Compositions of Weighted Extended Top-down Tree Transducers

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Contents

Weight structure

Definition

Commutative semiring $(C, +, \cdot, 0, 1)$ if

- $(C, +, 0)$ and $(C, \cdot, 1)$ commutative monoids
- $\cdot$ distributes over finite (incl. empty) sums

Idempotent if $c + c = c$

Example

- Boolean semiring $(\{0, 1\}, \max, \min, 0, 1)$ (idempotent)
- semiring $(\mathbb{N}, +, \cdot, 0, 1)$ of non-negative integers
- tropical semiring $(\mathbb{N} \cup \{\infty\}, \min, +, \infty, 0)$ (idempotent)
- any field, ring, etc.

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Syntax

Definition (Arnold, Dauchet 1976, Graehl, Knight 2004)

Weighted extended top-down tree transducer (WXTT) is a system $(Q, \Sigma, \Delta, I, R, \chi)$

- Q: finite set of *states*
- Σ and Δ: ranked alphabets of *input* and *output symbols*
- $I \subseteq Q$: *initial states*
- R: finite set of *rule identifiers*

- $\chi \colon R \to Q(T_\Sigma(X)) \times C \times T_\Delta(Q(X))$ such that
  - $\{\ell, r\} \not\subseteq Q(X)$
  - ℓ is linear (in X)
  - $\text{var}(r) \subseteq \text{var}(\ell)$

  for every $(\ell, c, r) = \chi(\rho)$ with $\rho \in R$

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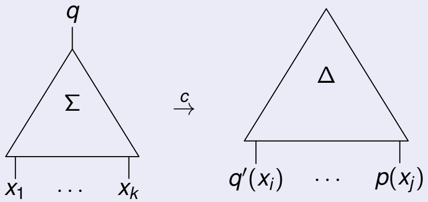
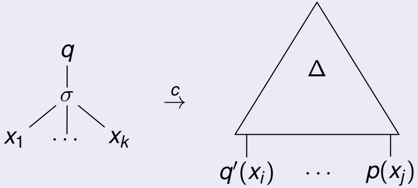


Syntactic Restrictions

Definition (ROUNDS 1970, THATCHER 1970)

- Weighted top-down tree transducer (WTT) if $\ell = q(\sigma(x_1, \ldots, x_k))$



- linear if r is linear
- nondeleting if $\text{var}(r) = \text{var}(\ell)$
- ε-free if $\ell \in Q(X)$
- producing if $r \notin Q(X)$

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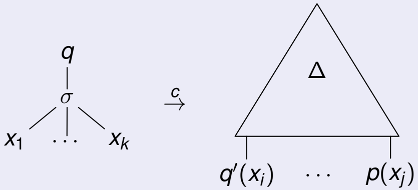

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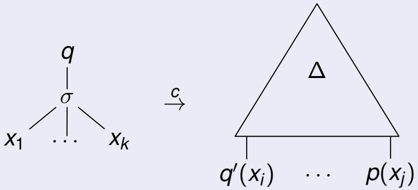

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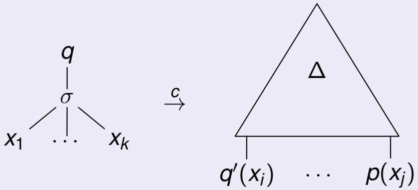

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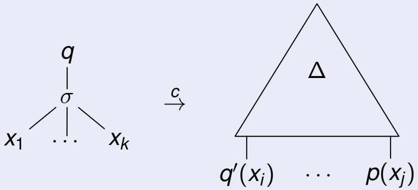

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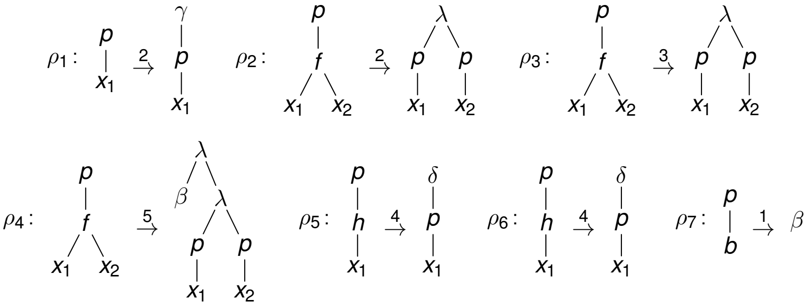

ε-rule	producing	linear	nondeleting
ρ_1	ρ_1–ρ_7	ρ_1–ρ_7	ρ_1–ρ_7

Semantics

WXTT $M = (Q, \Sigma, \Delta, I, R, \chi)$, sentential forms $\xi, \zeta \in T_{\Delta'}(Q(T_{\Sigma'}(X)))$ with $\Sigma \subseteq \Sigma'$ and $\Delta \subseteq \Delta'$

Definition

- position $w \in \text{pos}_Q(\xi)$ in ξ reducible if $\xi|_w = \ell\theta$ for some rule $\ell \stackrel{c}{\to} r \in \chi(R)$ and substitution $\theta \colon X \to T_{\Sigma'}(X)$
- Let $\rho \in R$ with $\chi(\rho) = \ell \stackrel{c}{\to} r$.

  ξ rewrites to ζ using ρ ($\xi \Rightarrow^{\rho}_{M} \zeta$) if
  - $\xi|_w = \ell\theta$
  - $\zeta = \xi[r\theta]_w$

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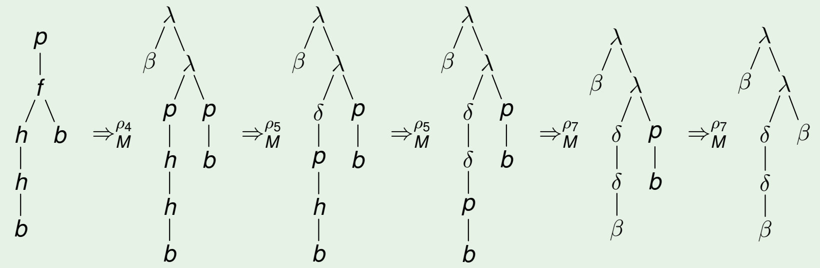

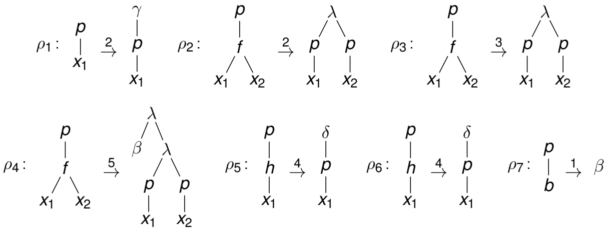

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Definition

- Rule weight: $\mathrm{wt}_M(\rho) = c$ iff $\chi(\rho) = \ell \xrightarrow{c} r$
- $\mathrm{wt}_M(\rho_1 \cdots \rho_k) = \prod_{i=1}^{k} \mathrm{wt}_M(\rho_i)$
- Derivation weight:

$$\mathrm{wt}_M(\xi, \zeta) = \sum_{\substack{\rho_1, \dots, \rho_k \in R \\ \xi \Rightarrow_M^{\rho_1} ; \cdots ; \Rightarrow_M^{\rho_k} \zeta}} \mathrm{wt}_M(\rho_1 \cdots \rho_k)$$

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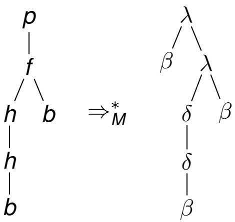

realized by the rule sequences:

$\rho_4\rho_5\rho_5\rho_7\rho_7$	$\mathrm{wt}_M(\rho_4) \cdot \mathrm{wt}_M(\rho_5) \cdot \mathrm{wt}_M(\rho_5) \cdot \mathrm{wt}_M(\rho_7)^2$
$\rho_4\rho_5\rho_6\rho_7\rho_7$	$\mathrm{wt}_M(\rho_4) \cdot \mathrm{wt}_M(\rho_5) \cdot \mathrm{wt}_M(\rho_6) \cdot \mathrm{wt}_M(\rho_7)^2$
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$$M(f(h(h(b)), b), \lambda(\beta, \lambda(\delta(\delta(\beta)), \beta))) = 80 + 80 + 80 + 80 = 360$$

Semantic Properties

Definition

WXTT $M = (Q, \Sigma, \Gamma, I, R, \chi)$

- functional (total) if for every $q \in Q$ and $t \in T_\Sigma$ there exists at most (at least) one $u \in T_\Gamma$ such that

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Remark

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Example Composition

Example

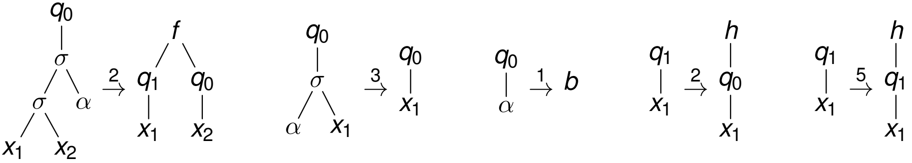
semiring $(\mathbb{R}, +, \cdot, 0, 1)$ and WXTT $M = (Q, \Sigma, \Gamma, I, R, \chi)$

- $Q = \{q_0, q_1\}$ and $I = \{q_0\}$
- $\Sigma = \{\sigma, \alpha\}$ and $\Gamma = \{f, h, b\}$
- the following rules:

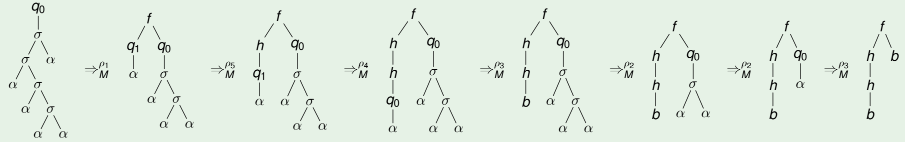
$$\begin{aligned}
\rho_1 : \quad & q_0(\sigma(\sigma(x_1, x_2), \alpha)) \xrightarrow{2} f(q_1(x_1), q_0(x_2)) \\
\rho_2 : \quad & q_0(\sigma(\alpha, x_1)) \xrightarrow{3} q_0(x_1) \\
\rho_3 : \quad & q_0(\alpha) \xrightarrow{1} b \\
\rho_4 : \quad & q_1(x_1) \xrightarrow{2} h(q_0(x_1)) \\
\rho_5 : \quad & q_1(x_1) \xrightarrow{5} h(q_1(x_1))
\end{aligned}$$

ε-rule	producing	linear	nondeleting
ρ_4, ρ_5	ρ_1, ρ_3–ρ_5	ρ_1–ρ_5	ρ_1–ρ_5

Example Composition



Example (Derivation)



$$\mathrm{wt}_M(\rho_1\rho_5\rho_4\rho_3\rho_2\rho_2\rho_3) = 2 \cdot 5 \cdot 2 \cdot 1 \cdot 3 \cdot 3 \cdot 1 = 180$$

A Second WXTT

Example

WXTT $N = (\{p\}, \Gamma, \Delta, \{p\}, R', \chi')$

- $\Gamma = \{f, h, b\}$ and $\Delta = \{\lambda, \gamma, \delta, \beta\}$
- the following rules:

$$
\begin{aligned}
\mu_1 : \quad & p(x_1) \xrightarrow{2} \gamma(p(x_1)) \\
\mu_2 : \quad & p(f(x_1, x_2)) \xrightarrow{5} \lambda(p(x_1), p(x_2)) \\
\mu_3 : \quad & p(f(x_1, x_2)) \xrightarrow{3} \lambda(\beta, \lambda(p(x_1), p(x_2))) \\
\mu_4 : \quad & p(h(x_1)) \xrightarrow{8} \delta(p(x_1)) \\
\mu_5 : \quad & p(b) \xrightarrow{1} \beta
\end{aligned}
$$

ε-rule	producing	linear	nondeleting
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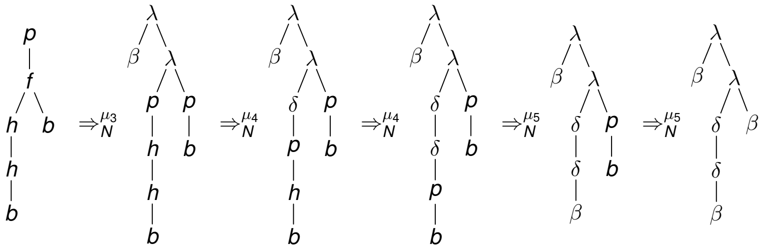
A Second WXTT



$$\mathrm{wt}_N(\mu_3\mu_4\mu_4\mu_5\mu_5) = 5 \cdot 8 \cdot 8 \cdot 1 \cdot 1 = 320$$

Formal Definition

$\tau_1 \colon T_\Sigma \times T_\Gamma \to A$ and $\tau_2 \colon T_\Gamma \times T_\Delta \to A$

$$(\tau_1 \,;\, \tau_2)(s, u) = \sum_{t \in T_\Gamma} \tau_1(s, t) \cdot \tau_2(t, u)$$

Restriction

(i) to permit infinite sums

(ii) to restrict the weighted relations

Here:

For all $s \in T_\Sigma$ and $u \in T_\Delta$

$$\{t \mid (s, t) \in \mathrm{supp}(\tau_1)\} \qquad \text{or} \qquad \{t \mid (t, u) \in \mathrm{supp}(\tau_2)\}$$

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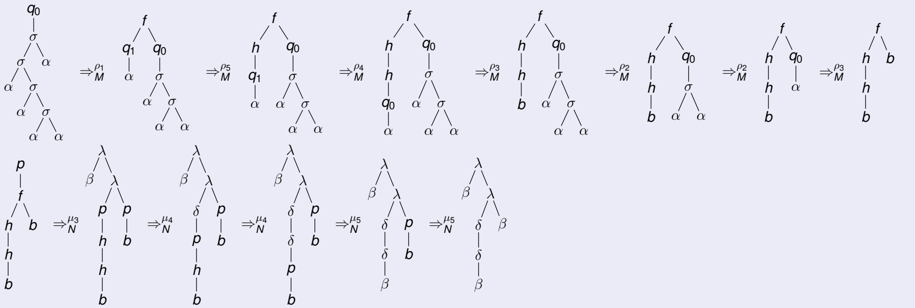
Composition Strategy

$\rho_1 \rho_5 \rho_4 \rho_3 \rho_2 \rho_2 \rho_3 \mu_3 \mu_4 \mu_4 \mu_5 \mu_5$

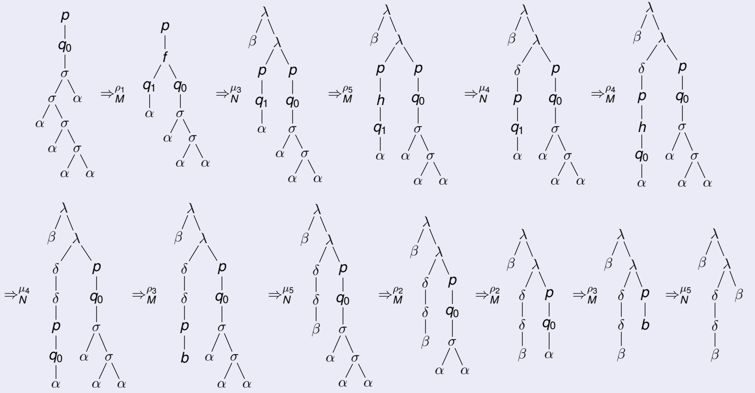
Original order



Composition Strategy

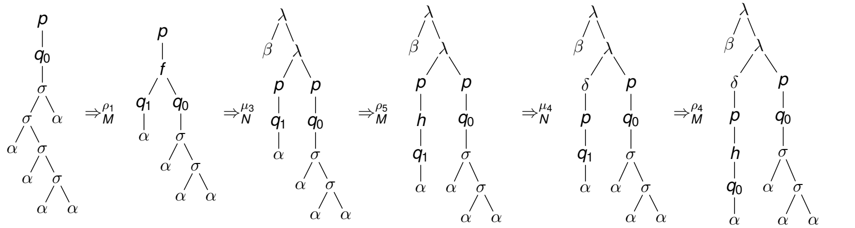
$\rho_1 \rho_5 \rho_4 \rho_3 \rho_2 \rho_2 \rho_3 \mu_3 \mu_4 \mu_4 \mu_5 \mu_5$

Reordered

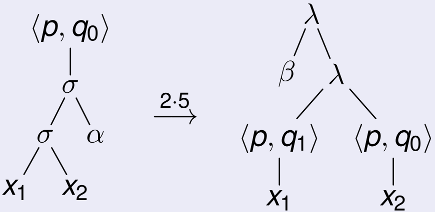


$\rho_1 \mu_3 \rho_5 \mu_4 \rho_4 \mu_4 \rho_3 \mu_5 \rho_2 \rho_2 \rho_3 \mu_5$

Composition Strategy



Composed rule



The Unweighted Case

Composition results

Case	M	N
(a)		linear and nondeleting
(b)	total	linear
(c)	functional	nondeleting
(d)	functional and total	

References:
[ENGELFRIET: *Bottom-up and top-down tree transformations — A comparison.* Math. Syst. Theory 9(3): 198-231 (1975)]
[BAKER: *Composition of top-down and bottom-up tree transductions.* Inf. Control 41(2): 186-213 (1979)]

Contents

Overview

Requirements

- WXTT $M = (Q, \Sigma, \Gamma, I_1, R_1, \chi_1)$ without ε-rules
- WTT $N = (P, \Gamma, \Delta, I_2, R_2, \chi_2)$

Constants

- Let $v \geq |\mathrm{pos}_x(r)|$ for all rules $\ell \to r$ of N and $x \in X$
- Let $s \geq |\mathrm{pos}_\Gamma(r)|$ for all rules $\ell \to r$ of M
- Let $m \geq v^s$

Construction

Definition

Composed WXTT $M \,;\, N = (P \times Q, \Sigma, \Delta, I_2 \times I_1, R, \chi)$

- $R = \{\langle \rho, p, w \rangle \mid \rho \in R_1, p \in P, w \in R_2^*, |w| \leq m\}$
- $\chi(\rho, p, \mu_1 \cdots \mu_k\rangle) = (p(\ell), a, r')$
  - for every $\rho \in R_1$ with $\chi_1(\rho) = \ell \to r$
  - $p \in P$
  - $\mu_1, \ldots, \mu_k \in R_2$ with $k \leq m$
  - $r' \in T_\Delta(P(Q(X)))$ and

$$a = \begin{cases} \text{wt}_M(\rho) \cdot \prod_{i=1}^{k} \text{wt}_N(\mu_i) & \text{if } p(\ell) \Rightarrow_M^{\rho} \,;\, \Rightarrow_N^{\mu_1 \cdots \mu_k} r' \\ 0 & \text{otherwise.} \end{cases}$$

Construction — Example

Example

- M has the rules

$$\rho_1: \quad q(\gamma(x_1)) \xrightarrow{2} \gamma(\gamma(q(x_1))) \qquad \rho_2: \quad q(\alpha) \xrightarrow{2} \alpha$$

- N has the rules

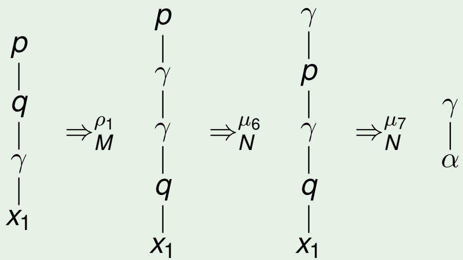
$$\begin{aligned}
\mu_1 &: \quad p_0(\gamma(x_1)) \xrightarrow{4} \sigma(p_0(x_1), p_0(x_1)) & \mu_6 &: \quad p(\gamma(x_1)) \xrightarrow{1} \gamma(p(x_1)) \\
\mu_2 &: \quad p_0(\gamma(x_1)) \xrightarrow{2} \sigma(p_0(x_1), p(x_1)) & \mu_7 &: \quad p(\gamma(x_1)) \xrightarrow{3} \alpha \\
\mu_3 &: \quad p_0(\gamma(x_1)) \xrightarrow{2} \sigma(p(x_1), p_0(x_1)) & \mu_8 &: \quad p(\alpha) \xrightarrow{1} \alpha \\
\mu_4 &: \quad p_0(\gamma(x_1)) \xrightarrow{1} \sigma(p(x_1), p(x_1)) & & \\
\mu_5 &: \quad p_0(\alpha) \xrightarrow{1} \alpha & &
\end{aligned}$$

ε-rule	producing	linear	nondeleting
	ρ_1, ρ_2	ρ_1, ρ_2	ρ_1, ρ_2
	μ_1–μ_8	μ_5–μ_8	μ_1–μ_6, μ_8

Construction — Example

Example

rule for identifier $\imath = \langle \rho_1, p, \mu_6\mu_7 \rangle$

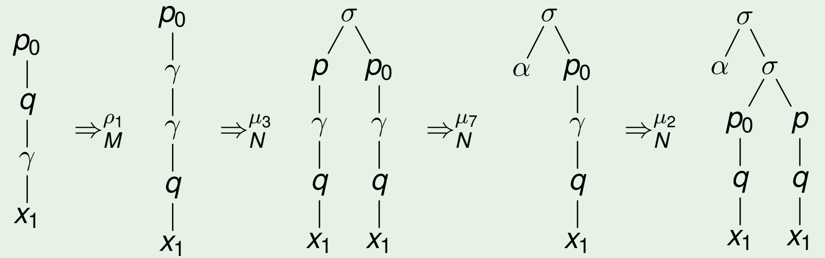


$$\chi(\imath) = \Big(\langle p, q\rangle(\gamma(x_1)), 2 \cdot 1 \cdot 3, \gamma(\alpha)\Big)$$

Construction — Example

Example

rule for identifier $\imath = \langle \rho_1, p_0, \mu_3\mu_7\mu_2 \rangle$



$$\chi(\imath) = \Big(\langle p_0, q\rangle(\gamma(x_1)), 2 \cdot 2 \cdot 3 \cdot 2, \sigma(\alpha, \sigma(\langle p_0, q\rangle(x_1), \langle p, q\rangle(x_1)))\Big)$$

First Result

Unweighted composition results

Case	M	N
(a)		linear and nondeleting
(b)	total	linear
(c)	functional	nondeleting
(d)	functional and total	

Theorem

If the WTT N is linear and nondeleting, then $\tau_{M;N} = \tau_M \,;\, \tau_N$.

[∼: *Compositions of tree series transformations.*
Theor. Comput. Sci. 366(3): 248–271 (2006)]

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Towards a Second Result

Unweighted composition results

Case	M	N
(a)		linear and nondeleting
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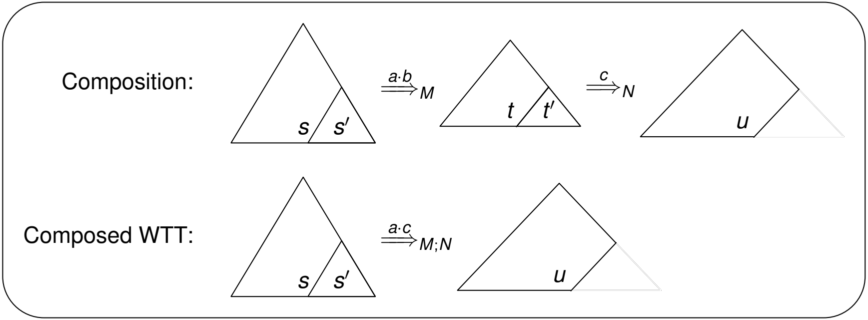
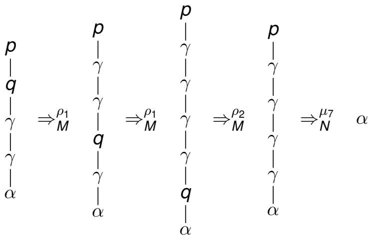
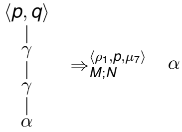


Towards a Second Result

Unweighted composition results

Case	M	N
(a)		linear and nondeleting
(b)	total	linear
(c)	functional	nondeleting
(d)	functional and total	

Composition: 

Composed WTT: 

Towards a Second Result

Definition

$q \in Q$ constant if there is $c \in C$

$$\sum_{t \in T_\Gamma} \sum_{\substack{\rho_1, \ldots, \rho_k \in R_1 \\ q(s) \Rightarrow_M^{\rho_1} ; \cdots ; \Rightarrow_M^{\rho_k} t}} \mathrm{wt}_M(\rho_1 \cdots \rho_k) = c$$

for every $s \in T_\Sigma$

Example

1-constant states:

- in every total WTT over the BOOLEAN semiring
- in every BOOLEAN and total WTT over an idempotent semiring
- in every functional, total, and BOOLEAN WTT

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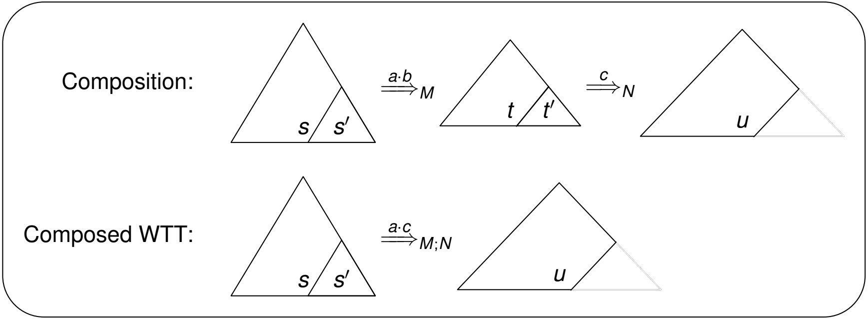
A Second Result

Conjecture

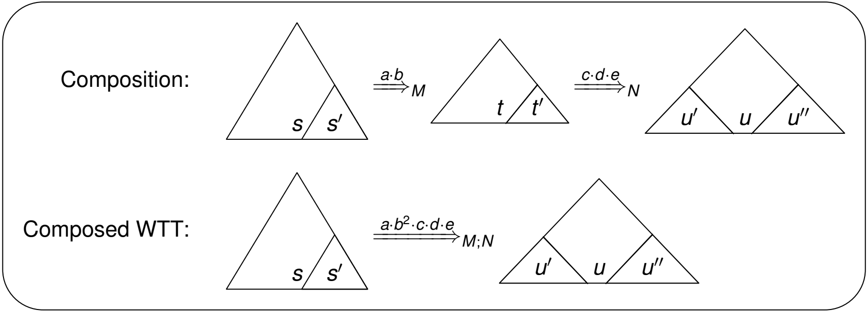
If the WXTT M is constant and the WTT N is linear, then $\tau_M ; \tau_N$ can be computed by a WXTT.



The Third Case

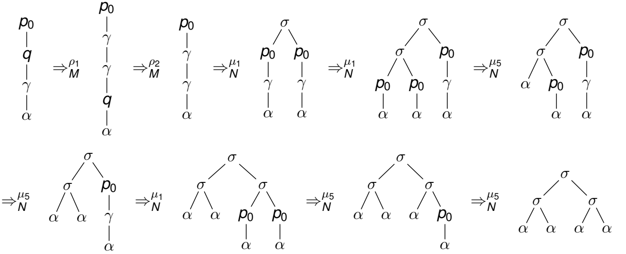
Unweighted composition results

Case	M	N
(a)		linear and nondeleting
(b)	total	linear
(c)	functional	nondeleting
(d)	functional and total	

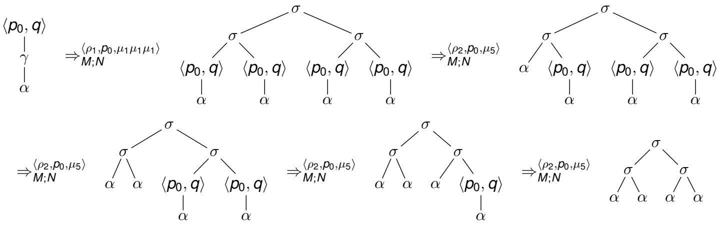


The Third Case

Composition:



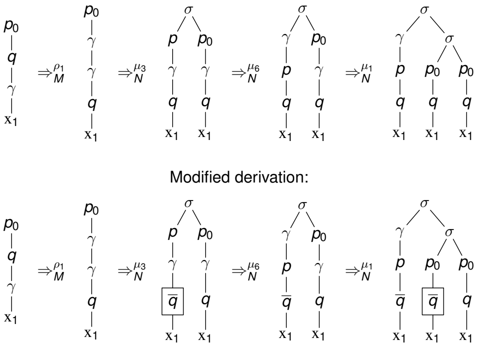
Composed WTT:



The Third Case

Original derivation:

Modified derivation:



Conjecture

If the WXTT M is functional and the WTT N is nondeleting, then $\tau_M ; \tau_N$ can be computed by a WXTT.

The Last Case

Unweighted composition results

Case	M	N
(a)		linear and nondeleting
(b)	total	linear
(c)	functional	nondeleting
(d)	functional and total	

Conjecture

If the WXTT M is functional and constant and N is a WTT, then $\tau_M ; \tau_N$ can be computed by an WXTT.

The Last Case

Unweighted composition results

Case	M	N
(a)		linear and nondeleting
(b)	total	linear
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(d)	functional and total	

Conjecture

If the WXTT M is functional and constant and N is a WTT, then $\tau_M ; \tau_N$ can be computed by an WXTT.

essentially a combination of Case (b) and (c)

Summary

Case	M	N	Reference
(a)	linear, nondel.	linear, nondel.	[1] Thm. 2.4
	functional	func., lin., nondel.	[2] Thm. 5.18
		linear, nondel.	[3] Thm. 26
(b)	constant	linear	Conj.
(c)	functional	nondeleting	Conj.
(d)	BOOL., func., total	functional	[2] Thm. 5.18
	BOOL., func., total	linear	[3] Thm. 30
	constant, func.		Conj.

[1] KUICH: *Full Abstract Families of Tree Series I.* In "Jewels are Forever", pp. 145–156 (1999)

[2] ENGELFRIET, FÜLÖP, VOGLER: *Bottom-up and top-down tree series transformations.* J. Autom. Lang. Combin. 7(1): 11–70 (2002)

[3] ∼: *Compositions of tree series transformations.* Theor. Comput. Sci. 366(3): 248–271 (2006)

Contents

Requirements

Definition

- WXTT M shallow if $|\mathrm{pos}_\Gamma(r)| \leq 1$ for every rule $\ell \to r$ of M
- WXTT N WTT with ε-rules if $|\mathrm{pos}_\Gamma(\ell)| \leq 1$ for every rule $\ell \to r$ of N

[∼, VOGLER: *Compositions of top-down tree transducers with ε-rules.* In Proc. FSMNLP 2009, pp. 69–80 (2010)]

Why Another Construction

Example

This derivation cannot be simulated by *M* ; *N* because the rules constructed for *M* ; *N* always trigger a rule of *M* first

$$\begin{array}{c} p \\ | \\ q_0 \\ | \\ \alpha \end{array} \Rightarrow_N^{\mu_1} \begin{array}{c} \gamma \\ | \\ p \\ | \\ q_0 \\ | \\ \alpha \end{array} \Rightarrow_N^{\mu_1} \begin{array}{c} \gamma \\ | \\ \gamma \\ | \\ p \\ | \\ q_0 \\ | \\ \alpha \end{array} \Rightarrow_M^{\rho_3} \begin{array}{c} \gamma \\ | \\ \gamma \\ | \\ p \\ | \\ b \end{array} \Rightarrow_N^{\mu_5} \begin{array}{c} \gamma \\ | \\ \gamma \\ | \\ \beta \end{array}$$

$$\begin{array}{c} p \\ | \\ q_0 \\ | \\ \alpha \end{array} \Rightarrow_M^{\rho_3} \begin{array}{c} p \\ | \\ b \end{array} \Rightarrow_N^{\mu_1} \begin{array}{c} \gamma \\ | \\ p \\ | \\ b \end{array} \Rightarrow_N^{\mu_1} \begin{array}{c} \gamma \\ | \\ \gamma \\ | \\ p \\ | \\ b \end{array} \Rightarrow_N^{\mu_5} \begin{array}{c} \gamma \\ | \\ \gamma \\ | \\ \beta \end{array}$$

New Construction

Definition

$M \,;_\varepsilon N = (P \times Q, \Sigma, \Delta, I_2 \times I_1, R, \chi)$

- rule identifiers:

$$
\begin{aligned}
R = \{\langle \rho, p, \varepsilon\rangle \mid \text{erasing } \rho \in R_1, p \in P\} &\cup \\
\cup \{\langle \rho, p, \mu\rangle \mid \text{producing } \rho \in R_1, p \in P,\ \text{consuming } \mu \in R_2\} &\cup \\
\cup \{\langle \varepsilon, q, \mu\rangle \mid q \in Q,\ \varepsilon\text{-rule } \mu \in R_2\}
\end{aligned}
$$

- $\chi(\rho, p, \varepsilon\rangle) = (p(\ell), \mathrm{wt}_M(\rho), p(r))$ for all erasing $\rho = \ell \to r$, $p \in P$
- $\chi(\rho, p, \mu\rangle) = (p(\ell), a, r')$

$$
a = \begin{cases} \mathrm{wt}_M(\rho) \cdot \mathrm{wt}_N(\mu) & \text{if } p(\ell) \Rightarrow^{\rho}_{M} ; \Rightarrow^{\mu}_{N} r' \\ 0 & \text{otherwise} \end{cases}
$$

  for producing $\rho = \ell \to r$ of M, $p \in P$, and consuming $\mu \in R_2$
- $\chi(\langle \varepsilon, q, \mu\rangle) = \dots$

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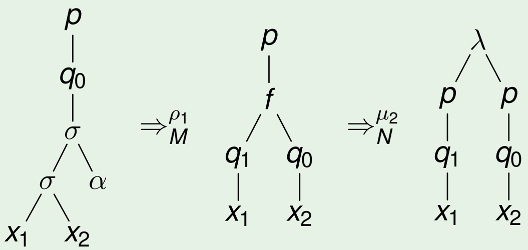
  for producing $\rho = \ell \to r$ of M, $p \in P$, and consuming $\mu \in R_2$
- $\chi(\langle \varepsilon, q, \mu\rangle) = \dots$

Construction — Example

Example



Unweighted Setting

Theorem

M shallow, N is a WTT with ε-rules. If

- *N is linear, and*
- *M is total or N is nondeleting,*

then $\tau_M \,;\, \tau_N$ can be computed by a WXTT.

Case	M	N	Reference
(a)		linear, nondeleting	[1] Thm. 17
(b)	total	linear	[1] Thm. 17

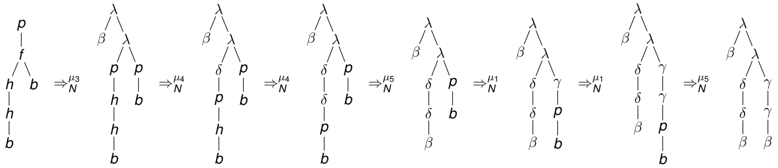
[1] ∼, VOGLER: *Compositions of top-down tree transducers with ε-rules.* In Proc. FSMNLP 2009, pp. 69–80 (2010)

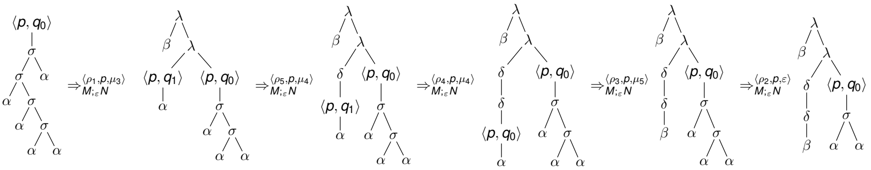
The First Case

Theorem

If M is shallow and N is a linear and nondeleting WTT with ε-rules, then $\tau_{M ;_\varepsilon N} = \tau_M ; \tau_N$.

The First Case





Step	1	2	3	4	5	6	7
d_M:	ρ_1	ρ_5	ρ_4	ρ_3	ρ_2	ρ_2	ρ_3
d_N:	μ_3	μ_4	μ_4	μ_5			$\mu_1\ \mu_1\ \mu_5$
d:	$\langle \rho_1, p, \mu_3 \rangle$	$\langle \rho_5, p, \mu_4 \rangle$	$\langle \rho_4, p, \mu_4 \rangle$	$\langle \rho_3, p, \mu_5 \rangle$	$\langle \rho_2, p, \varepsilon \rangle$	$\langle \rho_2, p, \varepsilon \rangle$	$\langle \varepsilon, q_0, \mu_1 \rangle\ \langle \varepsilon, q_0, \mu_1 \rangle\ \langle \rho_3, p, \mu_5 \rangle$

The Second Case

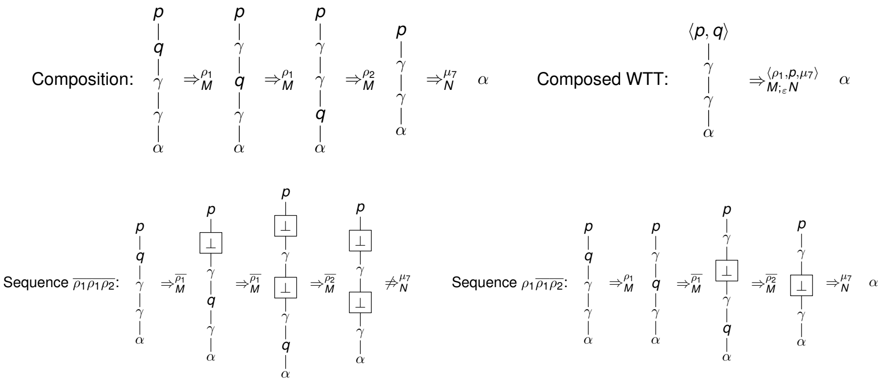
Theorem

If the shallow WXTT M is total and BOOLEAN*, the WTT N with ε-rules is linear, and the semiring C is idempotent, then $\tau_{M ;_\varepsilon N} = \tau_M ; \tau_N$.*

The Second Case

Theorem

If the shallow WXTT M is total and BOOLEAN, *the WTT N with ε-rules is linear, and the semiring C is idempotent, then* $\tau_{M;_\varepsilon N} = \tau_M \,;\, \tau_N$.



Conjecture

If the shallow WXTT M is constant and the WTT N with ε-rules is linear, then $\tau_M \,;\, \tau_N$ can be computed by a WXTT.

Case	M	N
(a)		linear, nondeleting
(b)	total, BOOLEAN	linear
	constant	linear

It's over!

Thank you for your attention!