Minimization of Weighted Automata

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In Collaboration with

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- Jonathan May, USC, Los Angeles, CA, USA
- Heiko Vogler, TU Dresden, Germany

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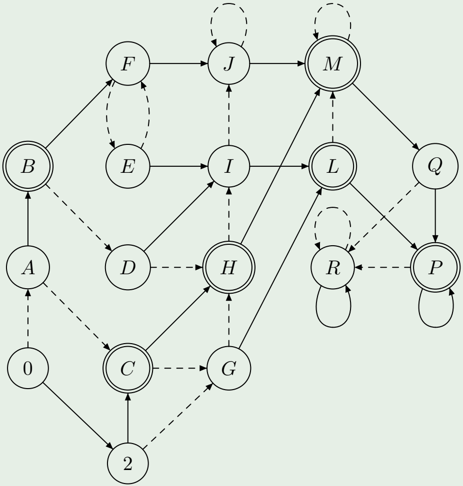
Unweighted automata

Example (Unweighted automaton)



Unweighted automata (Cont'd)

Minimization methods

- HOPCROFT's algorithm for DFA $O(m \log n)$ [HOPCROFT 1971]
- TARJAN's algorithm for NFA $O(m \log n)$ (bisimulation minimization) [TARJAN 1987]
- NFA minimization is PSPACE-complete [JIANG, RAVIKUMAR 1993]

Special cases

- Cover automata $O(m \log n)$ [KÖRNER 2002]
- Hyper-minimization $O(m \log n)$ [GAWRYCHOWSKI, JEŻ 2009; HOLZER, $\sim$ 2010]

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Minimization Procedures

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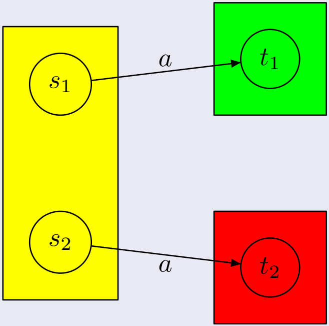

Tarjan (for NFA)

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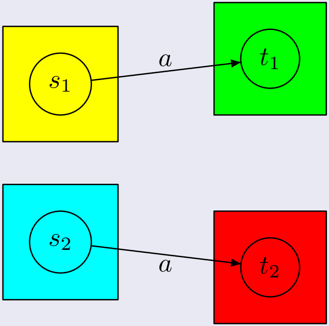

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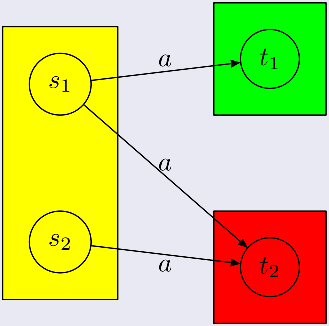

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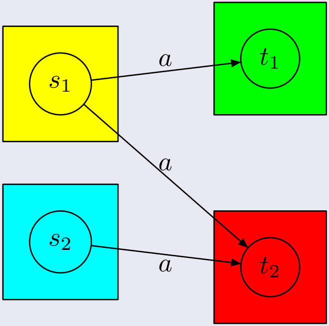

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Syntax

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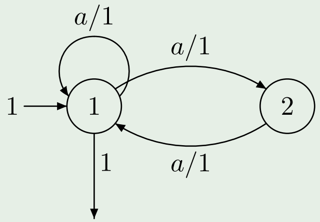

Definition (Weighted automaton)

(Q, Σ, μ, I, F)

- Q finite set of states
- Σ alphabet
- $\mu \colon Q \times \Sigma \times Q \to A$
- $I \colon Q \to A$ initial weights
- $F \colon Q \to A$ final weights

References

- BERSTEL, REUTENAUER: *Rational series and their languages*. Springer, 1988
- KUICH, SALOMAA: *Semirings, automata, languages*. Springer, 1986

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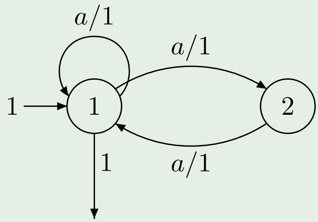

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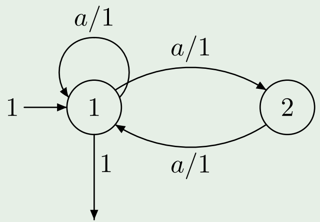
Semantics

Weight structure: Semiring $A = (A, +, \cdot, 0, 1)$

Example



Definition (Semantics)

$h_\mu \colon \Sigma^* \to A^Q$

$$h_\mu(\varepsilon)_q = I(q)$$

$$h_\mu(wa)_q = \sum_{p \in Q} h_\mu(w)_p \cdot \mu(p, a, q)$$

Example (using natural numbers)

$$h_\mu(a) = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \qquad h_\mu(aa) = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \qquad h_\mu(aaa) = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$$

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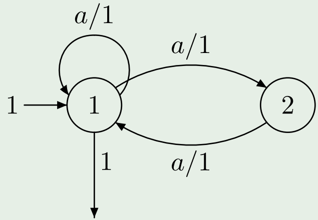

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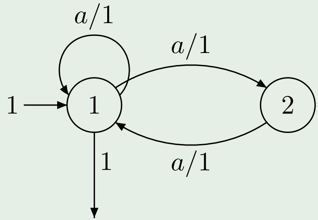

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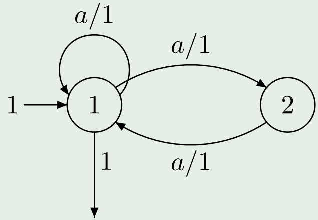

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Overview

Results

Method	Nondet.	Det.	Complexity	Reference
Pushing & HOPCROFT	–	x	$O(m \log n)$	MOHRI
Forward Bisimulation	x	x	$O(m \log n)$	BUCHHOLZ
Backward Bisimulation	x	–	$O(m \log n)$	BUCHHOLZ
Backward Simulation	x	–	$O(mn)$	RANZATO, ...
Full minimization	x	x	$O(mn^2)$	BERSTEL, ...

Notation

- m: number of transitions
- n: number of states

Pushing & HOPCROFT [MOHRI]

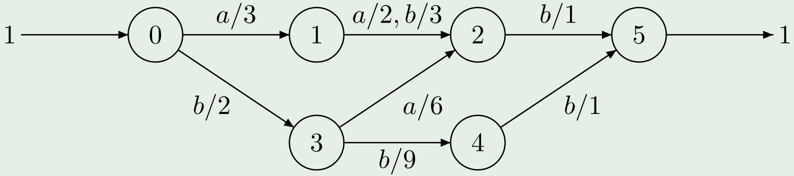
1. Push

Move weights toward the front

2. Minimize

Minimize as unweighted automaton; treat weight as part of label

Example (ala Eisner)



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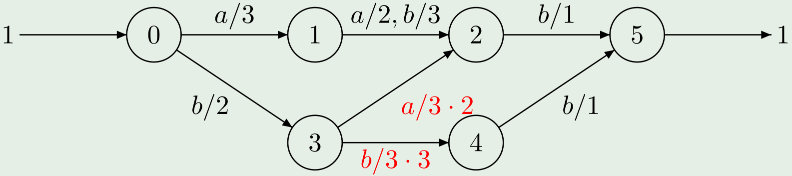

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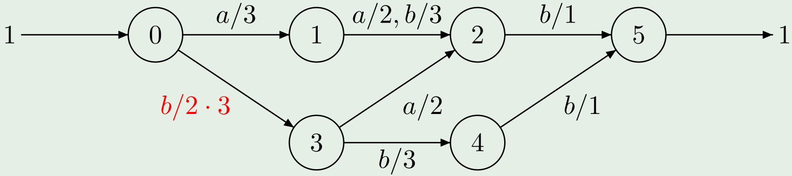

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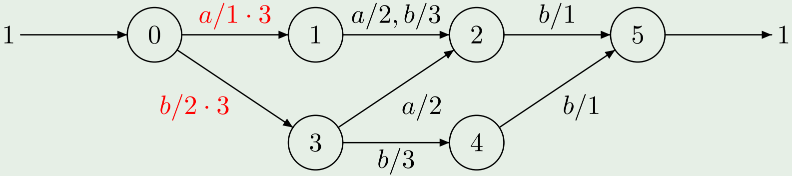

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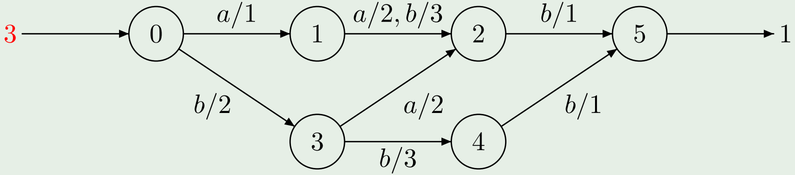

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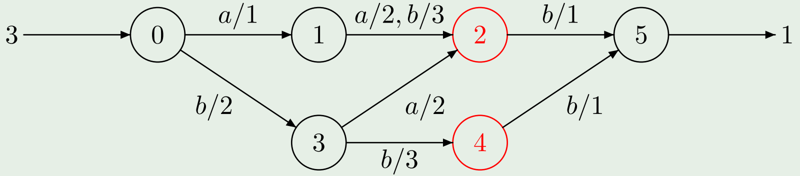

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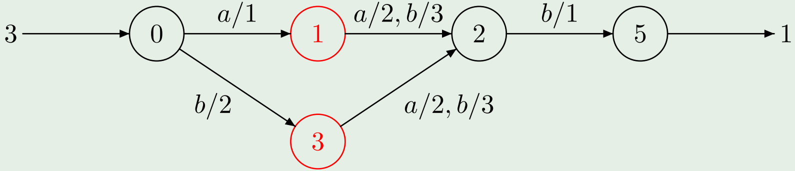

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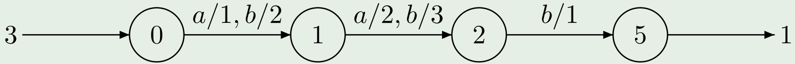

Pushing & HOPCROFT [MOHRI]

Prerequisites (ala EISNER)

1. Automaton deterministic
2. Semiring multiplicatively cancellative
3. Semiring allows greedy factorization

Definition (Greedy factorization)

There exists a mapping $f\colon A^2 \to A$ such that for all $a, b, c \in A$ with $c \neq 0$:

If $a|c$ and $b|c$, then $\dfrac{c}{a \cdot f(a,b)} = \dfrac{c}{b \cdot f(b,a)}$

$$\begin{tikzcd}
& \bullet \arrow[dl, "a^{-1}"'] \arrow[dr, "b^{-1}"] & \\
\bullet \arrow[dr, "f(a,b)^{-1}"'] & & \bullet \arrow[dl, "f(b,a)^{-1}"] \\
& \bullet &
\end{tikzcd}$$

Forward Bisimulation [BUCHHOLZ]

Definition (Forward bisimulation)

Equivalence relation $\equiv$ on states such that $F(p) = F(p')$ and

$$\sum_{r \in [q]} \mu(p, a, r) = \sum_{r \in [q]} \mu(p', a, r)$$

for every $p \equiv p'$, state q, and symbol a.

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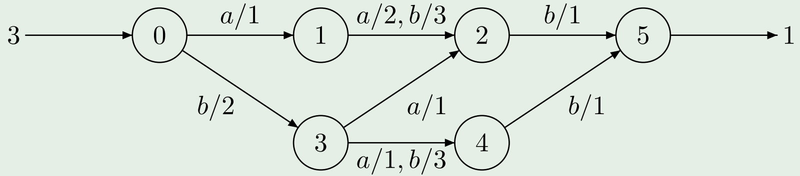

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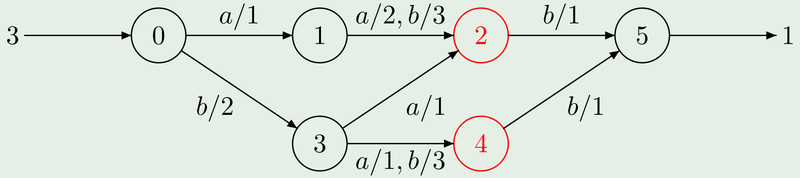

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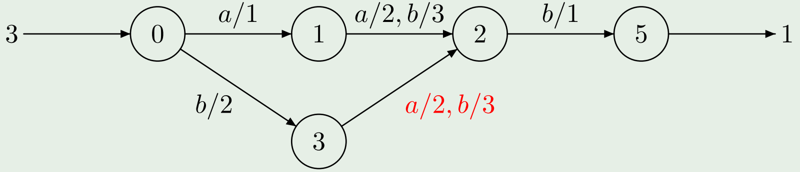

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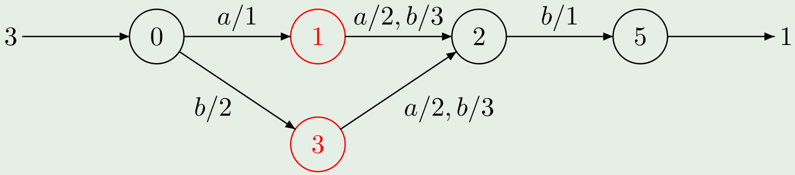

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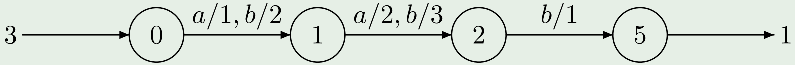

Unweighted Forward Simulation [ABDULLA et. al.]

Definition (Forward bisimulation unweighted)

Reflexive, symmetric, and transitive relation $\equiv$ on states such that $F(p) = F(p')$ and

$$\exists q' \equiv q \colon \mu(p, a, q) \leq \mu(p', a, q')$$

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Unweighted Forward Simulation [ABDULLA et. al.]

Definition (Forward simulation)

Pre-order $\leq$ on states such that $F(p) \leq F(p')$ and

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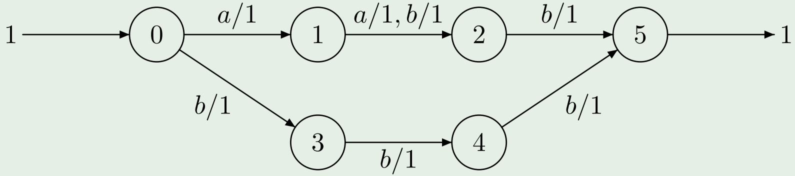
for every $p \leq p'$, state q, and symbol a.

Example



$0 \quad 3 \leq 1 \quad 4 \leq 2 \leq 4 \quad 5$

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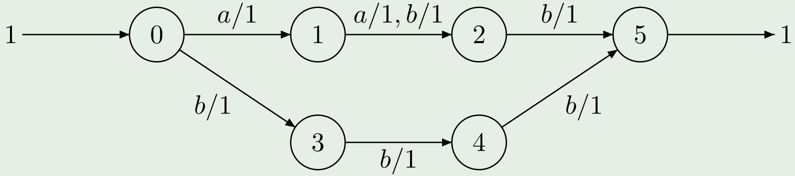

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Note

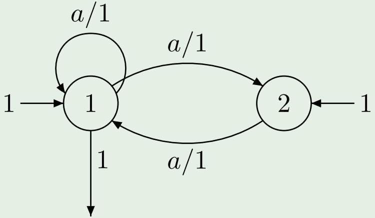
Does in general not preserve the language!

Backward Simulation [ABDULLA et. al.]

Definition (Backward (bi)simulation)

A forward (bi)simulation on the reversed automaton.

Example



$1 \leq 2 \leq 1$

Theorem

Reducing automaton by $\leq \cap \geq$ with $\leq$ a backward simulation preserves the language.

Note

Slightly more general than backward bisimulation.

Backward Simulation [ABDULLA et. al.]

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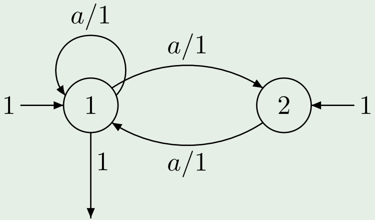

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Full Minimization [Berstel et. al.]

Requirements

- weight structure is a field

Procedure

- prefix compression: select prefix-closed set W such that all $h_\mu(w)$ with $w \in W$ are linearly independent
- suffix compression

References

- Berstel, Reutenauer: *Rational series and their languages*. Springer, 1988
- Berstel, Reutenauer: *Noncommutative Rational Series With Applications*, Cambridge Univ. Press, 2010

Simulation vs. Equivalence

Theorem

Two equivalent automata can be joined by forward and backward bisimulation (and weight inversion).

Extensions

- natural numbers
- integers
- rings, ...

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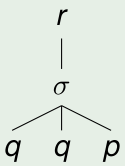

Definition (Weighted tree automaton)

$(Q, \Sigma, (\mu_k)_{k \in \mathbb{N}}, F)$

- Q finite set of states
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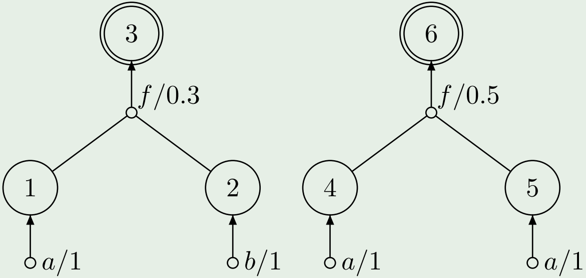
- $\mu_k \colon Q^k \times \Sigma_k \times Q \to A$
- $F \colon Q \to A$ final weights

References

- BERSTEL, REUTENAUER: *Recognizable formal power series on trees*. Theor. Comput. Sci. 18, 1982
- BORCHARDT: *The theory of recognizable tree series*. PhD thesis, 2004

Syntax — Illustration

Example



Semantics

Definition

Let $t \in T_\Sigma(Q)$ and $W = \text{pos}(t)$.

- Run on t: map $r\colon W \to Q$ with $r(w) = t(w)$ if $t(w) \in Q$
- Weight of r

$$\text{wt}(r) = \prod_{\substack{w \in W \\ t(w) \in \Sigma}} \mu_k(r(w1), \dots, r(wk), t(w), r(w))$$

- Recognized tree series

$$(\|M\|, t) = \sum_{r \text{ run on } t} F(r(\varepsilon)) \cdot \text{wt}(r)$$

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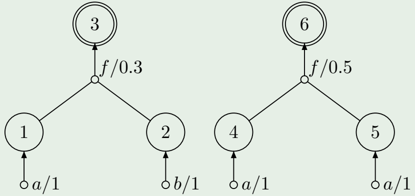


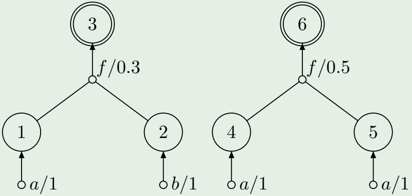
Example (Runs)

Input tree: $f(a, b)$

Runs: $6(1, 2)$ with weight 0

Semantics — Illustration

Example (Automaton)



Example (Runs)

Input tree: *f*

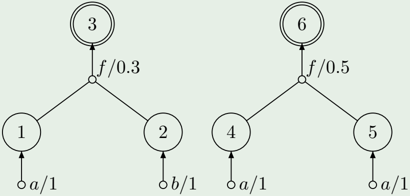
a *b*

Runs: *f* with weight

a *b*

Semantics — Illustration

Example (Automaton)



Example (Runs)

Input tree: $f(a, b)$

Runs: $f(1, b)$ with weight 1

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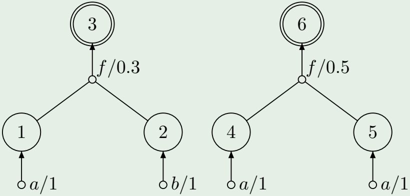

Example (Runs)

Input tree: f

$a \quad b$

Runs: f with weight 1

1 2

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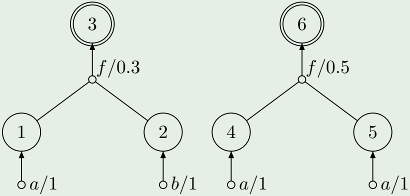

Example (Runs)

Input tree: $f(a, b)$

Runs: $\textcolor{red}{3}(1, 2)$ with weight 0.3

Overview

Results

Method	Nondet.	Det.	Complexity	Reference
Det. Minimization	–	x	$O(rmn)$	$\sim$
Forw. Bisimulation	x	x	$O(rm \log n)$	HÖGBERG, ...
Backw. Bisimulation	x	–	$O(r^2 m \log n)$	HÖGBERG, ...
Backw. Simulation	x	–	$O(r^2 mn)$	ABDULLA, ...
Full minimization	x	x	P	BOZAPALIDIS

Notation

- m: number of transitions
- n: number of states
- r: maximal rank of the input symbols

Forward Bisimulation [Högberg, et. al.]

Definition (Forward bisimulation)

Equivalence relation $\equiv$ on states such that $F(p) = F(p')$ and

$$\sum_{r \in [q]} \mu(\dots, p, \dots, a, r) = \sum_{r \in [q]} \mu(\dots, p', \dots, a, r)$$

for every $p \equiv p'$, symbol a, and states q and $\dots$.

Backward Bisimulation [HÖGBERG, et. al.]

Definition (Backward bisimulation)

Equivalence relation $\equiv$ on states such that

$$\sum_{q_1 \cdots q_k \in B_1 \cdots B_k} \mu(q_1, \ldots, q_k, a, p) = \sum_{q_1 \cdots q_k \in B_1 \cdots B_k} \mu(q_1, \ldots, q_k, a, p')$$

for every $p \equiv p'$, symbol a, and blocks $B_1, \ldots, B_k$.

Det. Minimization — Overview

Applicability

- Deterministic wta
- Commutative semifield (i.e. multiplicative inverses)

Roadmap

- MYHILL-NERODE congruence relation [BORCHARDT 2003]
- Determine signs of life
- Refinement

MYHILL-NERODE congruence

Definition

$p \equiv q$: there exists nonzero a such that for every context C

$$(\|M\|, C[p]) = a \cdot (\|M\|, C[q])$$

Notes

- Semifields are zero-divisor free
- Element a is unique if p is not dead

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Notes

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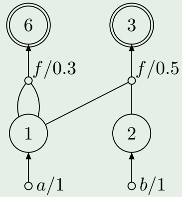
Signs of Life

Definition

Sign of life of $q \in Q$: context C such that $(\|M\|, C[q]) \neq 0$

Example



State	Sign of life	State	Sign of life
1	$f(\square, b)$	2	$f(1, \square)$
3	$\square$	6	$\square$

Stages

Definition

Stage $(\Pi, \mathrm{sol}, f, r)$:

(i) $\equiv$ refinement of $\equiv_\Pi$

(ii) $\mathrm{sol}(F) = \{\Box\}$

(iii) for live q with $p = r([q])$

$$(\|M\|, \mathrm{sol}(p)[q]) = f(q) \cdot (\|M\|, \mathrm{sol}(p)[p])$$

(iv) $\equiv_\Pi$ congruence

(v) for symbol σ and context C with live $\delta_\sigma(C[q])$

$$f(q)^{-1} \cdot c_\sigma(C[q]) \cdot f(\delta_\sigma(C[q])) = c_\sigma(C[p]) \cdot f(\delta_\sigma(C[p]))$$

where $\delta_\sigma \colon Q^k \to Q$ and $c_\sigma \colon Q^k \to A$

Stages

Definition

Stable stage $(\Pi, \mathrm{sol}, f, r)$:

(i) $\equiv$ refinement of $\equiv_\Pi$

(ii) $\mathrm{sol}(F) = \{\Box\}$

(iii) for live q with $p = r([q])$

$$(\|M\|, \mathrm{sol}(p)[q]) = f(q) \cdot (\|M\|, \mathrm{sol}(p)[p])$$

(iv) $\equiv_\Pi$ congruence

(v) for symbol σ and context C with live $\delta_\sigma(C[q])$

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where $\delta_\sigma \colon Q^k \to Q$ and $c_\sigma \colon Q^k \to A$

Refining a Stage

Definition

Refinement of $(\Pi, \mathrm{sol}, f, r)$: Partition Π' with $p \equiv_{\Pi'} q$ if

(i) $p \equiv_\Pi q$

(ii) $\delta_\sigma(C[p]) \equiv_\Pi \delta_\sigma(C[q])$

(iii) if $\delta_\sigma(C[p])$ is live, then

$$f(p)^{-1} \cdot c_\sigma(C[p]) \cdot f(\delta_\sigma(C[p])) = f(q)^{-1} \cdot c_\sigma(C[q]) \cdot f(\delta_\sigma(C[q]))$$

for states p and q, symbol σ, and context C

Complete Algorithm

Algorithm

$(\Pi', \mathrm{sol}, D) \leftarrow \textsc{ComputeSol}(M)$
2: **repeat**
$\quad (\Pi, \mathrm{sol}, f, r) \leftarrow \textsc{Complete}(M, \Pi', \mathrm{sol}, D)$
4: $\quad \Pi' \leftarrow \textsc{Refine}(M, \Pi, \mathrm{sol}, f, r, D)$
until $\Pi' = \Pi$
6: **return** minimized wta

Notes

- Algorithm runs in $O(rmn^4)$
- Can be optimized to run in $O(rmn)$ [$\sim$ 2008]
- Returns equivalent minimal deterministic wta

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return minimized wta

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Simulation vs. Equivalence

Theorem

Two equivalent tree automata over fields can be joined by forward and backward bisimulation (and weight inversion).

Extensions

- natural numbers
- integers
- rings, ...

References

- BOZAPALIDIS: *Effective Construction of the Syntactic Algebra of a Recognizable Series on Trees.* Acta Inf. 1991
- ÉSIK, $\sim$: *Simulations of weighted tree automata*, 2010

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Table of Contents

Experiments

State Count

Original	Minimal	Reduction to
98	68	69%
394	308	78%
497	381	77%
727	515	71%
2701	1993	74%
3686	1766	48%

State & Transition Count

Error	Original	Minimal	Reduction to
10^{-4}	$(727, 6485)$	$(629, 6131)$	$(87\%, 95\%)$
10^{-2}	$(727, 6485)$	$(525, 3425)$	$(72\%, 53\%)$

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The End

Thank You!