

# Weighted Tree Transducers in Natural Language Processing

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- **KEVIN KNIGHT**, USC, Los Angeles, CA, USA
- **ERIC LILIN**, Université de Lille, France
- **GIORGIO SATTÀ**, University of Padua, Italy
- **HEIKO VOGLER**, TU Dresden, Germany

- 1 Machine Translation
- 2 Weighted Tree Transducer
- 3 Expressive Power
- 4 Standard Algorithms
- 5 Implementation

# Motivation

## Example (Input in Catalan)

*Benvolguda i benvolgut membre de la comunitat universitària, Avui dilluns es duu a terme el darrer Consell de Govern del meu mandat com a rector; el proper dia 6 de maig, com correspon, hi haurà una nova elecció on tota la comunitat universitària podrà escollir nou rector o rectora. Aquest darrer consell té, naturalment, un caràcter marcadament tècnic; l'ordre del dia complet el trobaràs adjunt al final d'aquest text. A continuació et comento només els punts que, al meu parer, poden ser més del teu interès.*

## Translation (GOOGLE TRANSLATE) to English

*Dear and beloved member of the university community, Today is Monday carried out by the Governing Council last of my term as rector, the next day, May 6, as appropriate, there will be another election where the entire university community can choose new rector. This last advice is, of course, a markedly technician complete agenda can be found attached to the end of this text. Then I said only the points that I believe may be of interest.*

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# Machine Translation System

Input sentence (*Benvolguda i benvolgut ...*)



Translation system



Output sentence (*Dear and beloved ...*)

# Machine Translation System

Input sentence (*Benvolguda i benvolgut ...*) **f**



Translation system



Output sentence (*Dear and beloved ...*) **e**

Statistical translation system

$$\mathbf{e} = \underset{\mathbf{e}}{\operatorname{argmax}} p(\mathbf{e}|\mathbf{f})$$

# Noisy Channel Viewpoint

Input sentence (*Benvolguda i benvolgut ...*) **f**



Identity translation



Error signal (Noise)



Output sentence (*Dear and beloved ...*) **e**

# Noisy Channel Viewpoint

Input sentence (*Benvolguda i benvolgut ...*) **f**



Identity translation

⇐ Error signal (Noise)



Output sentence (*Dear and beloved ...*) **e**

## Bayes' theorem

$$\mathbf{e} = \underset{e}{\operatorname{argmax}} p(\mathbf{e}|\mathbf{f}) = \underset{e}{\operatorname{argmax}} \frac{p(\mathbf{f}|\mathbf{e}) \cdot p(\mathbf{e})}{p(\mathbf{f})} = \underset{e}{\operatorname{argmax}} p(\mathbf{f}|\mathbf{e}) \cdot p(\mathbf{e})$$

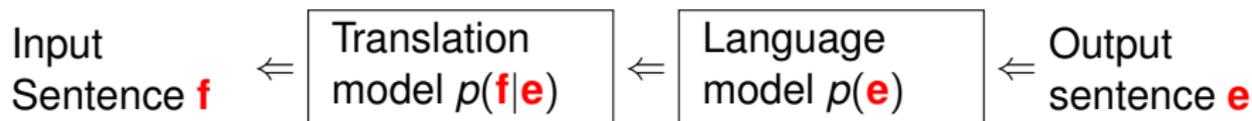
# Components

## Optimization problem

$$\mathbf{e} = \underset{e}{\operatorname{argmax}} p(\mathbf{f}|e) \cdot p(e)$$

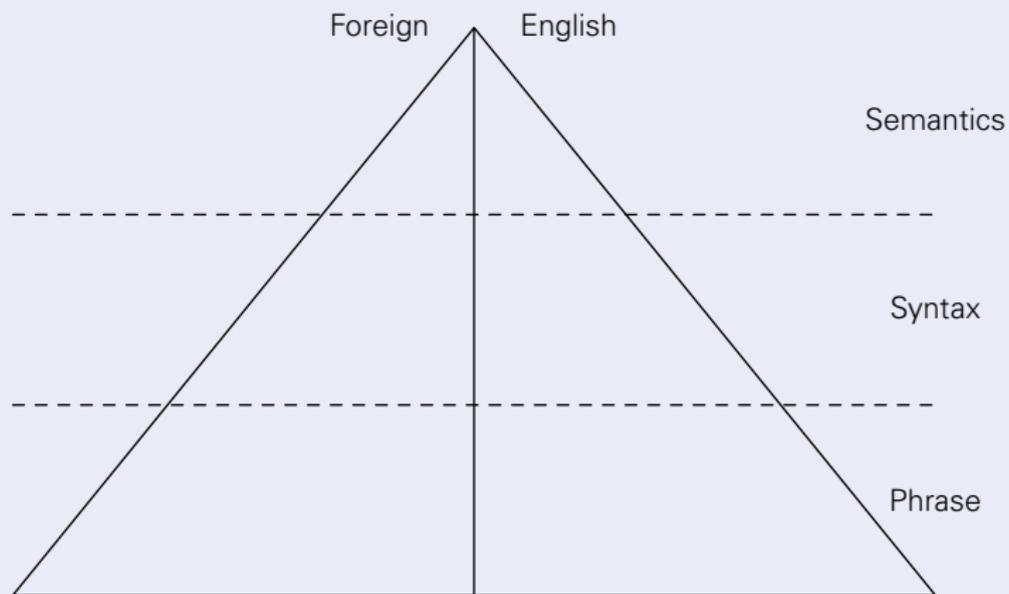
## Required models

- $p(e)$  — language model
- $p(\mathbf{f}|e)$  — translation model



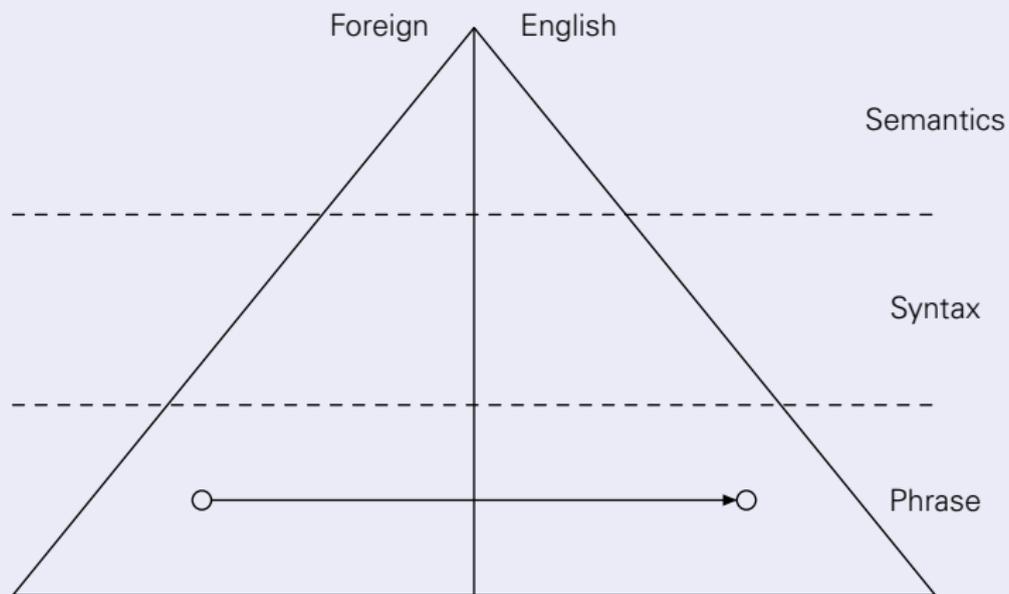
# Translation Approach

## Overview



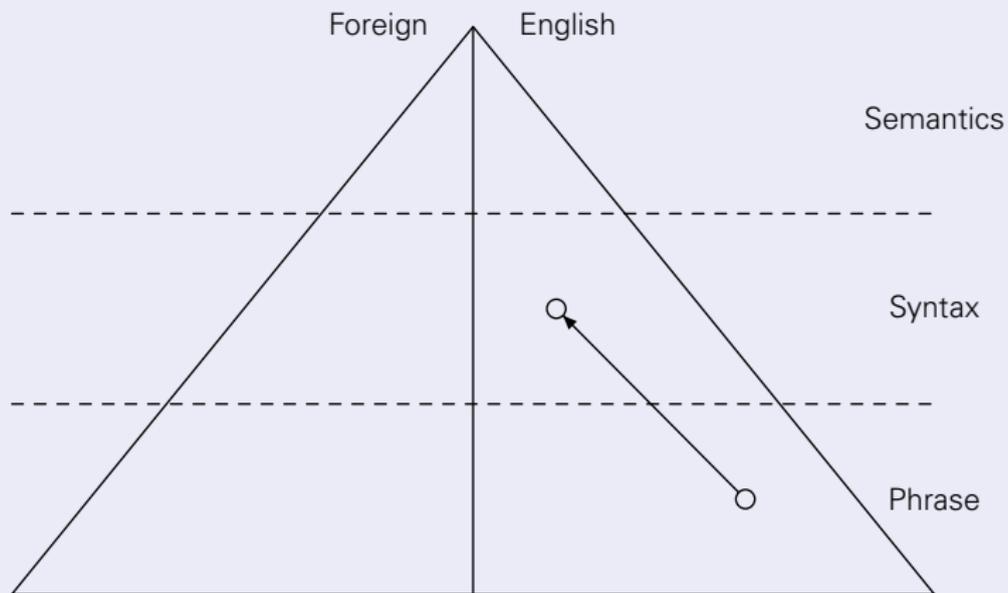
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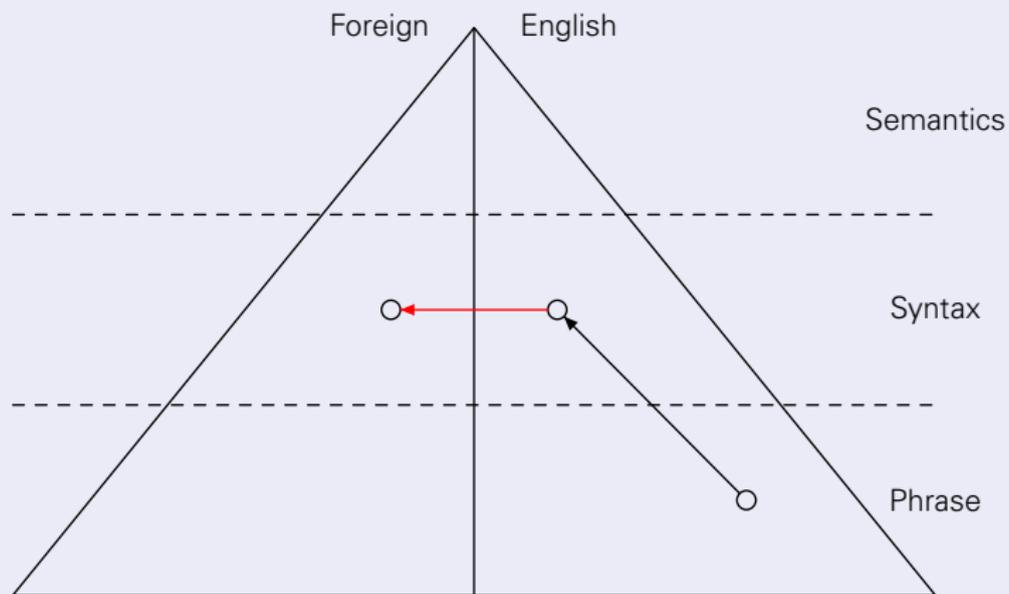
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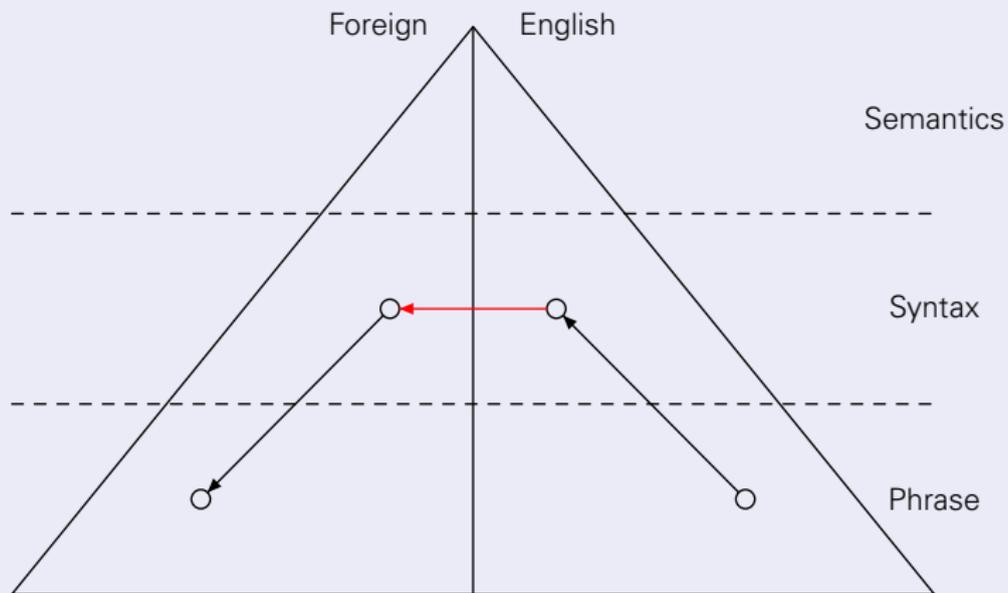
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# Translation Approach

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# Why Syntax?

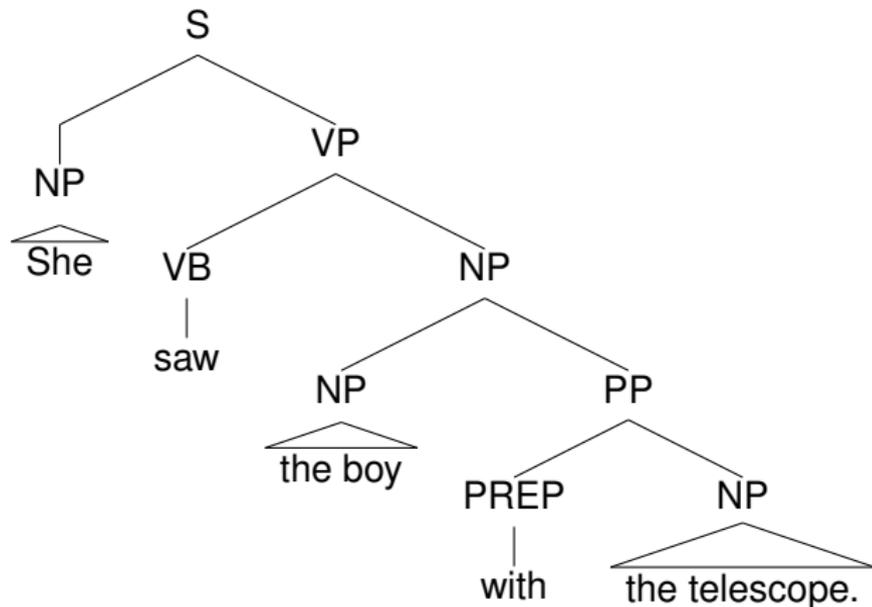
## Example

She saw the boy with the telescope.

# Why Syntax?

## Example

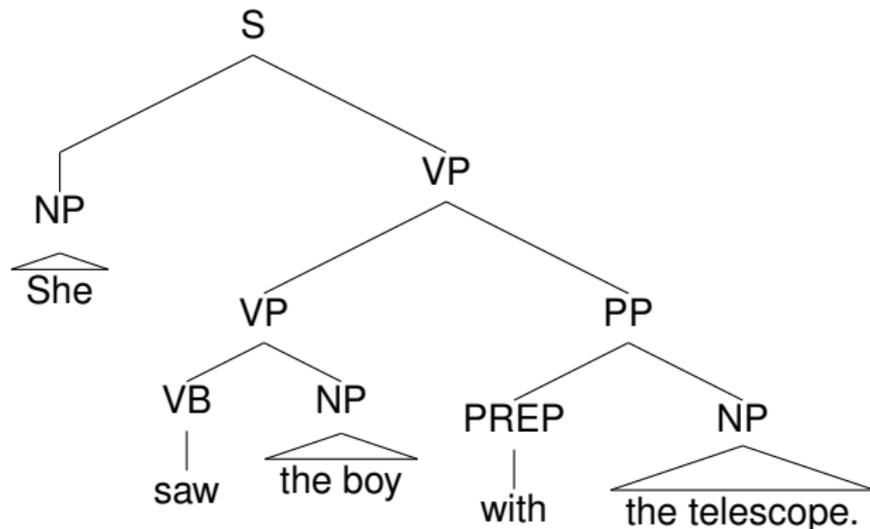
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# Why Syntax?

## Example

She saw the boy with the telescope.

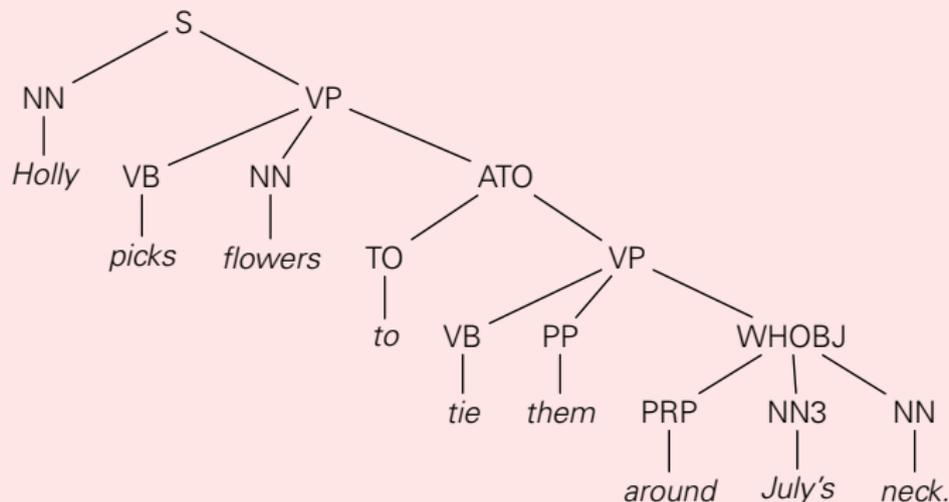


# Syntactic Analysis

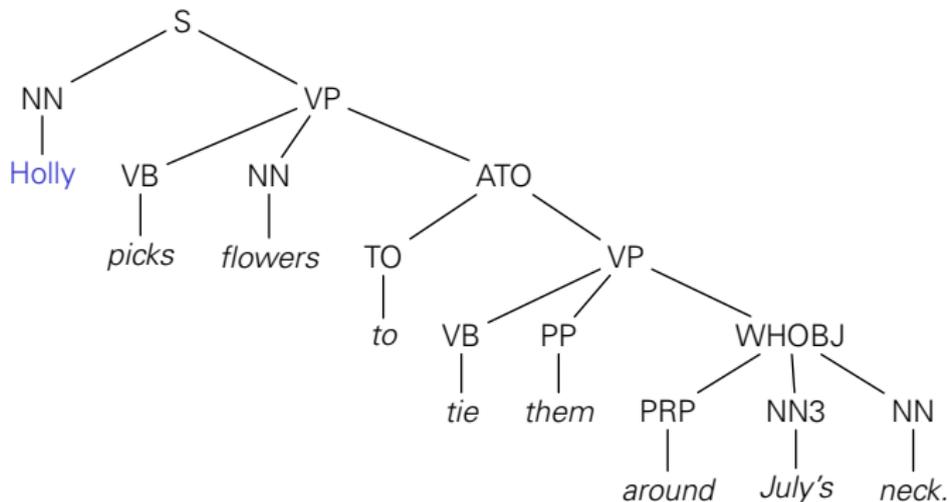
## Output sentence

Holly picks flowers to tie them around July's neck.

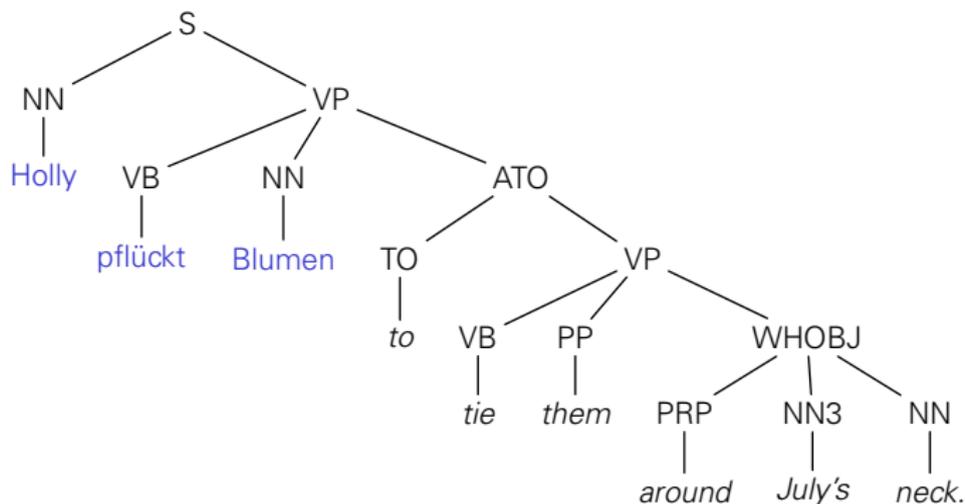
## Parser output



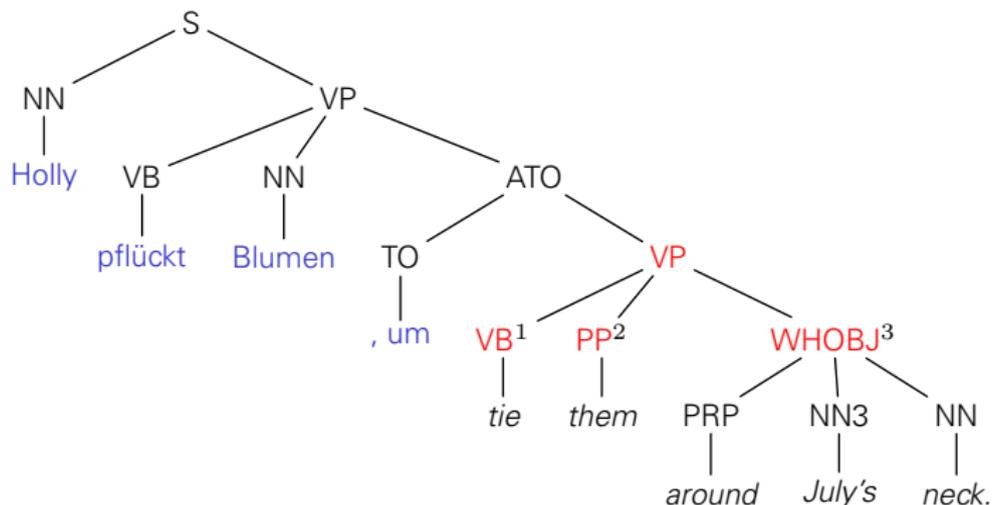
# Syntax-based Machine Translation



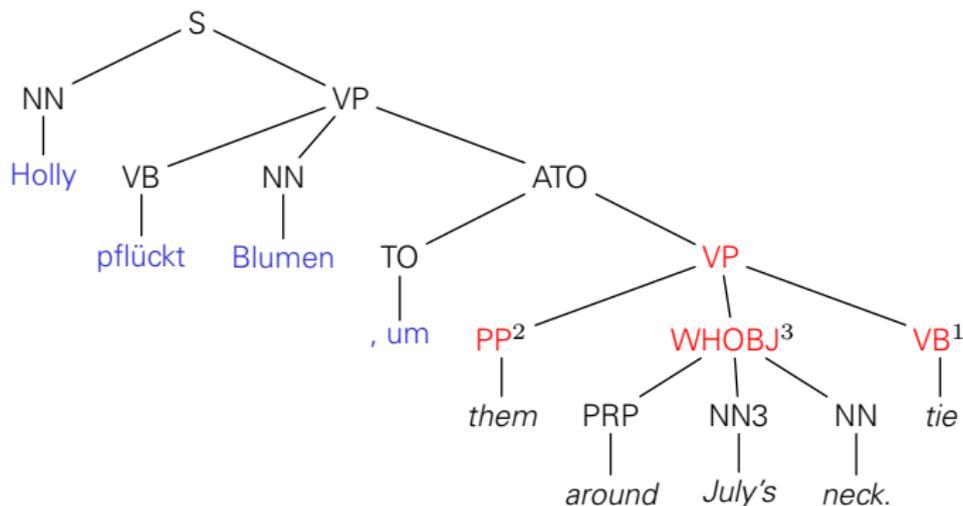
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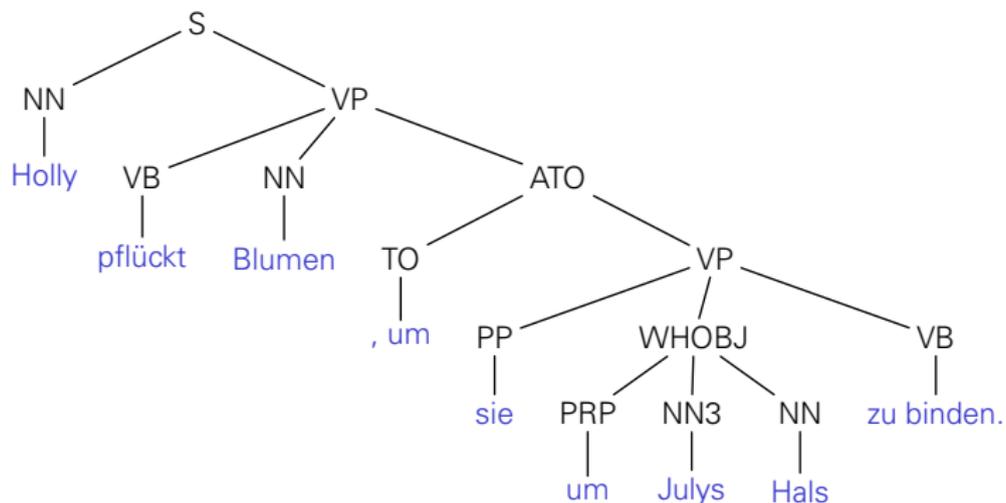
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# Syntax-based Machine Translation



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# Weight Structure

## Definition

$(A, +, \cdot, 0, 1)$  is a **(commutative) semiring** if

- $(A, +, 0)$  and  $(A, \cdot, 1)$  commutative monoids,
- $\cdot$  distributes over  $+$ , and
- $a \cdot 0 = 0$  for every  $a \in A$ .

## Example

- $(\{0, 1\}, \max, \min, 0, 1)$  BOOLEAN semiring
- $(\mathbb{R}, +, \cdot, 0, 1)$  semiring of real numbers
- $(\mathbb{N} \cup \{\infty\}, \min, +, \infty, 0)$
- any field, ring, etc.

## Definition

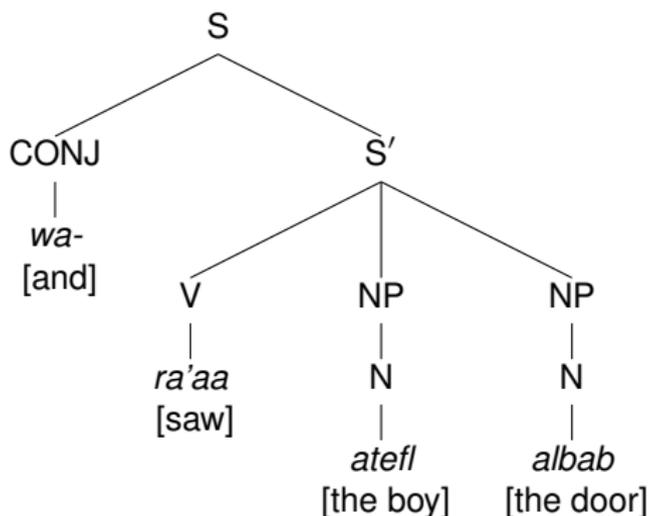
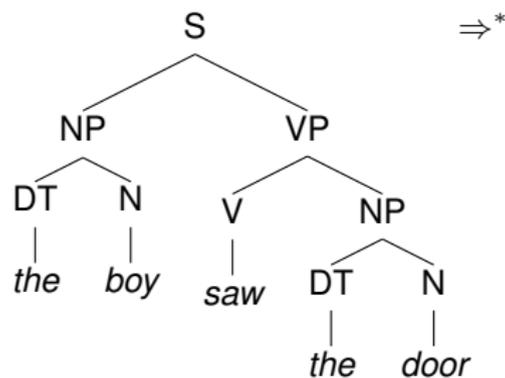
$(Q, \Sigma, \Delta, I, R)$  (**weighted extended (top-down) tree transducer** (xtt))

- $Q$  finite set of *states* (considered unary)
- $\Sigma$  and  $\Delta$  ranked alphabets
- $I: Q \rightarrow A$  *initial weight* distribution
- $R: Q(T_\Sigma(X)) \times T_\Delta(Q(X)) \rightarrow A$  *rule weight* assignment s.t.
  - ▶  $\text{supp}(R)$  is finite
  - ▶ for every  $(l, r) \in \text{supp}(R)$  there is  $k \in \mathbb{N}$  such that  $l \in Q(C_\Sigma(X_k))$  and  $r \in T_\Delta(Q(X_k))$
  - ▶  $\{l, r\} \not\subseteq Q(X)$  for every  $(l, r) \in \text{supp}(R)$

## References

- **ARNOLD, DAUCHET**: Bi-transductions de forêts. ICALP 1976
- **GRAEHL, KNIGHT**: Training tree transducers. HLT-NAACL 2004

# Syntax — Example



## Question

How to implement this English → Arabic translation using xtt?

# Syntax — Example (cont'd)

## Example

States  $\{q, q_S, q_V, q_{NP}\}$  of which only  $q$  is initial

$$q(x_1) \rightarrow q_S(x_1) \quad (r_1)$$

$$q(x_1) \rightarrow S(\text{CONJ}(wa-), q_S(x_1)) \quad (r_2)$$

$$q_S(S(x_1, VP(x_2, x_3))) \rightarrow S'(q_V(x_2), q_{NP}(x_1), q_{NP}(x_3)) \quad (r_3)$$

$$q_V(V(saw)) \rightarrow V(ra'aa) \quad (r_4)$$

$$q_{NP}(NP(DT(the), N(boy))) \rightarrow NP(N(ateff)) \quad (r_5)$$

$$q_{NP}(NP(DT(the), N(door))) \rightarrow NP(N(albab)) \quad (r_6)$$

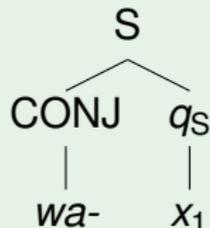
# Syntax — Example (cont'd)

## Example

- ① **Nondeterminism** and **epsilon rules** (rules  $r_1$  and  $r_2$ )

$$\begin{array}{c} q \\ | \\ x_1 \end{array} \rightarrow \begin{array}{c} q_S \\ | \\ x_1 \end{array}$$

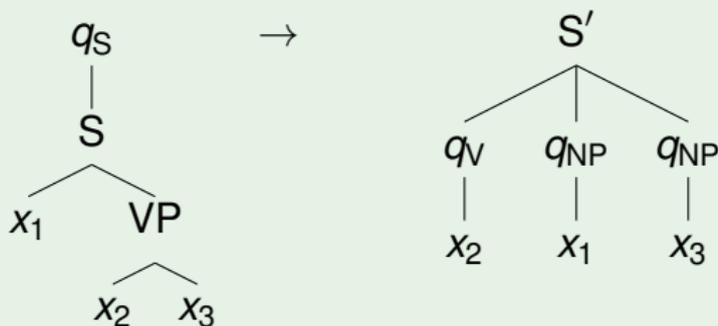
and

$$\begin{array}{c} q \\ | \\ x_1 \end{array} \rightarrow$$


# Syntax — Example (cont'd)

## Example

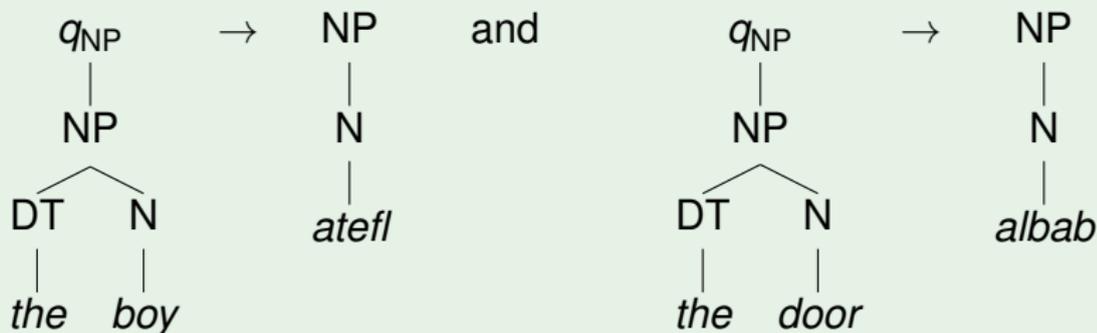
- 1 **Nondeterminism** and **epsilon rules** (rules  $r_1$  and  $r_2$ )
- 2 **Deep** attachment of variables (rule  $r_3$ )



# Syntax — Example (cont'd)

## Example

- 1 **Nondeterminism** and **epsilon rules** (rules  $r_1$  and  $r_2$ )
- 2 **Deep** attachment of variables (rule  $r_3$ )
- 3 **Finite** look-ahead (rules  $r_4$  and  $r_5$ )



## Definition

Let  $\xi, \zeta \in T_{\Delta}(Q(T_{\Sigma}))$ . Then  $\xi \xrightarrow{a}_M \zeta$  if there exist

- 1 a rule  $R(q(t), u) = a \neq 0$
- 2 a substitution  $\theta: X \rightarrow T_{\Sigma}$
- 3 a position  $w \in \text{pos}(\xi)$

such that  $\xi|_w = q(t\theta)$  and  $\zeta = \xi[u\theta]_w$

## Definition

Computed transformation ( $t \in T_{\Sigma}$  and  $u \in T_{\Delta}$ ):

$$\tau_M(t, u) = \sum_{\substack{q \in Q \\ q(t) \xrightarrow{a_1} \dots \xrightarrow{a_n} u \\ \text{left-most derivation}}} l(q) \cdot a_1 \cdot \dots \cdot a_n$$

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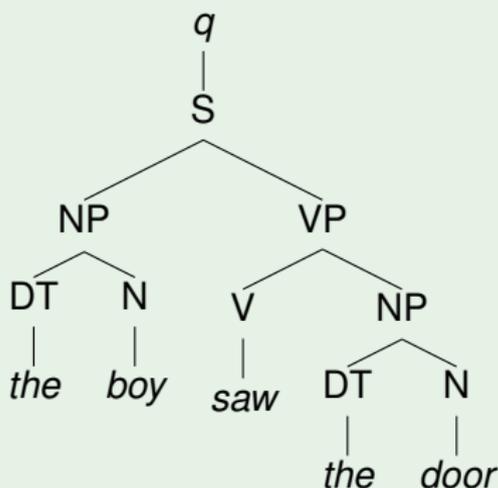
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## Example

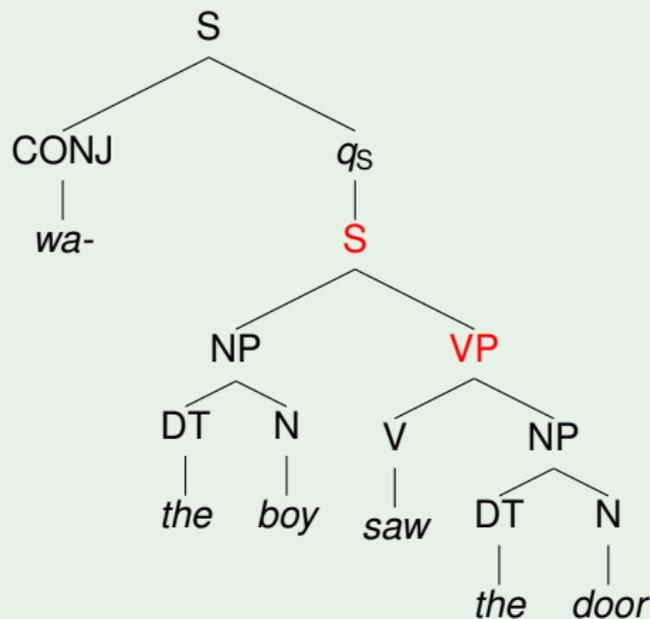


# Semantics — Example

## Rule

$$q_S(S(x_1, VP(x_2, x_3))) \rightarrow S'(q_V(x_2), q_{NP}(x_1), q_{NP}(x_3))$$

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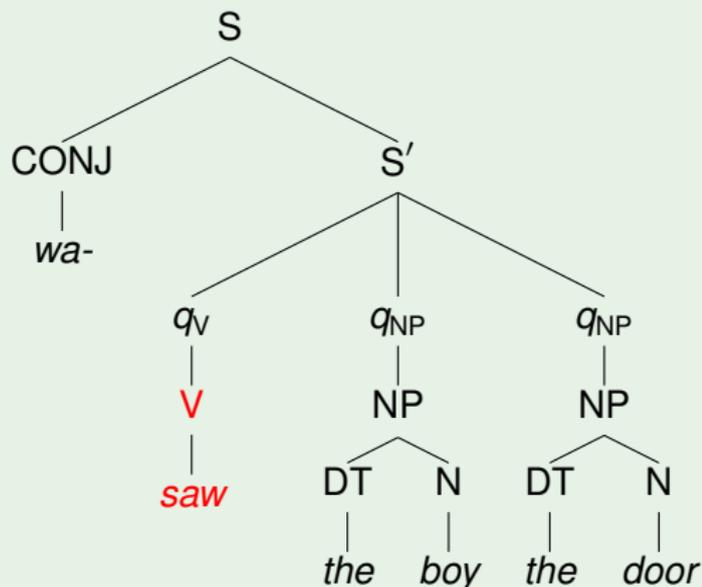


# Semantics — Example

## Rule

$q_V(V(\text{saw})) \rightarrow V(\text{ra'aa})$

## Example

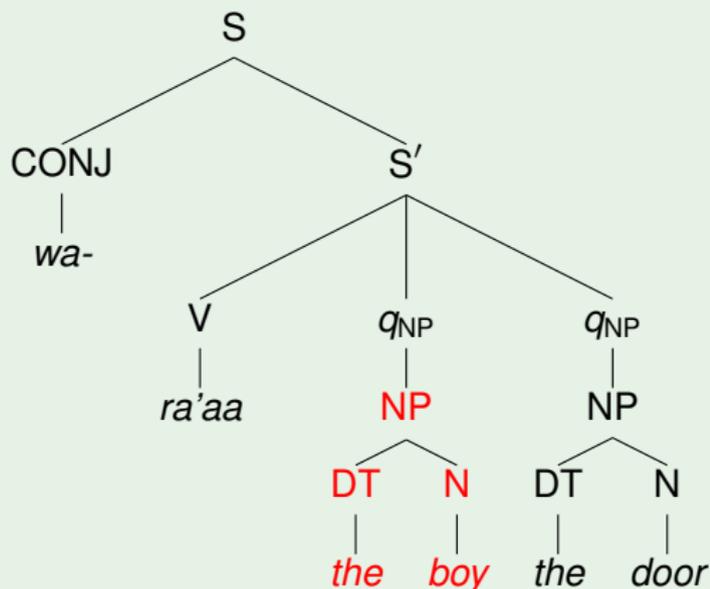


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$q_{NP}(NP(DT(\textit{the}), N(\textit{boy}))) \rightarrow NP(N(\textit{atefl}))$

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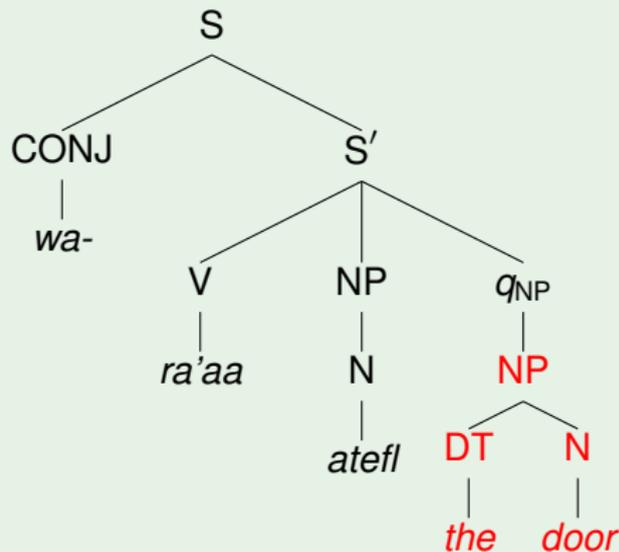


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## Rule

$q_{NP}(NP(DT(\textit{the}), N(\textit{door}))) \rightarrow NP(N(\textit{albab}))$

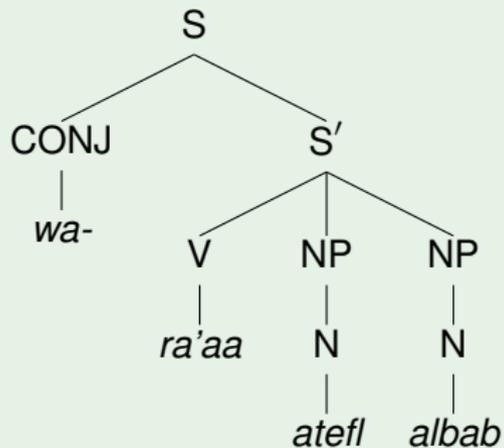
## Example



# Semantics — Example

## Rule

## Example



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# Syntactic Restrictions

## Definition

- **linear**: no right-hand side contains a duplicate variable
- **non-deleting**: all right-hand sides contain all variables of their left-hand side
- **epsilon-free**: no rules of the form  $q(x) \rightarrow u$

## Definition

- **tdtt**: every left-hand side is of the form  $q(\sigma(x_1, \dots, x_k))$

## Abbreviations

- **In-tdtt**: linear non-deleting tdtt
- **In-xtt**: linear non-deleting xtt

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# Wanted Expressivity

## Criteria

- 1 Generalize FST including epsilon rules (In-tdtt: **no**, In-xtt: **yes**)
- 2 Efficiently trainable (In-tdtt: **yes**, In-xtt: **yes**)
- 3 Can handle rotations (In-tdtt: **no**, In-xtt: **yes**)
- 4 Can handle flattenings (In-tdtt: **no**, In-xtt: **yes**)
- 5 Preservation of Recognizability (In-tdtt: **yes**, In-xtt: **yes**)
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Criterion \ Model	In-tdtt	In-xtt
FST generalization	—	X
trainable	X	X
rotations	—	X
flattenings	—	X
pres. recog.	X	X
composition	X	—

## Discriminative features

- **Finite** look-ahead
- **Epsilon** rules
- **Deep** attachment of variables

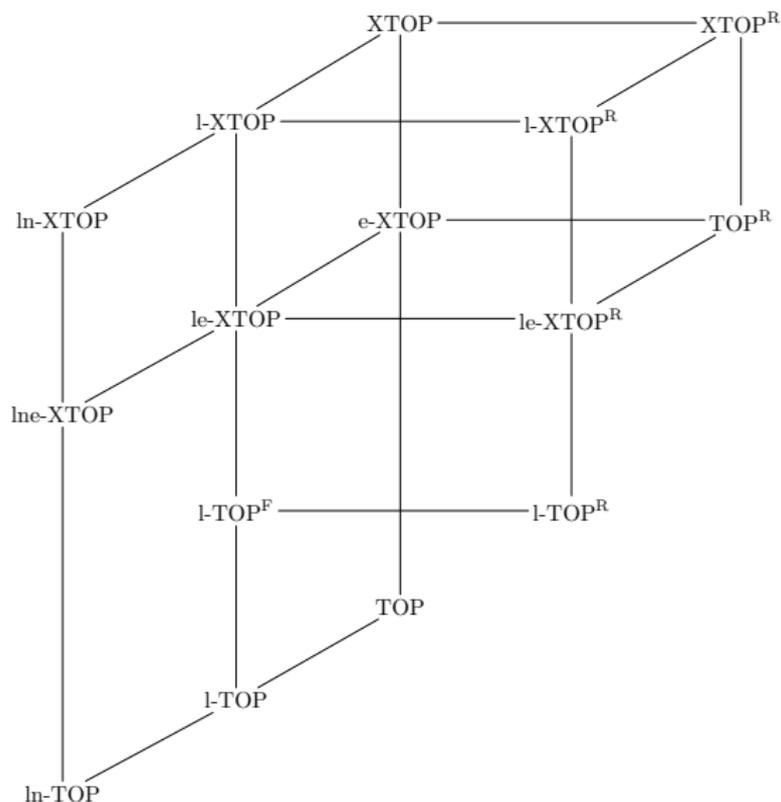
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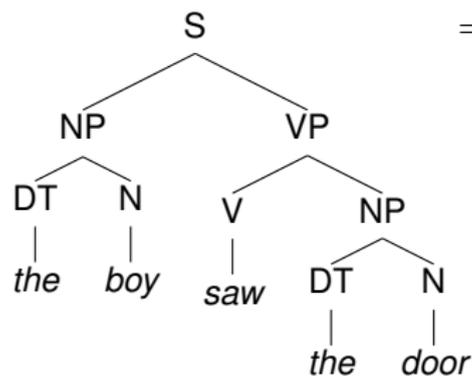
# Hasse Diagram (if the weight structure is not a ring)



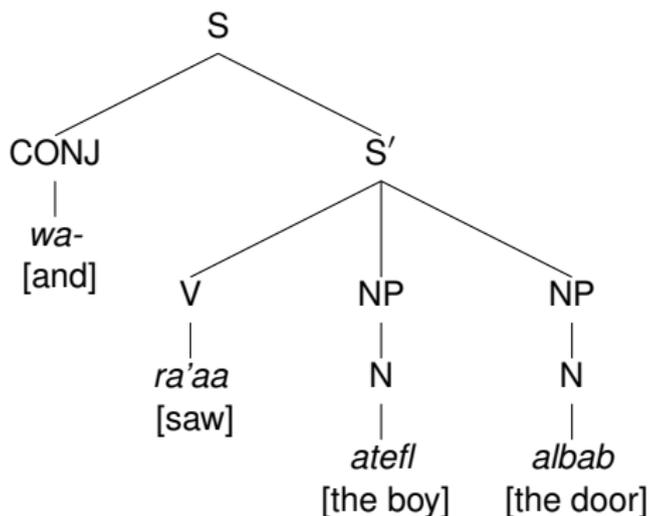
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# Training



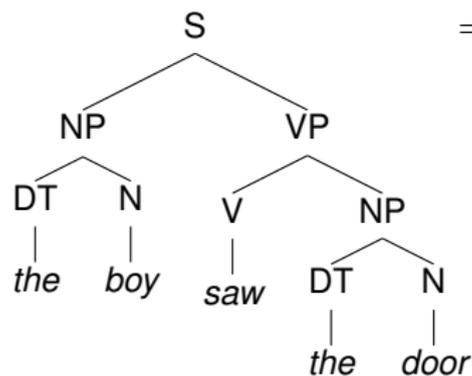
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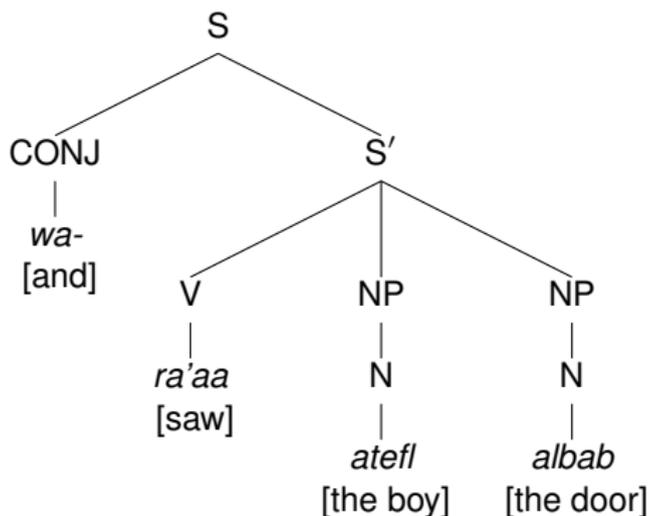
## References

- GALLEY, HOPKINS, KNIGHT, MARCU: What's in a translation rule? HLT-NAACL 2004
- GRAEHL, KNIGHT, MAY: Training tree transducers. Comput. Ling. 34, 2008

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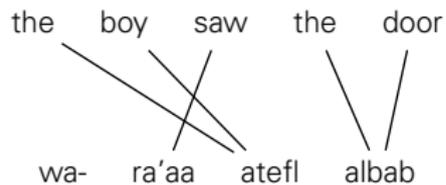
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## References

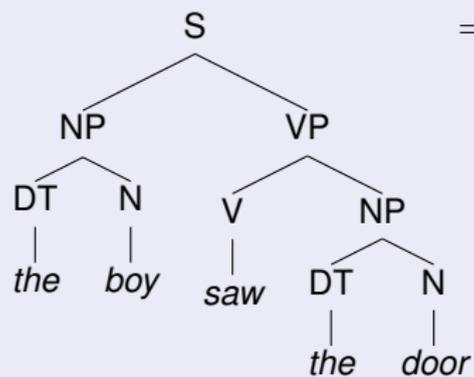
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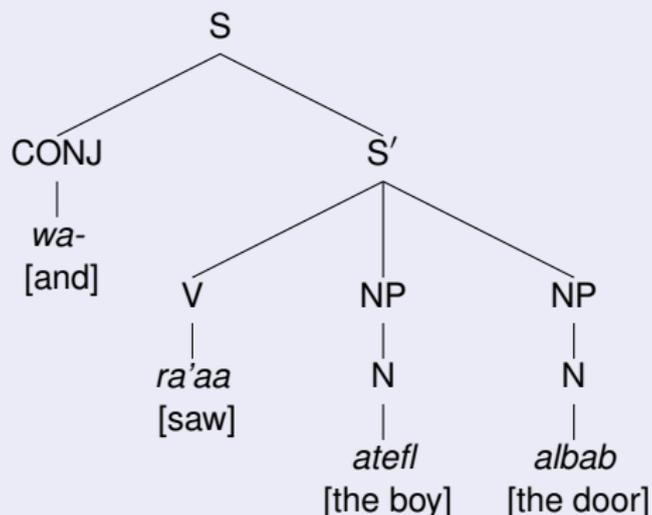


Alignment

## Generate rules

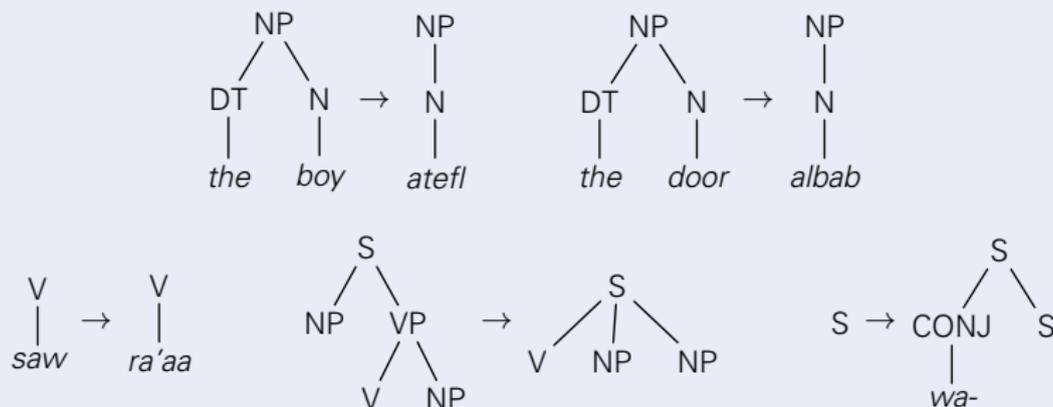


$\Rightarrow^*$



# Training (Cont'd)

## Generated STSG rules

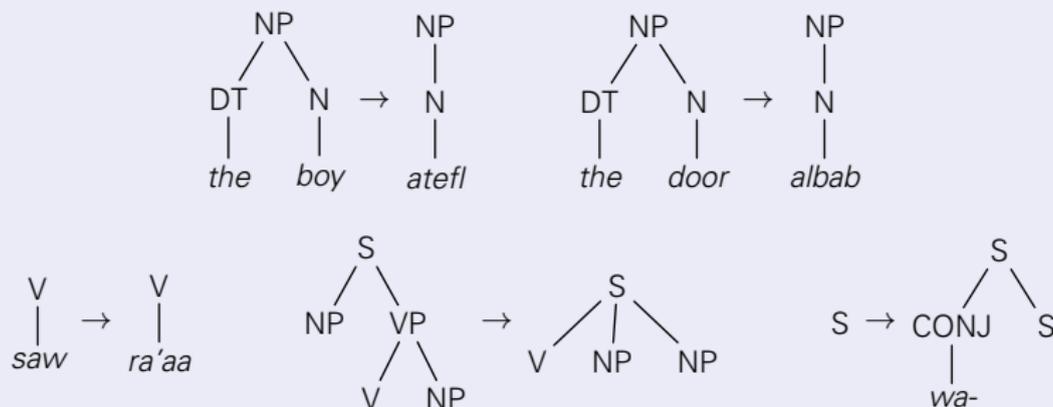


## Conclusion

- In-xtt efficiently trainable
- Can we use states? Nonlinearity? Deletion? ...

# Training (Cont'd)

## Generated STSG rules



## Conclusion

- In-xtt efficiently trainable
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# Training (Cont'd)

## Setting the weights

- count how often an obtained rule is used in a corpus (absolute frequency)
- set rule weight to relative frequency

## Optimize the weights

- EM algorithm [DEMPSTER, LAIRD, RUBIN 1977]
- Gradient descent
- etc.

# Composition

## Theorem

*Every  $1\text{-TOP} \subseteq \mathcal{L} \subseteq \text{XTOP}$  is not closed under composition.*

## Proof.

Composition closure of  $1\text{-TOP}$  is  $1\text{-TOP}^R$ . By the diagram,  
 $1\text{-TOP}^R \not\subseteq \text{XTOP}$ . □

## Reference

- **ARNOLD, DAUCHET**: Morphismes et bimorphismes d'arbres. Theoret. Comput. Sci. 20, 1982

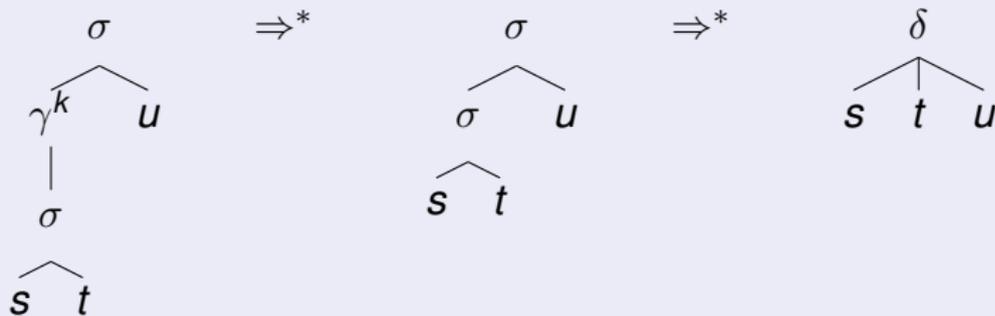
# Composition (Cont'd)

## Theorem

Every  $\text{In-TOP} \subseteq \mathcal{L} \subseteq \text{1-XTOP}^R$  that contains rotations or flattenings is not closed under composition.

## Proof.

Prove  $\text{In-TOP} ; \{\tau_{\text{flat}}\} \not\subseteq \text{1-XTOP}^R$  using, e.g.,



# Composition (Cont'd)

## Theorem

$XTOP^R$  is not closed under composition.

## Proof.

Follow classical proof for  $TOP^R$ . □

## Conclusion or Bad news

No (mentioned) class of xtt computes a closed class of transformation.

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## Problem

Compositions are extremely important (e.g., for a framework)!

## Questions

- 1 Identify suitable subclasses that are closed under composition (expressive vs. closure)
- 2 Determine whether two given l-xtt can be composed
- 3 What is the composition closure of l-XTOP
- 4 Identify superclasses that are closed under composition and still preserve recognizability (preservation vs. closure)

## Reference

- $\sim$ , GRAEHL, HOPKINS, KNIGHT: The power of extended top-down tree transducers. SIAM J. Comput. 39, 2009

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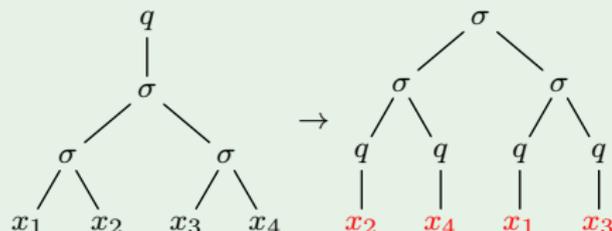
- $\sim$ , GRAEHL, HOPKINS, KNIGHT: The power of extended top-down tree transducers. SIAM J. Comput. 39, 2009

# Binarization

## Definition

A xtt is **binarized** if there are at most 3 states per rule.

## Example



## Conclusions

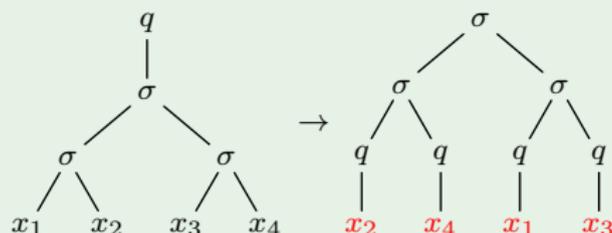
- linear xtt are **not binarizable** [AHO, ULLMAN 1972]
- What about non-linear xtt?

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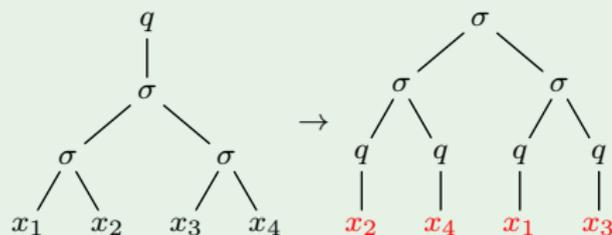


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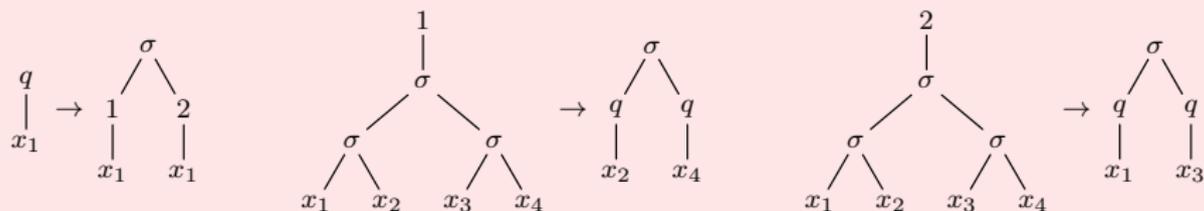
- linear xtt are **not binarizable** [AHO, ULLMAN 1972]
- What about non-linear xtt?

# Binarization (Cont'd)

## Example



## Binarization



$\Rightarrow$  xtt can be binarized using non-linearity

## Definition

Given  $\tau: T_\Sigma \times T_\Delta \rightarrow A$  and  $\varphi: T_\Sigma \rightarrow A$ , let  $\varphi \triangleleft \tau: T_\Sigma \times T_\Delta \rightarrow A$

$$(\varphi \triangleleft \tau)(t, u) = \varphi(t) \cdot \tau(t, u)$$

## Theorem

$\varphi \triangleleft \tau \in \text{n-XTOP}$  for every  $\varphi \in \text{Rec}$  and  $\tau \in \text{n-XTOP}$

## Parsing complexity

In-xtt  $M$  and input word  $w$ :  $O(|M| \cdot |w|^{2 \text{rk}(M)+5})$

## References

- $\sim$ , [SATA](#): Parsing and translation algorithms based on weighted extended tree transducers. ATANLP 2010
- $\sim$ : Why synchronous tree substitution grammars? HLT-NAACL 2010

# Input Product (Cont'd)

## Deleting xtt

How to obtain input products for deleting xtt?

## Partial solutions

- for idempotent semirings
- for rings

but they do not work for the (non-linear) xtt obtained from binarization

## References

- $\sim$ : Input products for weighted extended top-down tree transducers. DLT 2010

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# Preservation of Recognizability

## Definition

Given  $\tau: T_\Sigma \times T_\Delta \rightarrow A$  and  $\varphi: T_\Sigma \rightarrow A$ , let  $\tau(\varphi), \text{range}(\tau): T_\Delta \rightarrow A$

$$(\tau(\varphi))(u) = \sum_{t \in T_\Sigma} \varphi(t) \cdot \tau(t, u)$$

$$(\text{range}(\tau))(u) = \sum_{t \in T_\Sigma} \tau(t, u)$$

## References

- FÜLÖP, ~, VOGLER: Backward and forward application of extended tree series transformations. WATA 2010
- MAY, KNIGHT, VOGLER: Efficient inference through cascades of weighted tree transducers. ACL 2010

# Preservation of Recognizability (Cont'd)

## Theorem

Given  $\tau: T_\Sigma \times T_\Delta \rightarrow A$  and  $\varphi: T_\Sigma \rightarrow A$

$$\tau(\varphi) = \text{range}(\varphi \triangleleft \tau)$$

## Proof.

$$\begin{aligned}(\tau(\varphi))(u) &= \sum_{t \in T_\Sigma} \varphi(t) \cdot \tau(t, u) \\ &= \sum_{t \in T_\Sigma} (\varphi \triangleleft \tau)(t, u) \\ &= (\text{range}(\varphi \triangleleft \tau))(u)\end{aligned}$$



# Table of Contents

- 1 Machine Translation
- 2 Weighted Tree Transducer
- 3 Expressive Power
- 4 Standard Algorithms
- 5 Implementation**

# Tiburon

## Features

- Implements xtt (and tree automata; everything also weighted)
- Framework with command-line interface
- Optimized for machine translation

## Algorithms

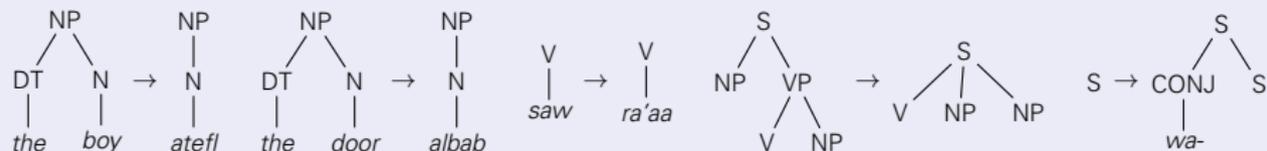
- Application of xtt to input tree/language
- Backward application of xtt to output language
- Composition (for some xtt)
- ...

## Reference

- **MAY, KNIGHT**: Tiburon: A Weighted Tree Automata Toolkit. CIAA 2006

# Tiburon (Cont'd)

## Generated STSG rules



## Example

q  
qNP.NP (DT (the) N (boy))  $\rightarrow$  NP (N (atefl))  
qNP.NP (DT (the) N (door))  $\rightarrow$  NP (N (albab))  
qV.V (saw)  $\rightarrow$  V (ra'aa)  
qS.S (x0: VP (x1: x2:))  $\rightarrow$  S (qV.x1 qNP.x0 qNP.x2)  
q.x0:  $\rightarrow$  S (CONJ (wa-) qS.x0)

# Summary

## Criteria

- (a) Generalize FST; in particular, epsilon-transitions
- (b) Efficient training
- (c) Handles rotation
- (d) Closed under composition
- (e) Preserves recognizability

## Models

Model \ Criterion	(a)	(b)	(c)	(d)	(e)
Top-down tree transducer	—	x	—	x	x
Synchronous context-free grammar	x	x	—	x	x
Synchronous tree substitution grammar	x	x	x	—	x
Synchronous tree adjoining grammar	x	x	x	—	—
Multi bottom-up tree transducer	x	?	x	x	—

# References

- **ARNOLD, DAUCHET**: Bi-transductions de forêts. ICALP 1976
- **BAKER**: Composition of top-down and bottom-up tree transducers. *Inform. Control* 41. 1979
- **ENGELFRIET**: Bottom-up and top-down tree transformations—a comparison. *Math. Syst. Theory* 9. 1975
- **ENGELFRIET**: Top-down tree transducers with regular look-ahead. *Math. Syst. Theory* 10. 1976
- **MAY, KNIGHT**: Tiburon: A Weighted Tree Automata Toolkit. CIAA 2006
- ~, **GRAEHL, HOPKINS, KNIGHT**: The power of extended top-down tree transducers. *SIAM J. Comput.* 2009

Thank You for your attention!