Weighted Multi Bottom-up Tree Transducers

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Synchronous Tree Substitution Grammars

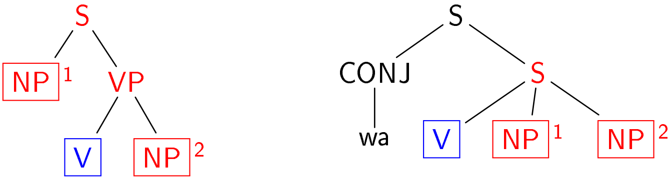
$\boxed{S}$ $\boxed{S}$

Weight: 1

Synchronous Tree Substitution Grammars

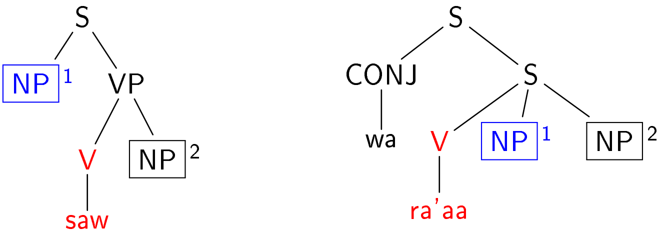
Weight: $1 \cdot 0.5$

Synchronous Tree Substitution Grammars



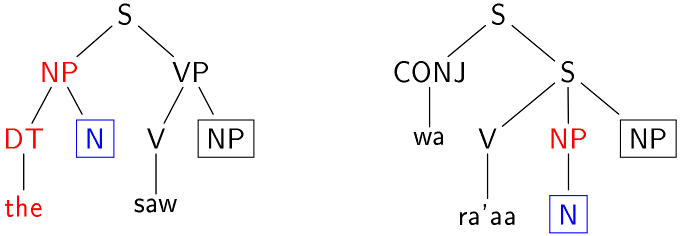
Weight: $1 \cdot 0.5 \cdot 0.25$

Synchronous Tree Substitution Grammars



Weight: $1 \cdot 0.5 \cdot 0.25 \cdot 0.03$

Synchronous Tree Substitution Grammars



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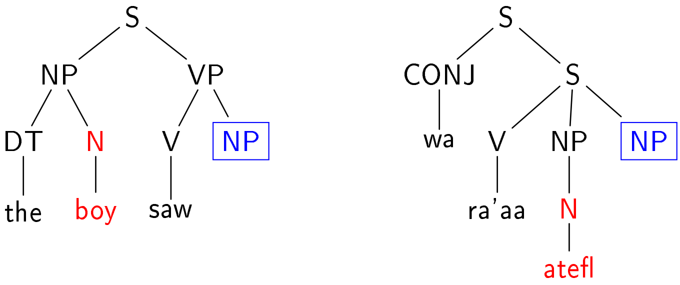

Weight: $1 \cdot 0.5 \cdot 0.25 \cdot 0.03 \cdot 0.25 \cdot 0.1$

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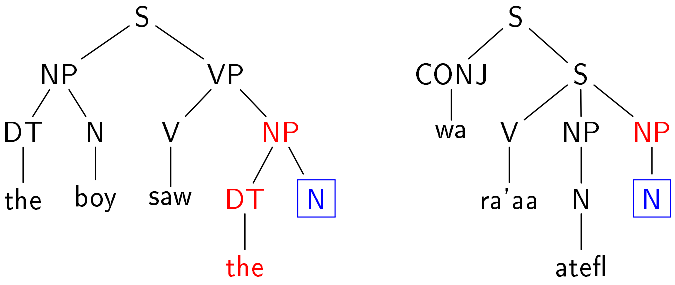

Weight: $1 \cdot 0.5 \cdot 0.25 \cdot 0.03 \cdot 0.25 \cdot 0.1 \cdot 0.25$

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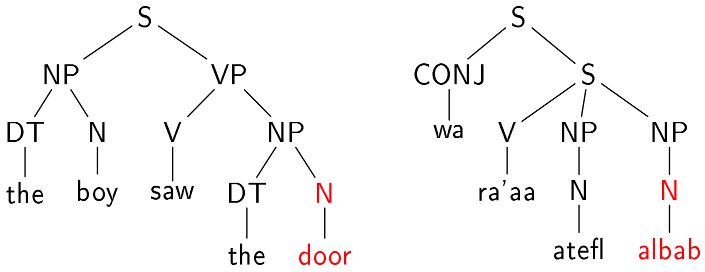

Weight: $1 \cdot 0.5 \cdot 0.25 \cdot 0.03 \cdot 0.25 \cdot 0.1 \cdot 0.25 \cdot 0.05$

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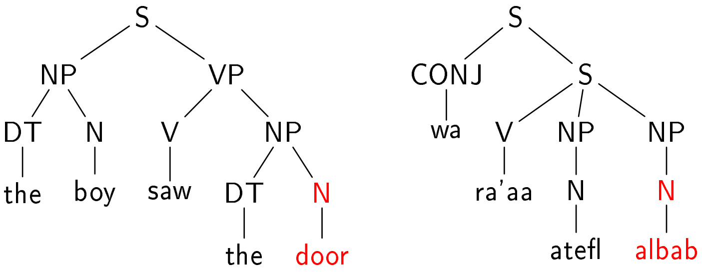

Weight: $1 \cdot 0.5 \cdot 0.25 \cdot 0.03 \cdot 0.25 \cdot 0.1 \cdot 0.25 \cdot 0.05$

Note

Popular model in machine translation.

Synchronous Tree Substitution Grammars (cont'd)

Advantages

- simple and natural model
- easy to train (from linguistic resources)
- symmetric

(Obvious) Disadvantages

- computes joint-probability ($\rightarrow$ *generative story*)
- no state behavior ($\rightarrow$ *local behavior*)

Implementation

- extended top-down tree transducer in Tiburon [May, Knight '06]

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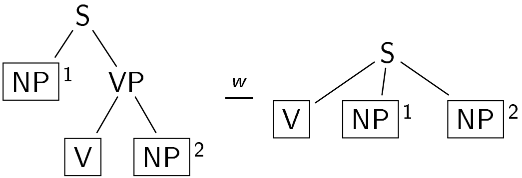
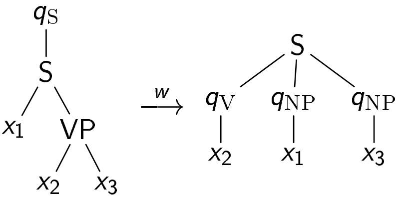
Synchronous Tree Substitution Grammars (cont'd)

Synchronous tree substitution grammar rule:



Corresponding extended top-down tree transducer rule:



Extended Top-down Tree Transducer

Advantages

- input-driven model (can easily compute conditional probability)
- state behavior

Disadvantages (also of STSG)

- not binarizable
  [Aho, Ullman '72; Zhang, Huang, Gildea, Knight '06]
- inefficient input/output restriction (Bar-Hillel construction)
  [M., Satta '10]
- not composable
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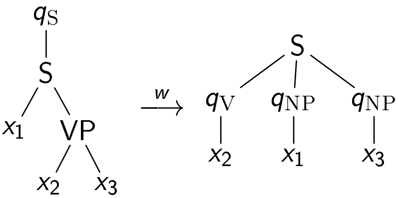
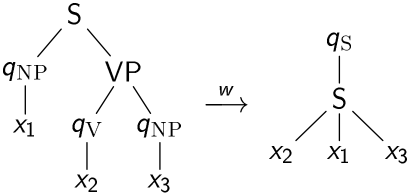
Extended Bottom-up Tree Transducer

Top-down tree transducer rule:



Corresponding extended bottom-up tree transducer rule:



Extended Bottom-up Tree Transducer (cont'd)

Theorem

For every STSG we can construct an equivalent extended bottom-up tree transducer in linear time.

Question

Do they have better properties?

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Roadmap

Syntax

Convention

Fix a commutative semiring $(S, +, \cdot, 0, 1)$.

Definition

Weighted extended multi bottom-up tree transducer (XMBOT) is a system $(Q, \Sigma, \Delta, F, R)$ with

- Q ranked alphabet of *states*
- Σ and Δ ranked alphabets of input and output symbols
- $F \subseteq Q_1$ final states
- R finite set of rules $l \xrightarrow{w} r$ with $w \in S$, linear $l \in T_\Sigma(Q(X))$, linear $r \in Q(T_\Delta(X))$ such that $\mathrm{var}(l) = \mathrm{var}(r)$

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Syntax (cont'd)

Definition

XMBOT $(Q, \Sigma, \Delta, F, R)$ is proper if $\{l, r\} \not\subseteq Q(X)$ for every $l \xrightarrow{w} r \in R$.

Syntax (cont'd)

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Example

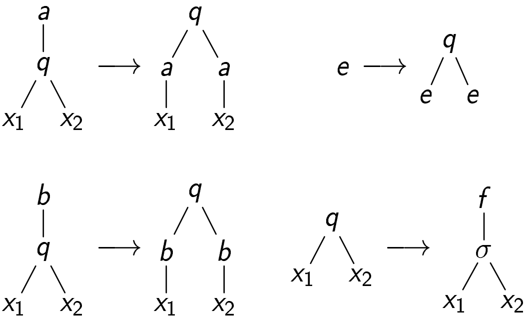
Disallowed rule for properness:

$$p(x_1, x_2, x_3) \xrightarrow{w} q(x_2, x_1, x_3)$$

Syntax — An Example

Example

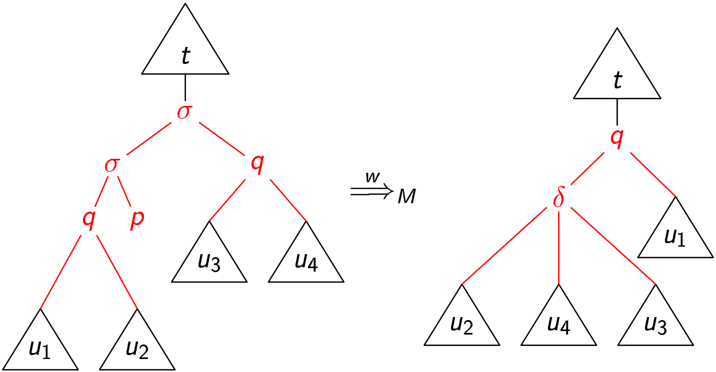
- $Q = \{f^{(1)}, q^{(2)}\}$ and $F = \{f\}$
- $\Sigma = \{a^{(1)}, b^{(1)}, e^{(0)}\}$ and $\Delta = \Sigma \cup \{\sigma^{(2)}\}$
- the following rules (all with weight 1)



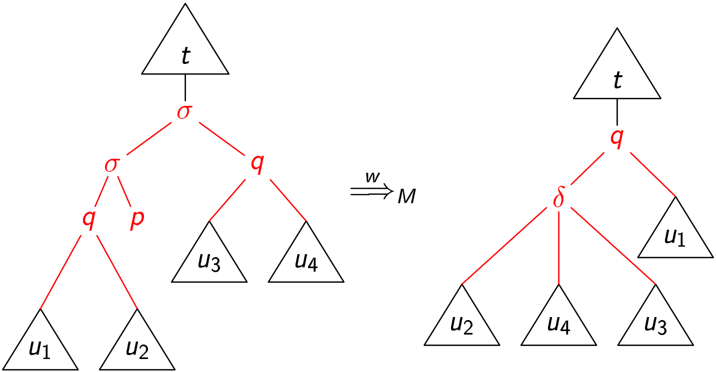
Semantics

$$\sigma(\sigma(q(x_1, x_2), p), q(x_3, x_4)) \xrightarrow{w} q(\delta(x_2, x_4, x_3), x_1)$$

Semantics



Semantics



Semantics

- $\mathrm{wt}(\xi_1 \overset{w_1}{\Longrightarrow}_M \cdots \overset{w_{n-1}}{\Longrightarrow}_M \xi_n) = w_1 \cdot \ldots \cdot w_{n-1}$
- $\mathrm{wt}(t, u) = \sum_{q \in F, d \colon t \Longrightarrow^*_M q(u)} \mathrm{wt}(d)$

Semantics — An Example

Example

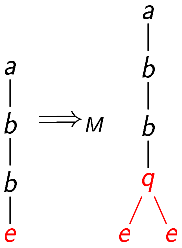
$$
\begin{array}{c}
a \\
| \\
b \\
| \\
b \\
| \\
e
\end{array}
$$

Remark

Properness guarantees well-definedness.

Semantics — An Example

Example



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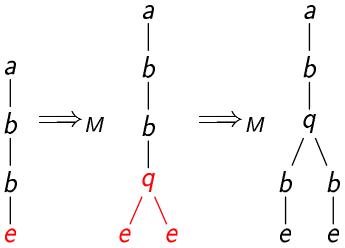

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Semantics — An Example

Example

$$a(b(b(\color{red}{e}))) \Longrightarrow_M a(b(b(\color{red}{q(e, e)}))) \Longrightarrow_M a(b(q(b(e), b(e)))) \Longrightarrow_M a(q(b(b(e)), b(b(e))))$$

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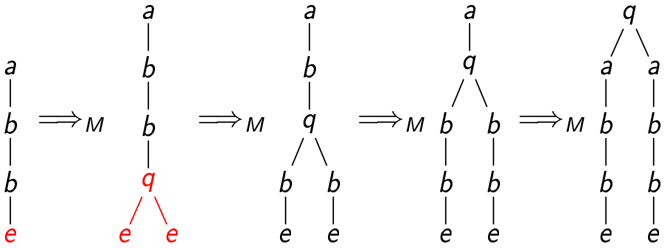
Semantics — An Example

Example



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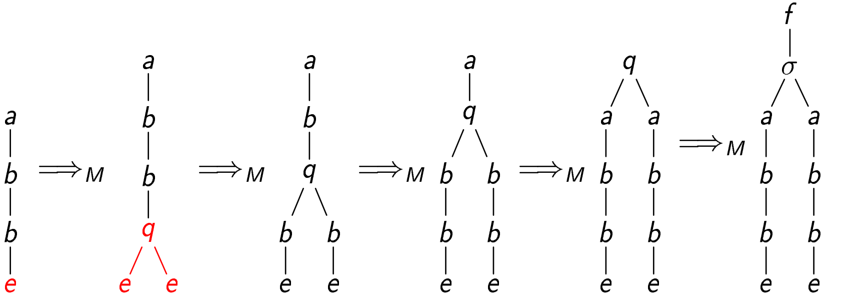

Remark

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Roadmap

One-Symbol Normal Form

Definition

XMBOT $(Q, \Sigma, \Delta, F, R)$ is in one-symbol normal form
if exactly one input or output symbol occurs in each rule.

Theorem

For every proper XMBOT there exists an equivalent XMBOT in one-symbol normal form. It can be constructed in linear time.

Corollary

For every proper XMBOT the transition from joint-distribution to conditional-distribution is linear time.

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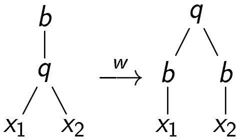
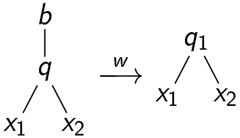
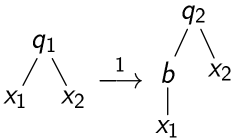
One-Symbol Normal Form (cont'd)

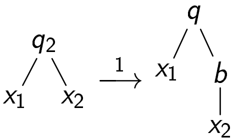
Rule not in one-symbol normal form:



Replacement rules for this rule:







Binarization

Definition

An XMBOT is fully binarized if each rule contains at most 3 states.
(≤ 2 in each left-hand side)

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Every proper XMBOT can be fully binarized in linear time.

Proof.

First binarize the trees in the rules and then transform into one-symbol normal form. □

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Binarization (cont'd)

$$\sigma(\sigma(x_1, x_2), \sigma(x_3, x_4)) \xrightarrow{w} \sigma(\sigma(x_3, x_1), \sigma(x_4, x_2))$$

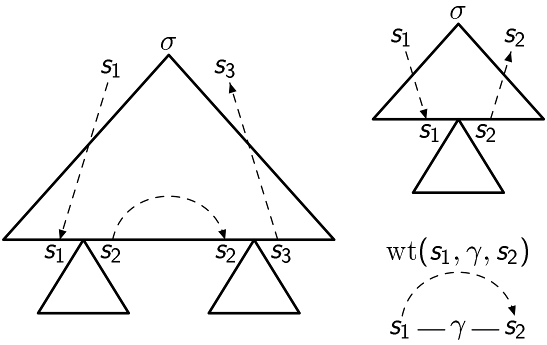
Comparison

In general, STSG cannot be binarized, but people try . . .
[Zhang, Huang, Gildea, Knight '06; DeNero, Pauls, Klein '09]

Bar-Hillel Construction

Definition

The input product of a weighted tree transformation $\tau\colon T_\Sigma \times T_\Delta \to S$ with a power series $\varphi\colon \Sigma^* \to S$ is $\tau'(s, t) = \tau(s, t) \cdot \varphi(\mathrm{yd}(s))$.



Bar-Hillel Construction (cont'd)

Theorem

The input product of an XMBOT M with a WSA S can be computed in time $O(|M| \cdot |S|^3)$.

Note

The output product of an XMBOT M with a WSA S can be computed in time $O(|M| \cdot |S|^{2\,\mathrm{rk}(M)+2})$.

Bar-Hillel Construction (cont'd)

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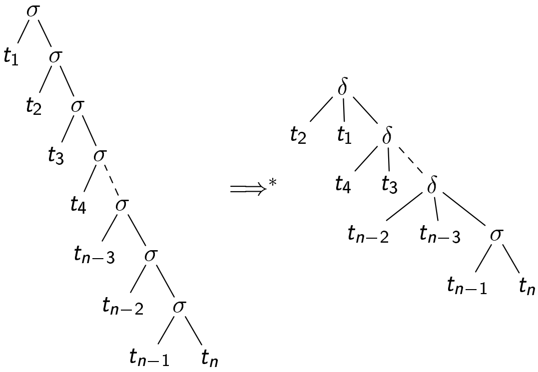
Roadmap

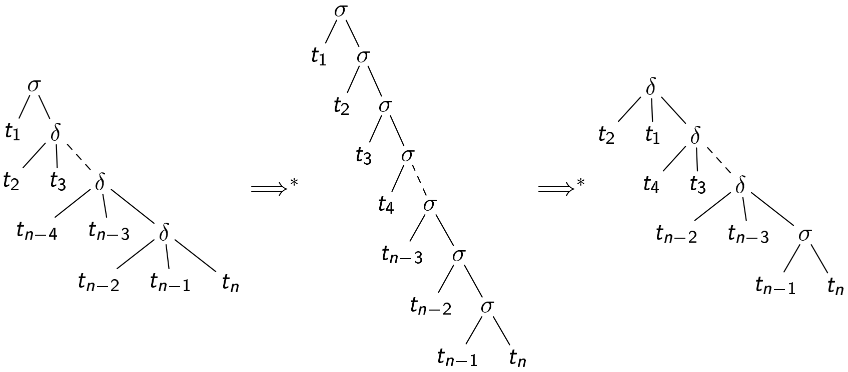
Composition of STSG



Conclusion

STSGs are not composable!

Composition of STSG



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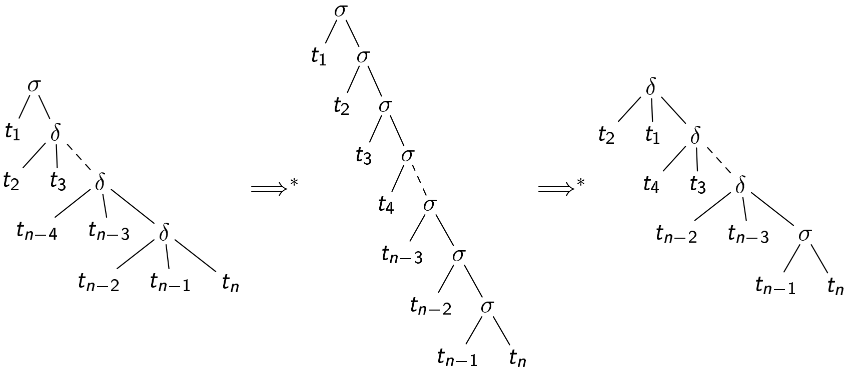

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Composition Construction

Definition

for XMBOT $M = (Q, \Sigma, \Gamma, F, R)$ and $N = (Q', \Gamma, \Delta, G, P)$ construct

$$M \,;\, N = (Q(Q'), \Sigma, \Delta, F(G), R')$$

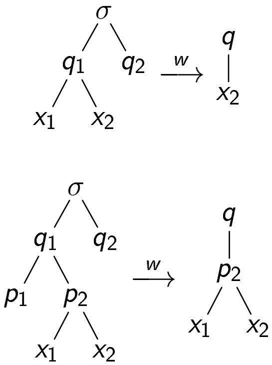
with three types of rules:

1. input-consuming rules constructed from input-consuming rules of R (with their weight)
2. epsilon rules constructed from epsilon-rules of P
3. epsilon rules constructed from an epsilon rule of R followed by an input consuming rule of P (product of the weights)

Composition construction (cont'd)

Example

Input consuming rule of R and resulting rule:



Composition construction (cont'd)

Example

Epsilon rule of P and resulting rule:

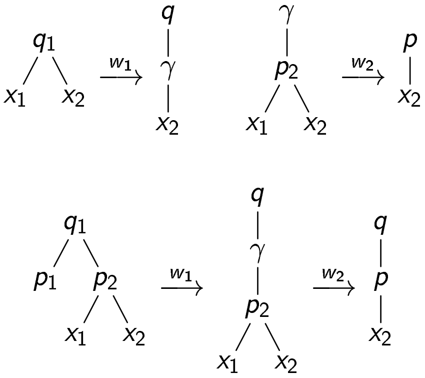
$$p_1 \xrightarrow{w} p(\alpha)$$

$$q_1(p_1, p_2(x_1, x_2)) \xrightarrow{w} q_1(p(\alpha), p_2(x_1, x_2))$$

Composition construction (cont'd)

Example

Epsilon rule of R and input consuming of P and resulting rule:



Composition construction (cont'd)

Note

The constructed XMBOT might be non-proper.

Theorem

For all proper XMBOTs M and N such that

- *M has no cyclic input epsilon rules or*
- *N has no cyclic output epsilon rules,*

then there exists a proper XMBOT that computes the composition of the transformations computed by M and N.

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Summary

Algorithm \ Device	STSG	XMBOT
Binarization		$O(\lvert M\rvert)$
Input product	$O(\lvert M\rvert \cdot \lvert S\rvert^{2\,\mathrm{rk}(M)+5})$	$O(\lvert M\rvert \cdot \lvert S\rvert^{3})$
Output product	$O(\lvert M\rvert \cdot \lvert S\rvert^{2\,\mathrm{rk}(M)+5})$	$O(\lvert M\rvert \cdot \lvert S\rvert^{2\,\mathrm{rk}(M)+2})$
Composition		$O(\lvert M_1\rvert \cdot \lvert M_2\rvert^{\mathrm{rk}(M_1)+1})$

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Algorithm \ Device	STSG	XMBOT
Binarization		$O(\|M\|)$
Input product	$O(\|M\| \cdot \|S\|^{2\,\mathrm{rk}(M)+5})$	$O(\|M\| \cdot \|S\|^3)$
Output product	$O(\|M\| \cdot \|S\|^{2\,\mathrm{rk}(M)+5})$	$O(\|M\| \cdot \|S\|^{2\,\mathrm{rk}(M)+2})$
Composition		$O(\|M_1\| \cdot \|M_2\|^{\mathrm{rk}(M_1)+1})$
Reversal	$O(\|M\|)$	
Pres. of REC		

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Thank you for your attention!