Hyper-minimisation of weighted finite automata

Daniel Quernheim and Andreas Maletti

Institute for Natural Language Processing, University of Stuttgart
<first.last>@ims.uni-stuttgart.de

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Outline

Unweighted case

Weighted case

Conclusion

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Unweighted case

Weighted case

Conclusion

Minimisation

Problem

given DFA, return

- equivalent DFA such that
- all equivalent DFA are larger

Theorem (Hopcroft 1971)

DFA minimisation can be done in time $O(n \log n)$

- n*: number of states*

Minimisation

Problem

given DFA, return

- equivalent DFA
- minimal

Theorem (Hopcroft 1971)

DFA minimisation can be done in time $O(n \log n)$

- n*: number of states*

Hyper-minimisation

Definition

DFA A, B almost equivalent if $L(A)$ and $L(B)$ have finite difference

Problem [Badr et al. 2009]

given DFA, return

- almost equivalent DFA such that
- all almost equivalent DFA are larger

Theorem (Holzer, $\sim$ 2009, Gawrychowsky, Jeż 2009)

DFA hyper-minimisation can be done in time $O(n \log n)$

Hyper-minimisation

Definition

DFA A, B almost equivalent if $L(A)$ and $L(B)$ have finite difference

Problem [Badr et al. 2009]

given DFA, return

- almost equivalent DFA
- hyper-minimal

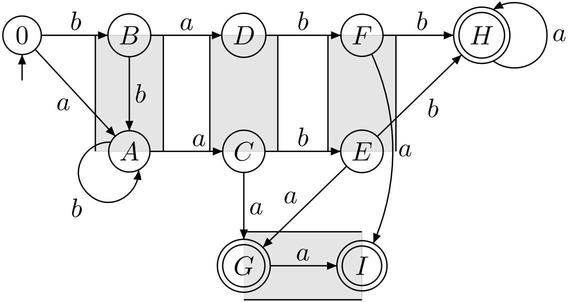
Theorem (Holzer, $\sim$ 2009, Gawrychowsky, Jeż 2009)

DFA hyper-minimisation can be done in time $O(n \log n)$

Almost equivalence

- States are almost equivalent if their right languages differ finitely

$$\overrightarrow{q} = \{w \in \Sigma^* \mid \delta(q, w) \in F\}$$

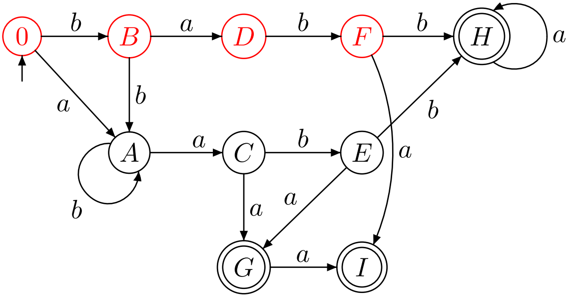


Kernel and preamble states

Definition

- preamble state: finite left language
- kernel state: infinite left language

$$\overleftarrow{q} = \{w \in \Sigma^* \mid \delta(q_0, w) = q\}$$



Structural characterisation

Theorem (Badr et al. 2009)

DFA is hyper-minimal iff

- *minimal*
- *no preamble state is almost equivalent to another state*

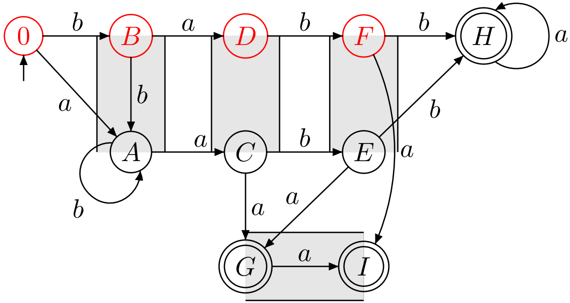
Structural characterisation

Theorem (Badr et al. 2009)

DFA is hyper-minimal iff

- *minimal*
- *no preamble state is almost equivalent to another state*

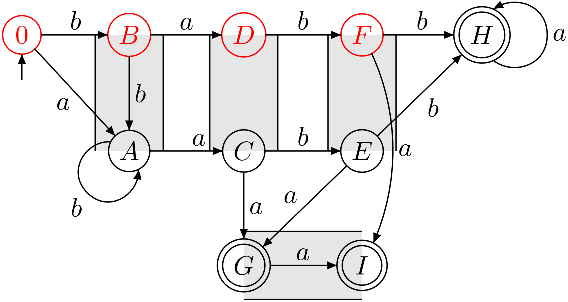
Example (Not hyper-minimal)



Hyper-minimisation

preamble	almost equivalent
B	A
D	C
F	E

Table: Merges



Hyper-minimisation

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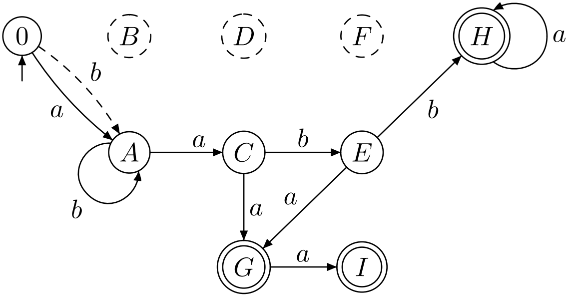
Table: Merges



Motivation

Application

In natural language processing

- Automata often deterministic for efficient evaluation
- DFA tend to be very big $\rightarrow$ good reduction potential
- Often approximative data $\rightarrow$ lossy compression ok

Motivation

Application

In natural language processing

- Automata often deterministic for efficient evaluation
- DFA tend to be very big $\rightarrow$ good reduction potential
- Often approximative data $\rightarrow$ lossy compression ok
- but DFA are weighted

Conclusion

Let's do hyper-minimisation for weighted DFA!

Outline

Unweighted case

Weighted case

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Weight structure

General approach

works for semifields (semirings with mult. inverses)

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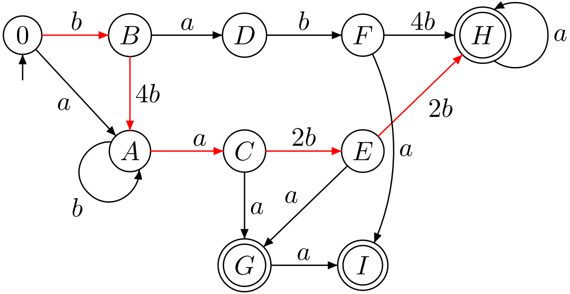
General approach

works for semifields (semirings with mult. inverses)

Presentation

here we use the field of real numbers

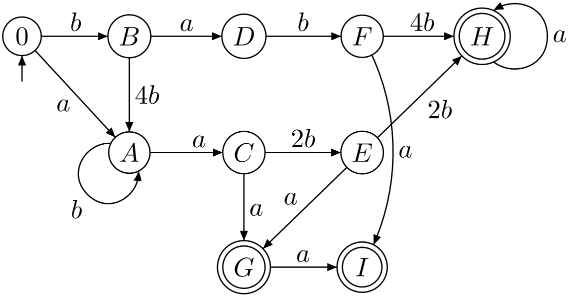
Example (WDFA)



weight of $bbabb$
is $1 \cdot 4 \cdot 1 \cdot 2 \cdot 2 = 16$

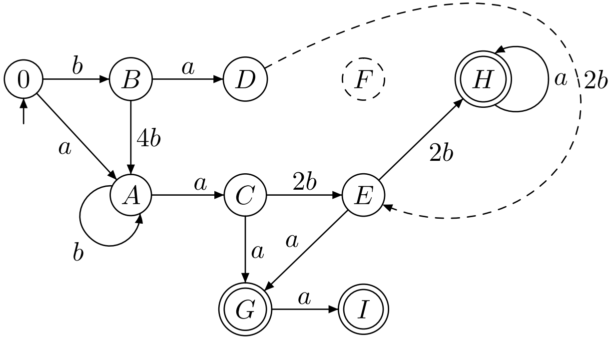
weight of ab is 0

Weighted merge



Weighted merge of F into E with factor 2

Weighted merge



Weighted merge of F into E with factor 2

Almost equivalence

Definition

Two WDFA A, B are almost equivalent if

$$A(w) \neq B(w) \quad \text{for only finitely many } w \in \Sigma^*$$

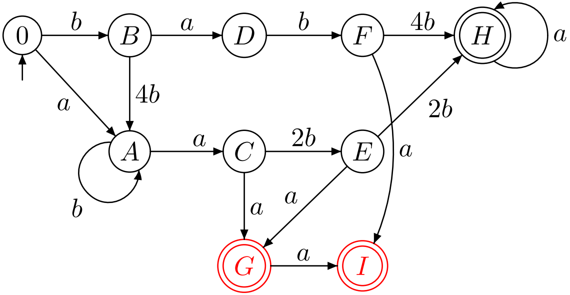
Definition

Two states p, q are almost equivalent if there is $k \in \mathbb{R} \setminus \{0\}$ such that

$$\overrightarrow{p}(w) \neq k \cdot \overrightarrow{q}(w) \quad \text{for only finitely many } w \in \Sigma^*$$

[Borchardt: The Myhill-Nerode theorem for recognizable tree series. DLT 2003]

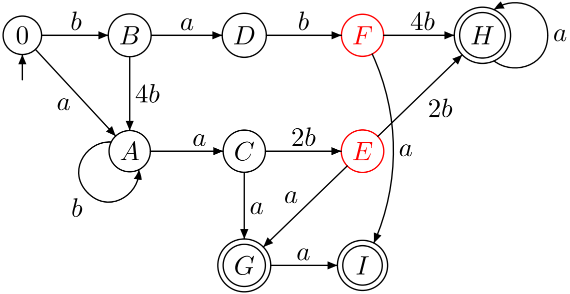
Example



	G	I
ϵ	1	1
a	1	0

- G and I almost equivalent

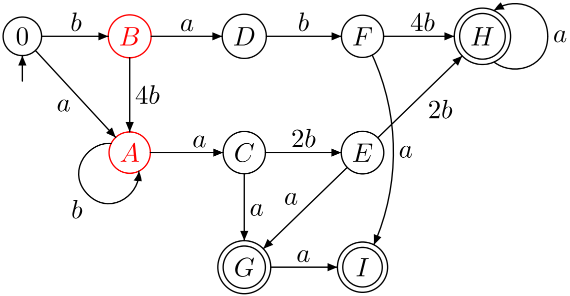
Example



	E	F
a	1	1
aa	1	0
b	2	4
ba	2	4
baa	2	4
$baaa$	2	4
$\ldots$		

- G and I almost equivalent
- E and F almost equivalent (with factor 2)

Example



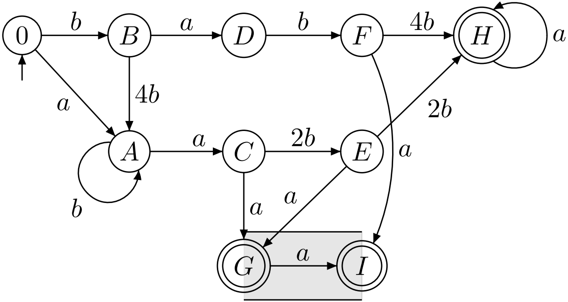
	A	B
abb	4	4
$abba$	4	4
$abbaa$	4	4
...		
$babb$	4	16
$babba$	4	16
$babbaa$	4	16
...		

- G and I almost equivalent
- E and F almost equivalent (with factor 2)
- A and B not almost equivalent!

Finding almost equivalent states

Definition

- co-kernel state: infinite right language
- co-preamble state: finite right language



Signature standardisation

Definition

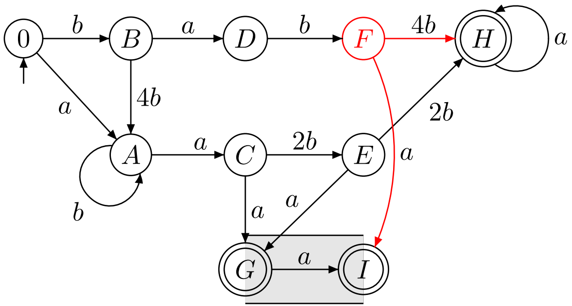
signature of q: $\big(\langle \delta(q,\sigma), c(q,\sigma)\rangle\big)_{\sigma\in\Sigma}$

Standardisation

- select minimal $\sigma_0 \in \Sigma$ such that $\delta(q,\sigma_0)$ is co-kernel
- adjust transition weights:

$$c'(q,\sigma) = \begin{cases} \frac{c(q,\sigma)}{c(q,\sigma_0)} & \text{if } \delta(q,\sigma) \text{ is co-kernel} \\ 1 & \text{otherwise} \end{cases}$$

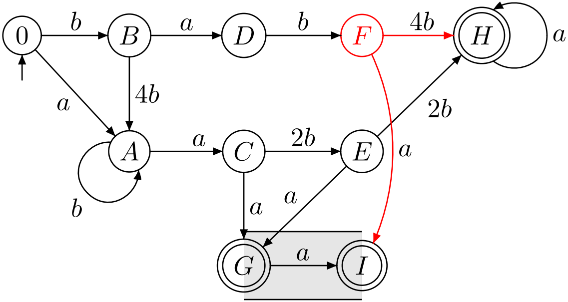
Signature standardisation



Standardised signature of F

- $F \xrightarrow{a} I$ leads to co-preamble

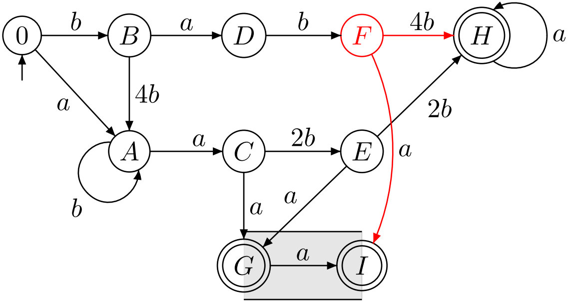
Signature standardisation



Standardised signature of F

- $F \stackrel{a}{\to} I$ leads to co-preamble $\quad \to \quad c'(F, a) = 1$

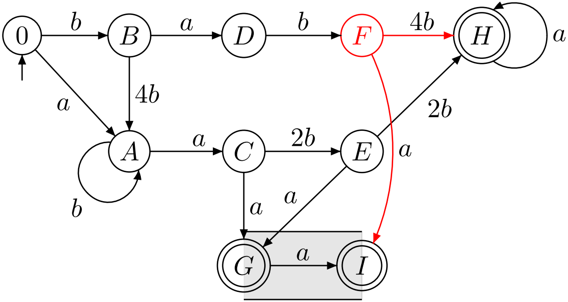
Signature standardisation



Standardised signature of F

- $F \xrightarrow{a} I$ leads to co-preamble $\quad \rightarrow \quad c'(F, a) = 1$
- $F \xrightarrow{4b} H$ leads to co-kernel

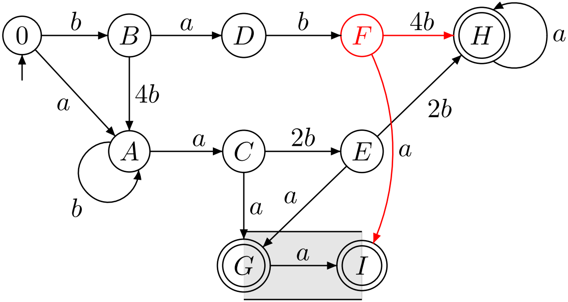
Signature standardisation



Standardised signature of F

- $F \stackrel{a}{\to} I$ leads to co-preamble $\quad \to \quad c'(F, a) = 1$
- $F \stackrel{4b}{\to} H$ leads to co-kernel $\quad \to \quad c'(F, b) = \frac{4}{4} = 1$

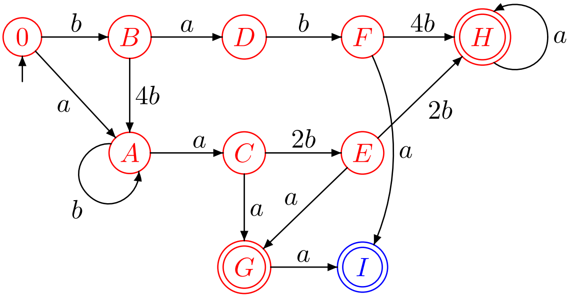
Signature standardisation



Standardised signature of F

- $F \xrightarrow{a} I$ leads to co-preamble $\rightarrow$ $c'(F, a) = 1$
- $F \xrightarrow{4b} H$ leads to co-kernel $\rightarrow$ $c'(F, b) = \frac{4}{4} = 1$
- Standardised signature $(\langle I, 1\rangle, \langle H, 1\rangle)$

Finding almost equivalent states



Signature map:

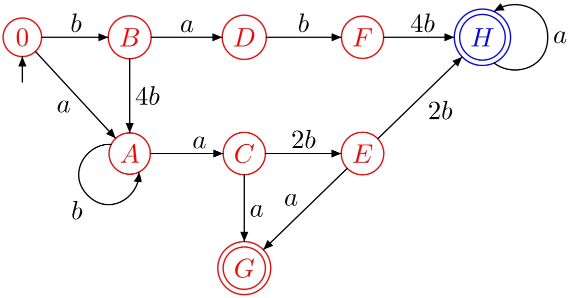
$\langle \bot, 1 \rangle, \langle \bot, 1 \rangle$	$\bot$

Blocks:

- $\{\bot, I\}$

- $\mathrm{sig}(I) = (\langle \bot, 1 \rangle, \langle \bot, 1 \rangle)$ in map!
- add I to block of $\bot$ (and merge)

Finding almost equivalent states



Signature map:

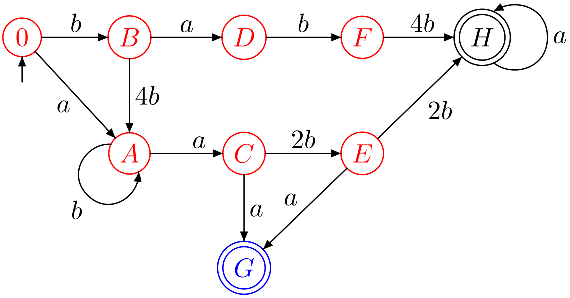
$\langle \bot, 1\rangle, \langle \bot, 1\rangle$	$\bot$
$\langle H, 1\rangle, \langle \bot, 1\rangle$	H

Blocks:

- $\{\bot, I\}$

- $\text{sig}(H) = (\langle H, 1\rangle, \langle \bot, 1\rangle)$
- add to map

Finding almost equivalent states



Signature map:

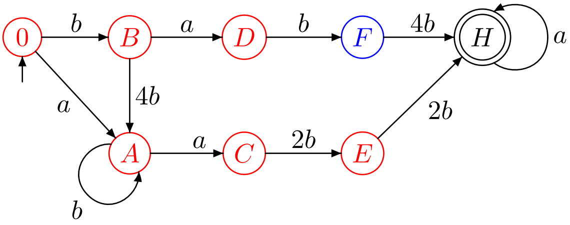
$\langle \bot, 1\rangle, \langle \bot, 1\rangle$	$\bot$
$\langle H, 1\rangle, \langle \bot, 1\rangle$	H

Blocks:

- $\{\bot, I, G\}$

- $\text{sig}(G) = (\langle \bot, 1\rangle, \langle \bot, 1\rangle)$ in map!
- add G to $\bot$ (and merge)

Finding almost equivalent states



Signature map:

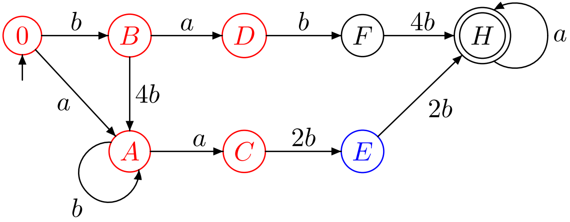
$\langle\bot, 1\rangle, \langle\bot, 1\rangle$	$\bot$
$\langle H, 1\rangle, \langle\bot, 1\rangle$	H
$\langle\bot, 1\rangle, \langle H, 1\rangle$	F

Blocks:

- $\{\bot, I, G\}$

- $\mathrm{sig}(F) = (\langle\bot, 1\rangle, \langle H, 1\rangle)$
- add to map

Finding almost equivalent states



Signature map:

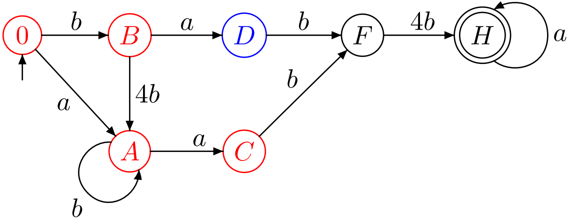
$\langle \bot, 1\rangle, \langle \bot, 1\rangle$	$\bot$
$\langle H, 1\rangle, \langle \bot, 1\rangle$	H
$\langle \bot, 1\rangle, \langle H, 1\rangle$	F

Blocks:

- $\{\bot, I, G\}$
- $\{F, E\}$

- $\mathrm{sig}(E) = (\langle \bot, 1\rangle, \langle H, 1\rangle)$ in map!
- add E to F (and merge with factor $\frac{1}{2}$)

Finding almost equivalent states



Signature map:

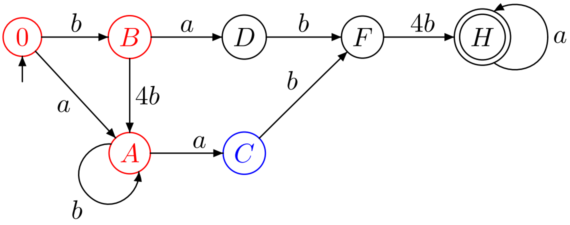
$\langle\bot,1\rangle,\langle\bot,1\rangle$	$\bot$
$\langle H,1\rangle,\langle\bot,1\rangle$	H
$\langle\bot,1\rangle,\langle H,1\rangle$	F
$\langle\bot,1\rangle,\langle F,1\rangle$	D

Blocks:

- $\{\bot, I, G\}$
- $\{F, E\}$

- $\text{sig}(D) = (\langle\bot,1\rangle,\langle F,1\rangle)$
- add to map

Finding almost equivalent states



Signature map:

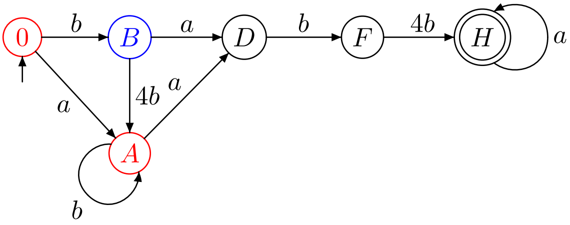
$\langle\bot, 1\rangle, \langle\bot, 1\rangle$	$\bot$
$\langle H, 1\rangle, \langle\bot, 1\rangle$	H
$\langle\bot, 1\rangle, \langle H, 1\rangle$	F
$\langle\bot, 1\rangle, \langle F, 1\rangle$	D

Blocks:

- $\{\bot, I, G\}$
- $\{F, E\}$
- $\{D, C\}$

- $\mathrm{sig}(C) = (\langle\bot, 1\rangle, \langle F, 1\rangle)$ in map!
- Add C to D (and merge)

Finding almost equivalent states



Signature map:

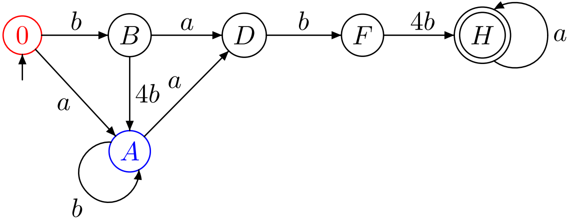
$\langle\bot,1\rangle,\langle\bot,1\rangle$	$\bot$
$\langle H,1\rangle,\langle\bot,1\rangle$	H
$\langle\bot,1\rangle,\langle H,1\rangle$	F
$\langle\bot,1\rangle,\langle F,1\rangle$	D
$\langle D,1\rangle,\langle A,4\rangle$	B

Blocks:

- $\{\bot, I, G\}$
- $\{F, E\}$
- $\{D, C\}$

- $\mathrm{sig}(B) = (\langle D,1\rangle,\langle A,4\rangle)$
- add to map

Finding almost equivalent states



Signature map:

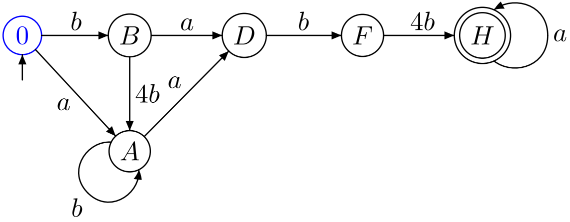
$\langle\bot,1\rangle,\langle\bot,1\rangle$	$\bot$
$\langle H,1\rangle,\langle\bot,1\rangle$	H
$\langle\bot,1\rangle,\langle H,1\rangle$	F
$\langle\bot,1\rangle,\langle F,1\rangle$	D
$\langle D,1\rangle,\langle A,4\rangle$	B
$\langle D,1\rangle,\langle A,1\rangle$	A

Blocks:

- $\{\bot, I, G\}$
- $\{F, E\}$
- $\{D, C\}$

- $\text{sig}(A) = (\langle D,1\rangle,\langle A,1\rangle)$
- add to map

Finding almost equivalent states



Signature map:

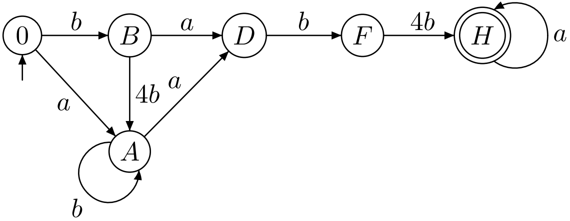
$\langle \bot, 1\rangle, \langle \bot, 1\rangle$	$\bot$
$\langle H, 1\rangle, \langle \bot, 1\rangle$	H
$\langle \bot, 1\rangle, \langle H, 1\rangle$	F
$\langle \bot, 1\rangle, \langle F, 1\rangle$	D
$\langle D, 1\rangle, \langle A, 4\rangle$	B
$\langle D, 1\rangle, \langle A, 1\rangle$	A
$\langle A, 1\rangle, \langle B, 1\rangle$	0

Blocks:

- $\{\bot, I, G\}$
- $\{F, E\}$
- $\{D, C\}$

- $\mathrm{sig}(0) = (\langle A, 1\rangle, \langle B, 1\rangle)$
- add to map

Finding almost equivalent states



Signature map:

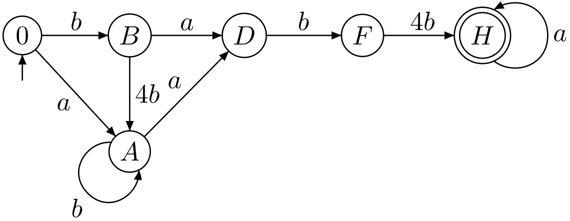
$\langle\bot, 1\rangle, \langle\bot, 1\rangle$	$\bot$
$\langle H, 1\rangle, \langle\bot, 1\rangle$	H
$\langle\bot, 1\rangle, \langle H, 1\rangle$	F
$\langle\bot, 1\rangle, \langle F, 1\rangle$	D
$\langle D, 1\rangle, \langle A, 4\rangle$	B
$\langle D, 1\rangle, \langle A, 1\rangle$	A
$\langle A, 1\rangle, \langle B, 1\rangle$	0

Blocks:

- $\{\bot, I, G\}$
- $\{F, E\}$
- $\{D, C\}$

- Blocks represent almost equivalence
- Scaling factors used in merges

Finding almost equivalent states



Signature map:

$\langle \bot, 1\rangle, \langle \bot, 1\rangle$	$\bot$
$\langle H, 1\rangle, \langle \bot, 1\rangle$	H
$\langle \bot, 1\rangle, \langle H, 1\rangle$	F
$\langle \bot, 1\rangle, \langle F, 1\rangle$	D
$\langle D, 1\rangle, \langle A, 4\rangle$	B
$\langle D, 1\rangle, \langle A, 1\rangle$	A
$\langle A, 1\rangle, \langle B, 1\rangle$	0

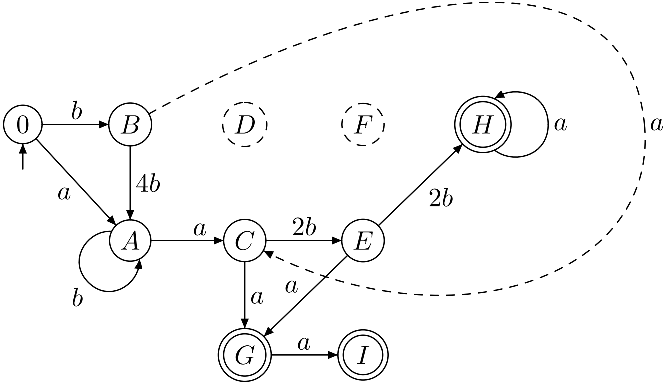
Blocks:

- $\{\bot, I, G\}$
- $\{F, E\}$
- $\{D, C\}$

- Blocks represent almost equivalence
- Scaling factors used in merges
- but we merged kernel states

Weighted merges

- merge of F into E with factor 2
- merge of D into C with factor 1



Conclusion

Solved

- hyper-minimisation for WDFA over semifields

Open

- Optimise number of errors
- Incremental construction
- Extensions to WNFA, more weight structures, etc.

Conclusion

Solved

- hyper-minimisation for WDFA over semifields

Open

- Optimise number of errors
- Incremental construction
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Thank you for your attention!

References

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