic tree automaton (DTA)

- Q finite set — *states*
- Σ ranked alphabet — *input symbols*
- $\delta\colon \Sigma(Q) \to Q$ — *transitions*
- $F \subseteq Q$ — *final states*

Definition

transition function extends to $\delta\colon T_\Sigma(Q) \to Q$ by

$$\delta(q) = q$$
$$\delta(\sigma(t_1, \ldots, t_k)) = \delta(\sigma(\delta(t_1), \ldots, \delta(t_k)))$$

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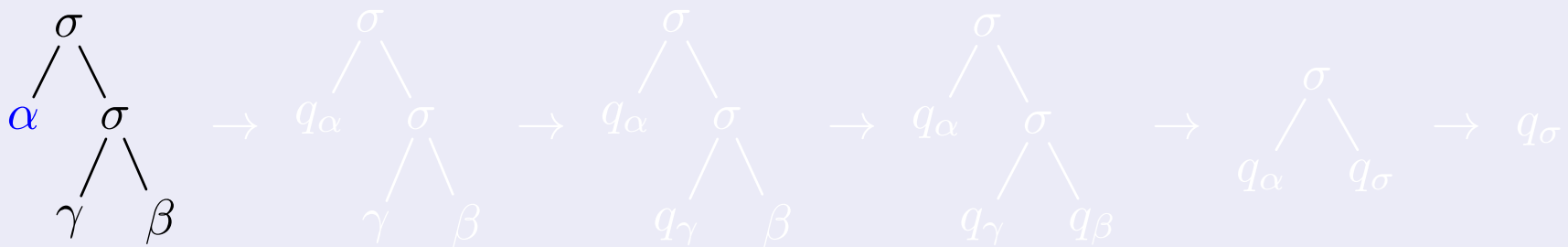
Deterministic Tree Automaton

Example

- states q_α, q_β (nonfinal) and q_γ, q_σ (final)
- nullary input symbols α, β, γ and binary σ
- for all nullary symbols π, π'

$$\pi \mapsto q_\pi \qquad \sigma(q_\pi, q_{\pi'}) \mapsto q_\sigma \qquad \sigma(q_\alpha, q_\sigma) \mapsto q_\sigma$$

Evaluating a tree



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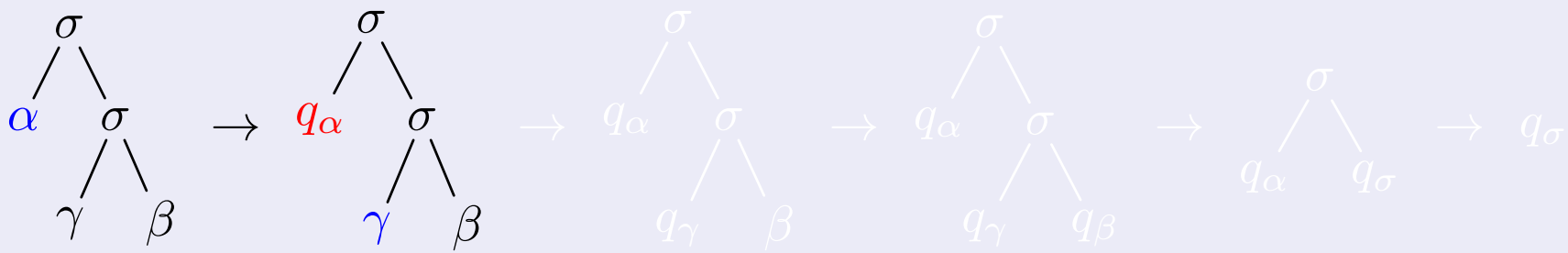

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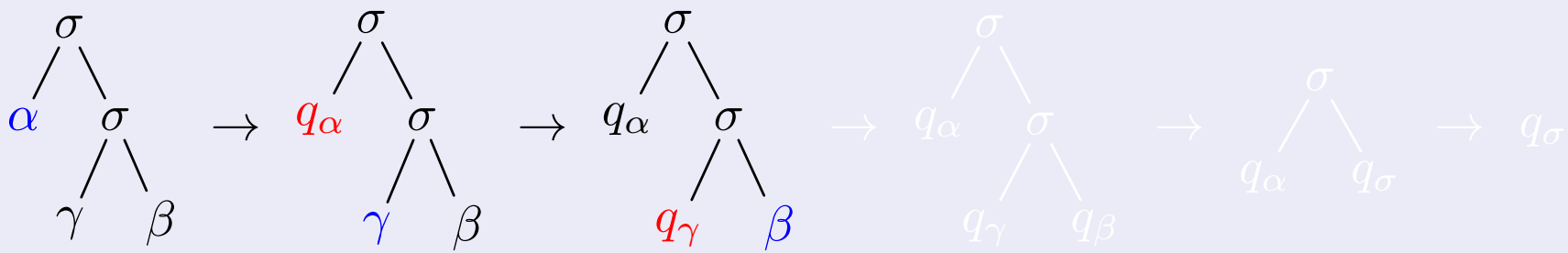

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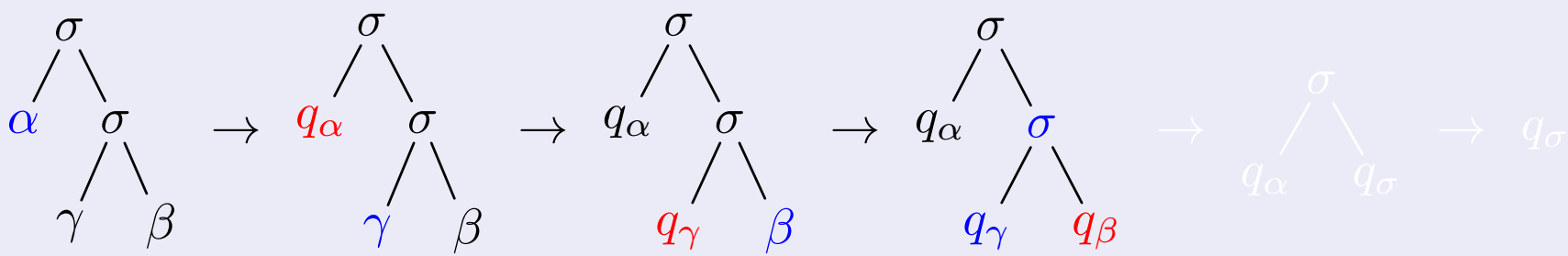

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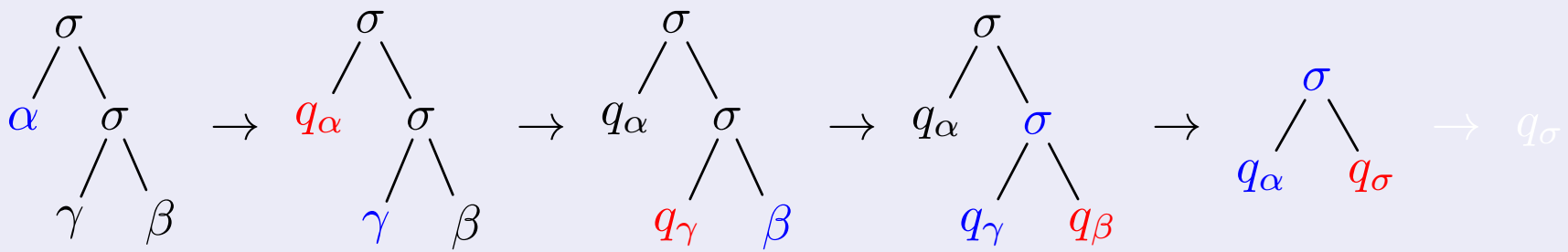

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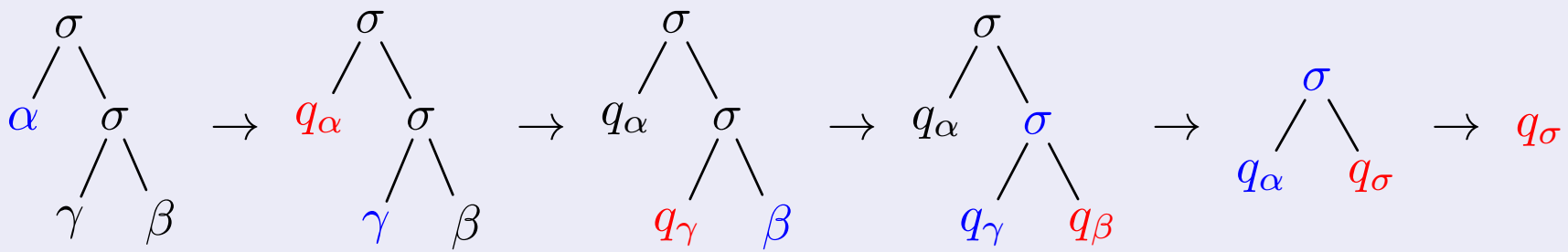
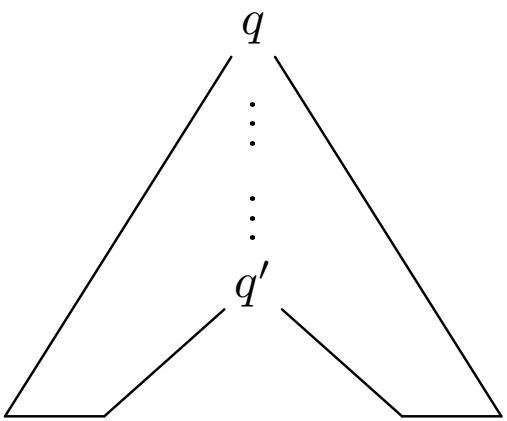


Deterministic Tree Automaton

Shorthands

- $L(M)^q_{q'} = \{c \in C_\Sigma \mid \delta(c[q']) = q\}$
- $L(M)_{q'} = \bigcup_{f \in F} L(M)^f_{q'}$
- $L(M)^q = \delta^{-1}(q) \cap T_\Sigma$



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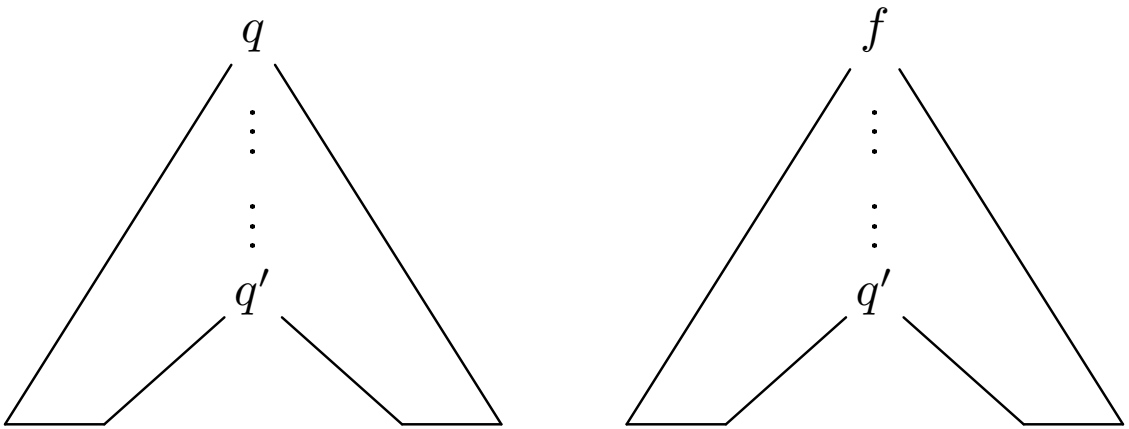

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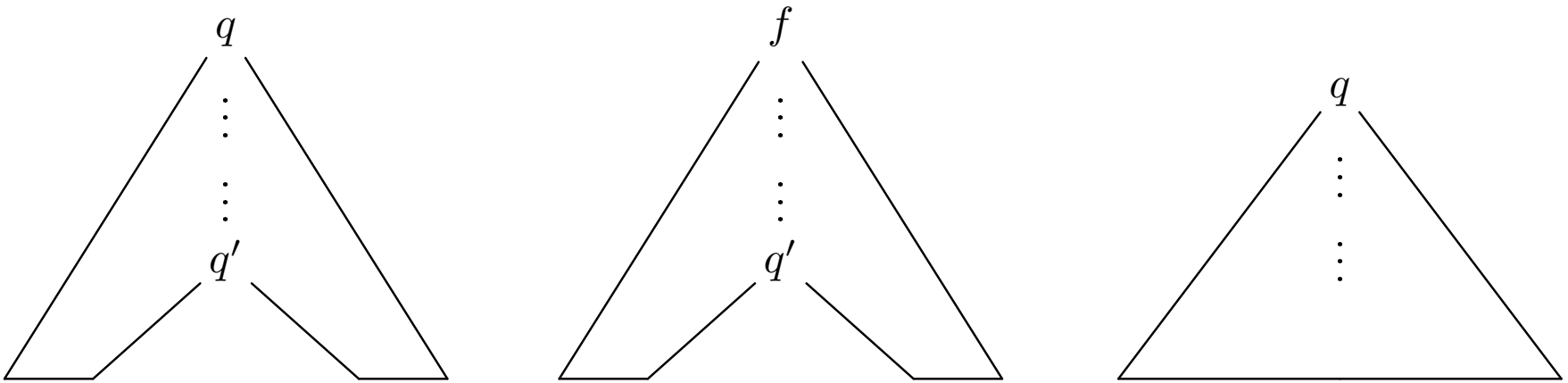

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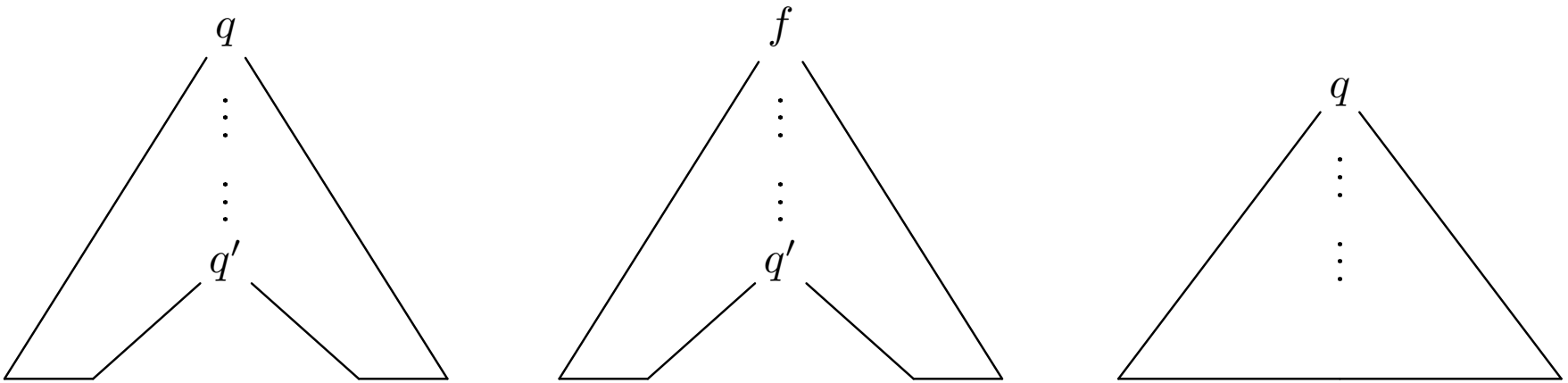

Definition

Recognized tree language $L(M) = \bigcup_{f \in F} L(M)^f$

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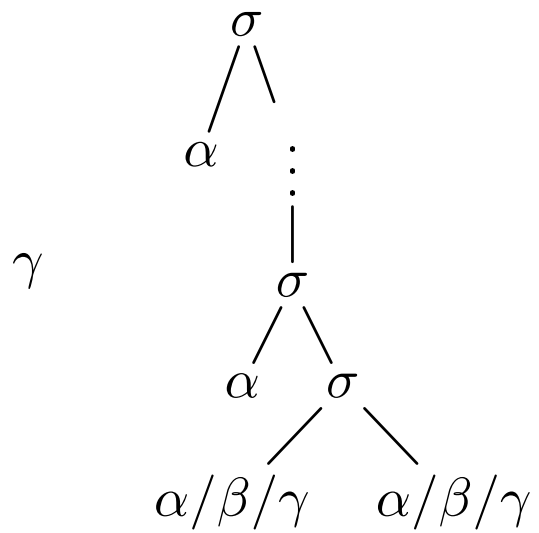
Recognized tree language

With $c = \sigma(\alpha, \square)$

$$\{\gamma\} \cup \{c^n[\sigma(\pi, \pi')] \mid n \in \mathbb{N},\ \text{nullary } \pi, \pi'\}$$



Minimization

Definition

States q and q' are equivalent if $L(M)_q = L(M)_{q'}$

Theorem

DTA minimal $\iff$ no different, but equivalent states

Theorem (HÖGBERG et al. 2008)

For every DTA we can efficiently construct an equivalent minimal DTA

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if $L(M)_q$ and $L(M)_{q'}$ have finite difference

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$\rightarrow$ minimal, but q_β and q_γ *almost equivalent*

$$L(M)_{q_\beta} \ominus L(M)_{q_\gamma} = \{\square\}$$

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Properties of Almost Equivalence

Lemma

Almost equivalent states agree on deep contexts

$\delta(c[q_1]) = \delta(c[q_2])$ *for suitably deep* c

Contrast

On strings: the converse also holds

On trees: the converse is not true

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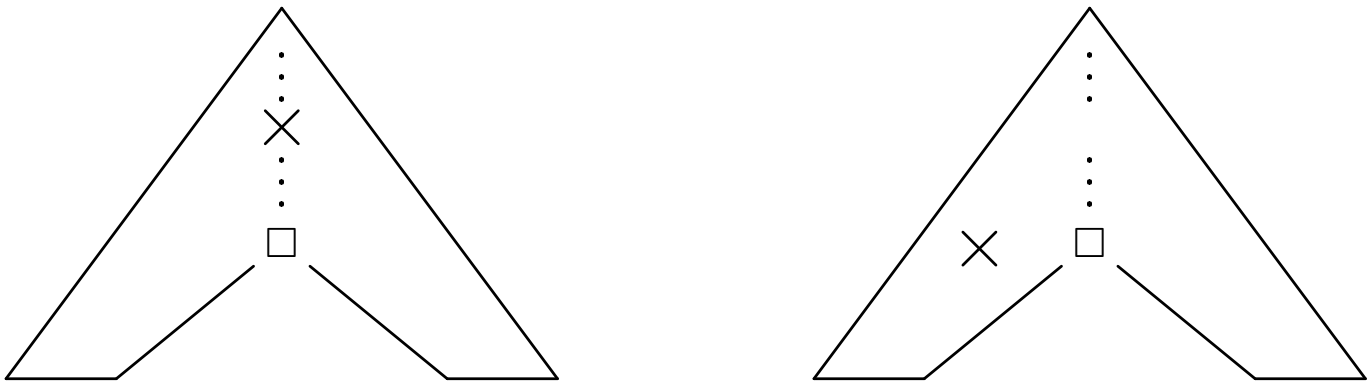

Properties of Almost Equivalence

Lemma (Lemma 2.10 of BADR et al. 2007)

- $\delta(c[q])$ *and* $\delta(c[q'])$ *are almost equivalent for almost equivalent* q *and* q' *and context* c
- *Almost equivalence is a congruence*

Definition

DTA are almost equivalent if they recognize tree languages with finite difference

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Almost equivalent DTA evaluate trees to almost equivalent states

$L(M)_{\delta(t)}$ *and* $L(N)_{\mu(t)}$ *are almost equal*

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Merge of q into q': redirect all transitions leading to q into q'

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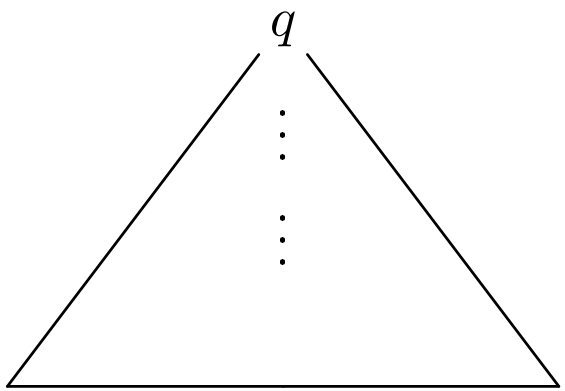

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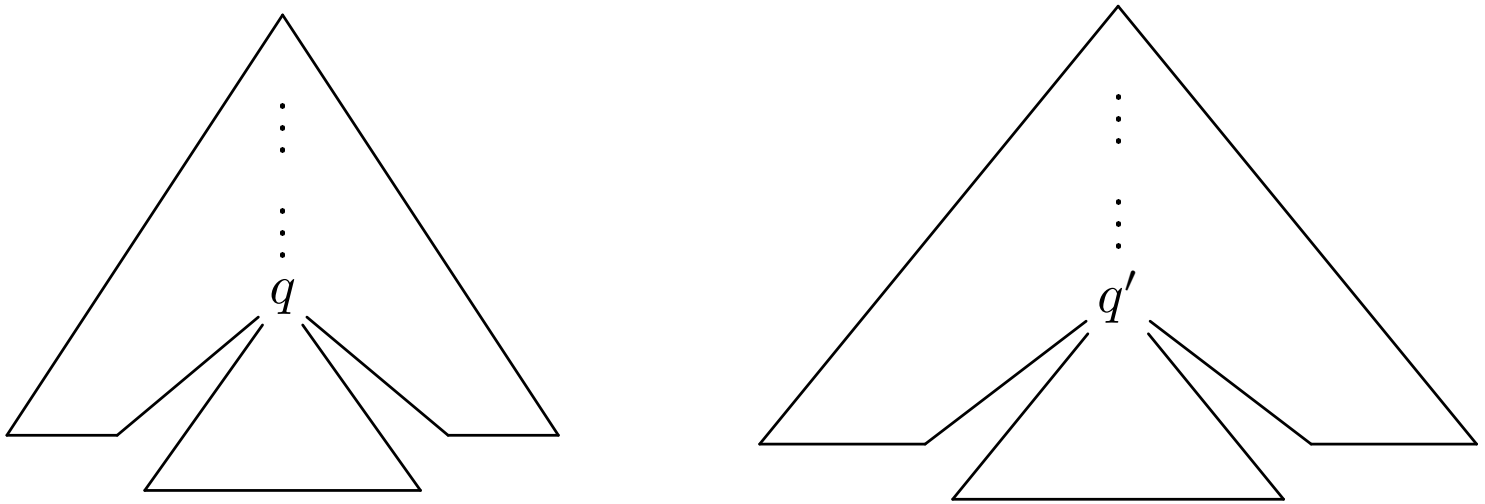
If

- *q and q' are almost equivalent*
- *q is a preamble state*

then merging q into q' yields an almost equivalent DTA



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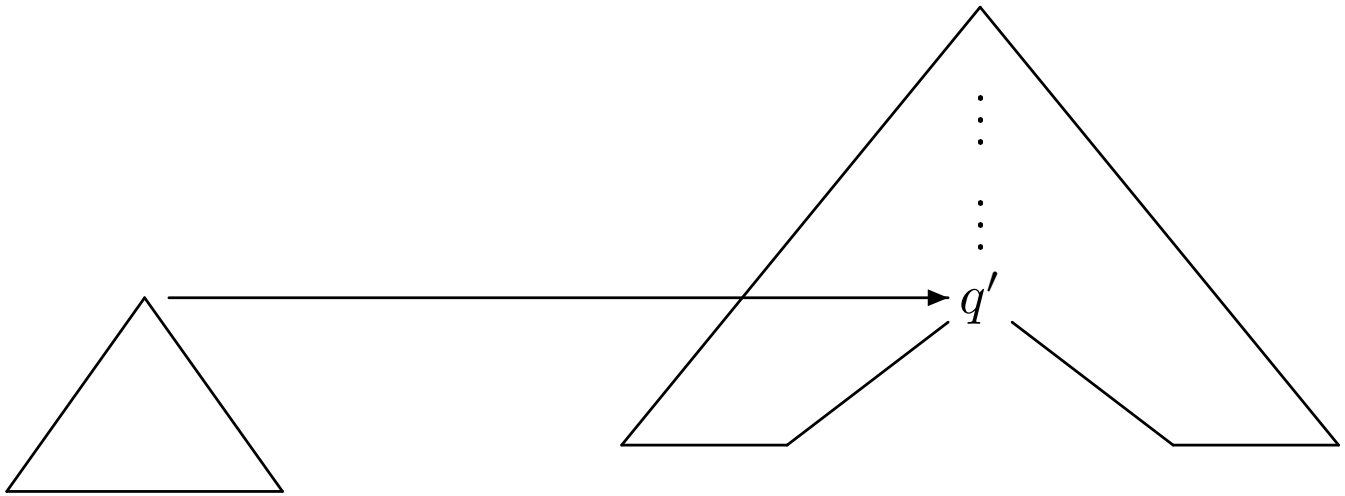

State Merging

Example (Original)

States q_α, q_β (nonfinal) and q_γ, q_σ (final)

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Example (Merged)

Merging q_β into q_γ yields

$$\alpha \mapsto q_\alpha \qquad \sigma(q_\pi, q_{\pi'}) \mapsto q_\sigma \qquad \sigma(q_\alpha, q_\sigma) \mapsto q_\sigma$$
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DTA hyper-minimal if there is no smaller DTA that is almost equivalent

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DTA hyper-minimal $\iff$ *no different, but almost equivalent states involving a preamble state*

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Overview

Require: DTA M
Return: almost equivalent hyper-minimal DTA

$M \leftarrow \textsc{Minimize}(M)$ // complexity: $\mathcal{O}(\ell m \log n)$
$P \leftarrow \textsc{ComputePreamble}(M)$ // complexity: $\mathcal{O}(\ell m)$
$\sim \leftarrow \{\langle q, q' \rangle \mid L(M_\otimes)_{\langle q,q' \rangle}$ is finite$\}$
for all $B \in (Q/\sim)$ **do**
  select $q_B \in B$ such that $q_B \notin P$ if possible
  merge each $q \in B \cap P$ into q_B // complexity: $\mathcal{O}(|B|)$
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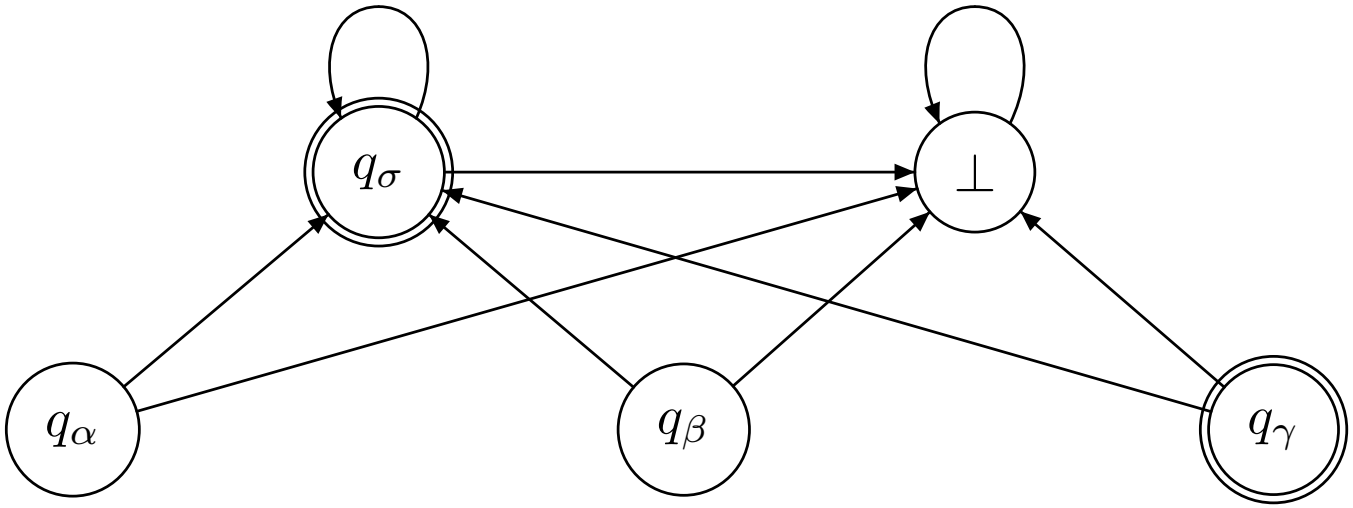

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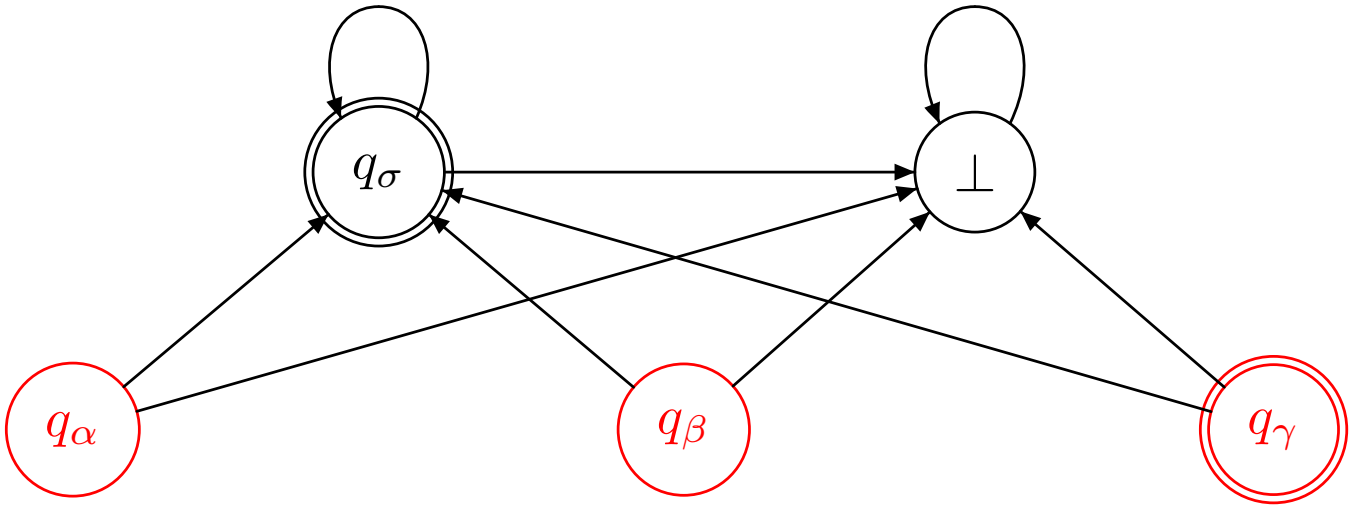

Computing Almost Equivalence

Definition

Exclusive-or single-point self-product

$$M_{\otimes} = (Q \cup Q^2, \Sigma, \delta \cup \delta', F')$$

- $F' = \{\langle q, q' \rangle \mid \text{ either } q \in F \text{ or } q' \in F\}$
- $\delta'(\sigma(c[\langle q, q' \rangle])) = \langle \delta(c[q]), \delta(c[q']) \rangle$

Lemma

We can construct $M_{\otimes}$ in time $\mathcal{O}(\ell mn)$.

Theorem

$L(M_{\otimes})_{\langle q,q' \rangle}$ is finite $\iff$ q and q' are almost equivalent

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Computing Almost Equivalence

Definition

Exclusive-or single-point self-product

$$M_{\otimes} = (Q \cup Q^2, \Sigma, \delta \cup \delta', F')$$

- $F' = \{\langle q, q' \rangle \mid \text{ either } q \in F \text{ or } q' \in F\}$
- $\delta'(\sigma(c[\langle q, q' \rangle])) = \langle \delta(c[q]), \delta(c[q']) \rangle$

Lemma

We can construct $M_{\otimes}$ in time $\mathcal{O}(\ell mn)$.

Theorem

$L(M_{\otimes})_{\langle q,q' \rangle}$ is finite $\iff$ q and q' are almost equivalent

Main Result

Theorem

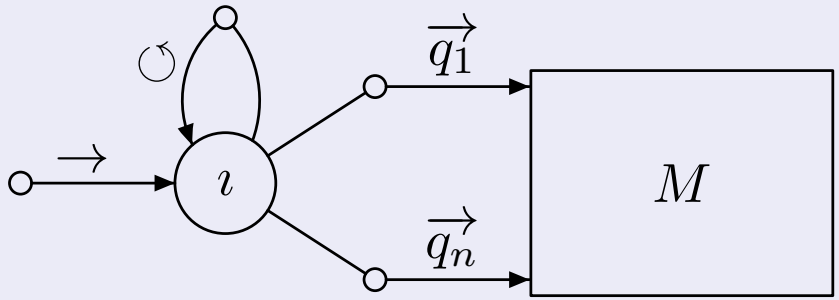
Hyper-minimization can be performed in time $\mathcal{O}(\ell mn)$

Contents

Remaining Issues

Reduction

Minimization linearly reduces to hyper-minimization



Efficiency Improvement

Using complex hashes we can improve hyper-minimization to

$$\mathcal{O}(\ell m \log n)$$

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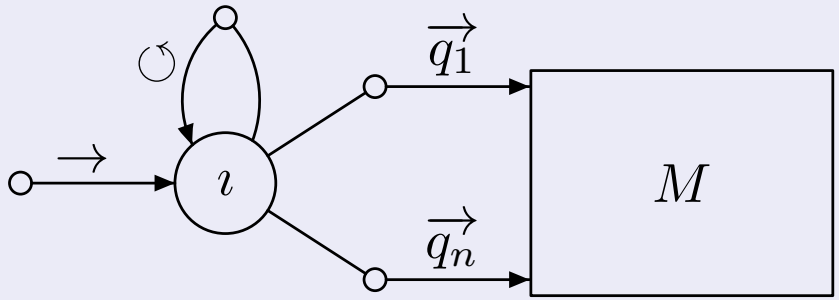

Efficiency Improvement

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Conclusion

Definition

States q and q' are **equivalent**
if $L(M)_q$ and $L(M)_{q'}$ are equal

Theorem

DTA minimal $\iff$ no different, but equivalent states

Theorem ()

We can efficiently construct an equivalent minimal DTA

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Theorem (HÖGBERG et al. 2008)

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Theorem (here)

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Solved problems

- Characterization of hyper-minimality
- Hyper-minimization algorithm $\mathcal{O}(\ell mn)$
- Minimization reduces to hyper-minimization

Open problems

- Can hashes be avoided in optimized version?
- Error optimization efficiently possible?
- Sub-quadratic error optimization?

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References

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