Compositions of Top-down Tree Transducers with ε-Rules

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Synchronous Tree Substitution Grammars

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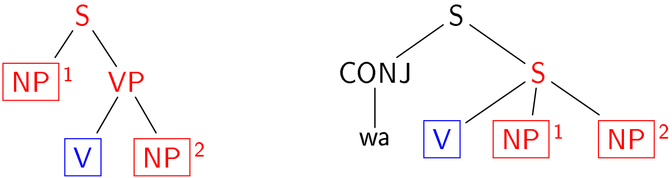

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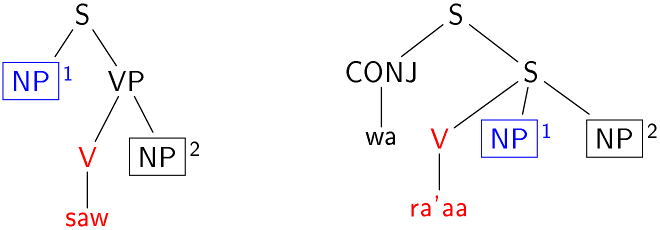

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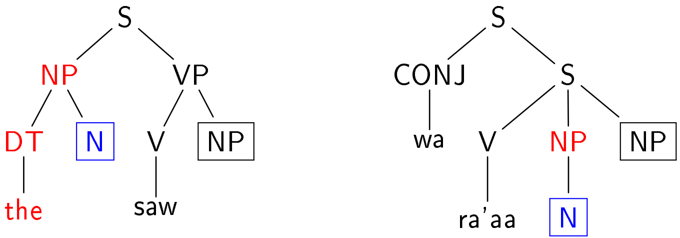

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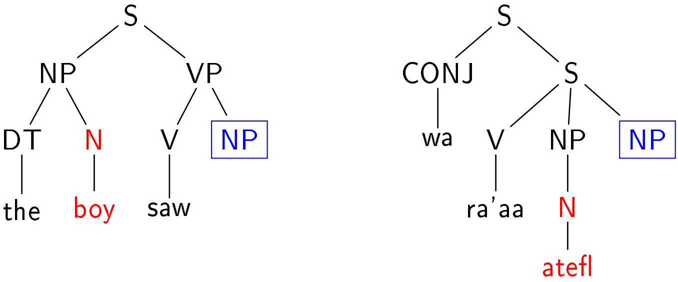

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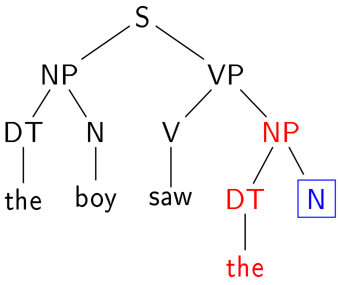

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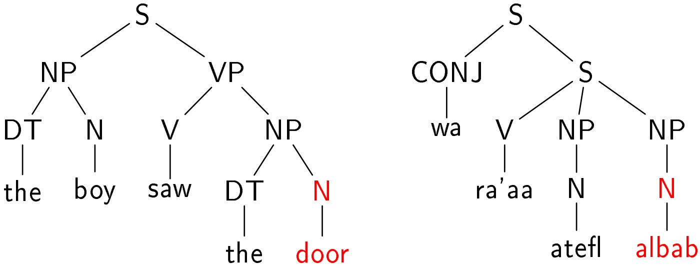

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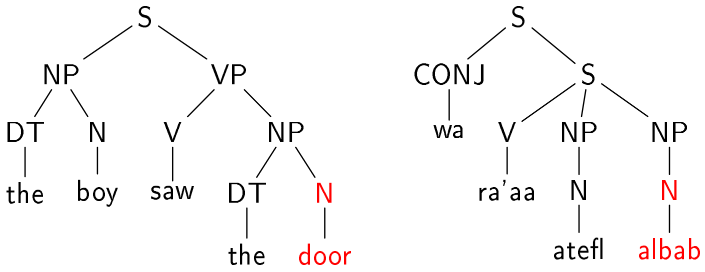
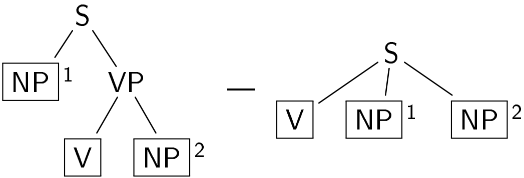


Implementation

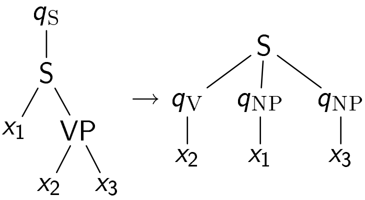
- input-oriented with hidden states ($\longrightarrow$ tree transducer)
- linear nondeleting extended top-down tree transducer in Tiburon [May, Knight '06]

Synchronous Tree Substitution Grammars (cont'd)

Synchronous tree substitution grammar rule:

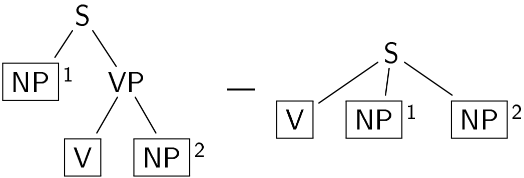


Tree transducer rule:



Synchronous Tree Substitution Grammars (cont'd)

Synchronous tree substitution grammar rule:



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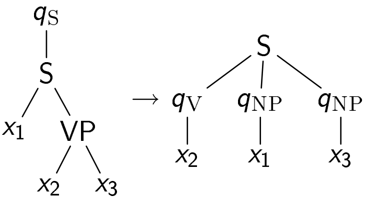
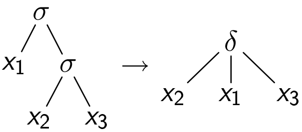
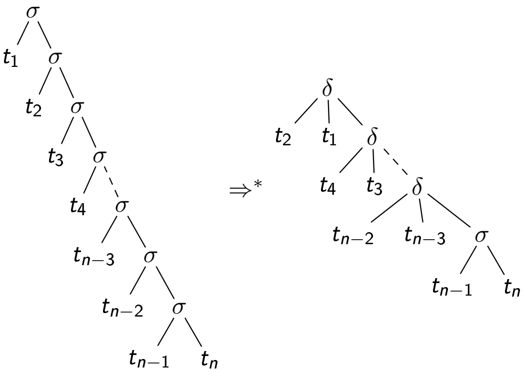


Simplified transducer rule:



Synchronous Tree Substitution Grammars (cont'd)

Derivation:



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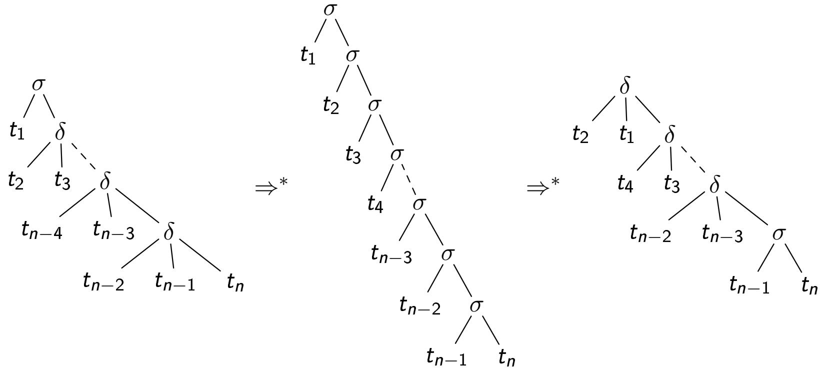

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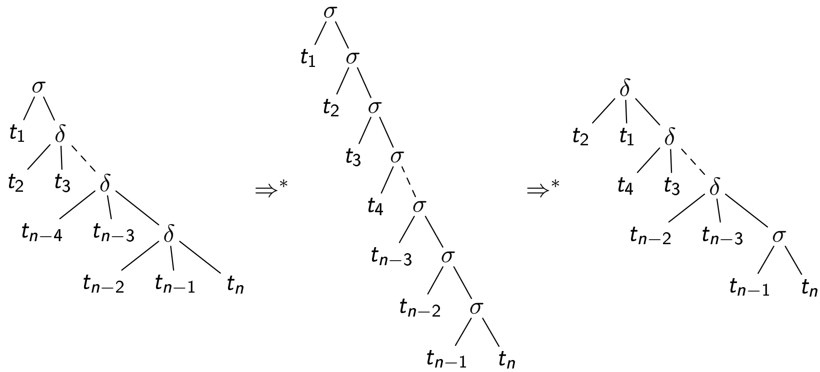


Conclusion

We cannot compose them!

Synchronous Tree Substitution Grammars (cont'd)

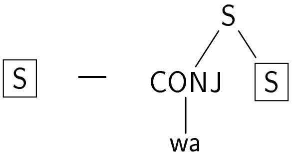
Why?

- linear nondeleting top-down tree transducers can be composed [Engelfriet '75, Baker '79]

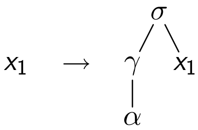
$\longrightarrow$ large left-hand sides cause problems

$\longrightarrow$ What about ε-rules?

STSG rule:



Simplified tree transducer rule:



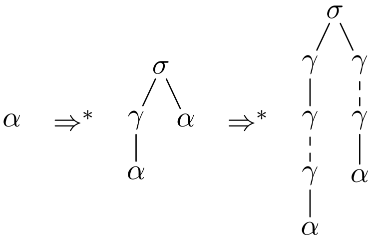
Synchronous Tree Substitution Grammars (cont'd)

Derivation:

$$\alpha \Rightarrow^* \sigma(\gamma(\alpha), \alpha)$$

Synchronous Tree Substitution Grammars (cont'd)

Derivation:



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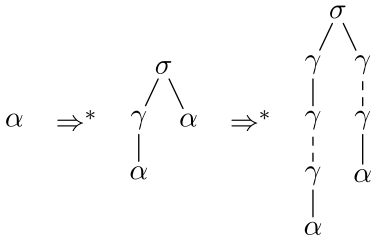
Derivation:



Conclusion

We cannot even compose linear nondeleting top-down tree transducers with ε-rules!

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Syntax

Definition (cf. [Rounds '70] & [Thatcher '70])

Top-down tree transducer (εtdtt) with ε-rules $(Q, \Sigma, \Delta, I, R)$

- Q alphabet of *states*
- Σ and Δ ranked alphabets of input and output symbols
- $I \subseteq Q$ *initial states*
- R finite set of *(rewrite) rules* $q(l) \to r$ with
  - (i) $q \in Q$,
  - (ii) $l = x_1$ or $l = \sigma(x_1, \ldots, x_k)$ for some $\sigma \in \Sigma_k$, and
  - (iii) $r \in T_\Delta(Q(\text{var}(l)))$

A full example

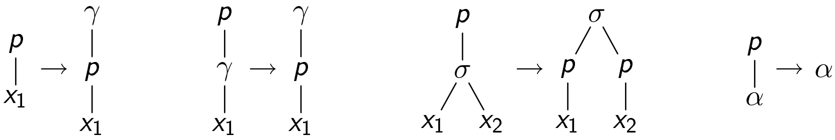
Example

εtdtt $M = (P, \Sigma, \Sigma, P, R)$ with

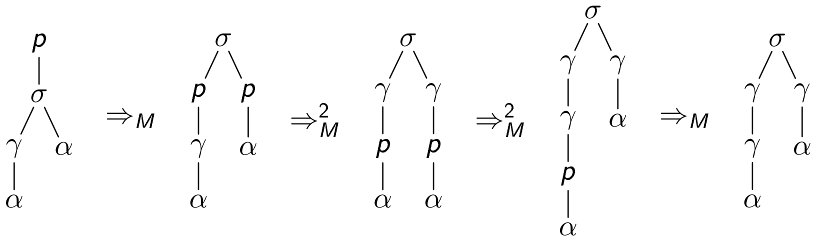
- $P = \{p\}$
- $\Sigma = \{\sigma^{(2)}, \gamma^{(1)}, \alpha^{(0)}\}$
- the following rules in R

$$p(x_1) \to \gamma(p(x_1)) \qquad p(\gamma(x_1)) \to \gamma(p(x_1)) \qquad p(\sigma(x_1, x_2)) \to \sigma(p(x_1), p(x_2)) \qquad p(\alpha) \to \alpha$$

A full example (cont'd)



Example



Semantics

Definition

$M = (Q, \Sigma, \Delta, I, R)$ εtdtt

$$\tau_M = \{(t, u) \mid \exists q \in I \colon q(t) \Rightarrow_M^* u\}$$

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Example

Our example εtdtt can include γ-symbols anywhere in the input tree.

Syntactical restrictions

Definition

εtdtt $M = (Q, \Sigma, \Delta, I, R)$ is

- linear if every right-hand side (of a rule) does not contain duplicate variables
- nondeleting, if every right-hand side contains all variables of its left-hand side
- total if for every $q \in Q$ and $t \in T_\Sigma$ there exists $u \in T_\Delta$ such that $q(t) \Rightarrow^*_M u$

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Example

Our example εtdtt is linear, nondeleting, and total.

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One-symbol form

Definition (cf. [Berstel '79] & [Engelfriet et al '08])

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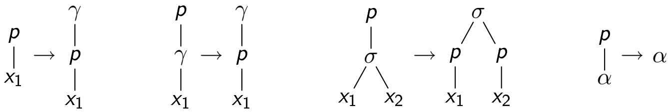

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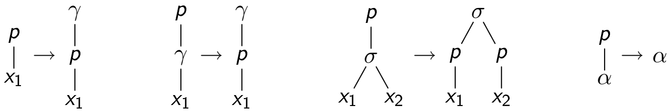


Theorem

For every εtdtt we can construct an equivalent one in one-symbol form.

Maximally output-separated

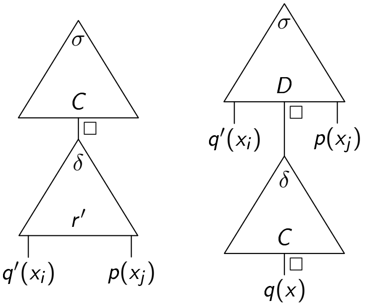
Definition

An εtdtt is maximally output-separated if for every rule $q(l) \to r \in R$

(i) $r \neq C[r']$ for every nontrivial context $C \in C_\Delta$ and $r' \in \Delta(T_\Delta(Q(X)))$,

(ii) $r \neq D[C[q(x)]]$ for every nontrivial context $D \in C_\Delta(Q(X))$ and nontrivial context $C \in C_\Delta$.

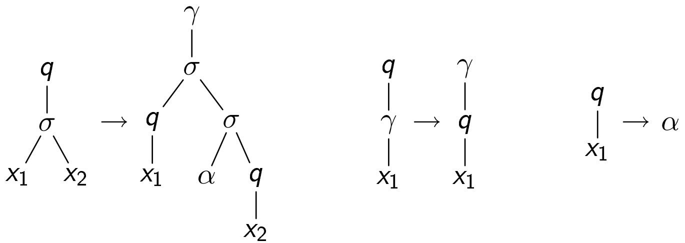
Illustration



(i) (ii)

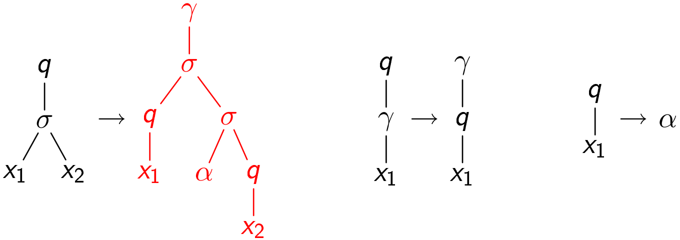
Maximally output-separated (cont'd)

Example



Maximally output-separated (cont'd)

Example



is not maximally output-separated because

(i) $r = C[r']$ with $C = \gamma(\Box)$ and $r' = \sigma(q(x_1), \sigma(\alpha, q(x_2)))$

(ii) $r = D[C[q(x_2)]]$ with $D = \gamma(\sigma(q(x_1), \Box))$ and $C = \sigma(\alpha, \Box)$

Maximally output-separated (cont'd)

Theorem

For every (linear, nondeleting) εtdtt there exists an equivalent maximally output-separated εtdtt with the same properties.

Proof.

Separate the context C from a rule creating two rules, which need to be applied one after the other by introducing a new state. $\square$

Maximally output-separated (cont'd)

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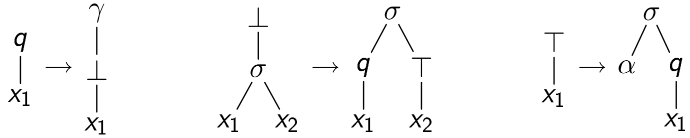
Example



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Composition construction

Definition

εtdtt $M = (P, \Sigma, \Gamma, I_1, R_1)$ and $N = (Q, \Gamma, \Delta, I_2, R_2)$ construct

$$M ; N = (Q \times P, \Sigma, \Delta, I_2 \times I_1, R_1' \cup R_2' \cup R')$$

with 3 types of rules:

1. rules R_1' constructed from rules of R_1 that do not produce output

$$R_1' = \{q(l) \to q(r) \mid q \in Q, l \to r \in R_1, r \in P(X)\}$$

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with 3 types of rules:

1. rules R_1' constructed from rules of R_1 that do not produce output
2. epsilon rules R_2' constructed from epsilon-rules of R_2

$$R_2' = \{l[p(x_1)] \to r[p(x_1)] \mid p \in P, l \to r \in R_2, l \in Q(X)\}$$

Composition construction

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$$M \,;\, N = (Q \times P, \Sigma, \Delta, I_2 \times I_1, R_1' \cup R_2' \cup R')$$

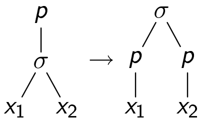
with 3 types of rules:

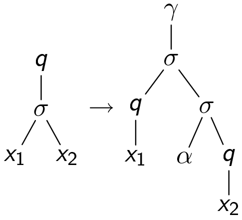
1. rules R_1' constructed from rules of R_1 that do not produce output
2. epsilon rules R_2' constructed from epsilon-rules of R_2
3. rules R' constructed from rules of R_1 that contain an output symbol that is immediately consumed by a rule of R_2

$$R' = \{q(l) \to r' \mid q \in Q, l \to r \in R_1, r \in \Gamma(P(X)), q(r) \Rightarrow_N r'\}$$

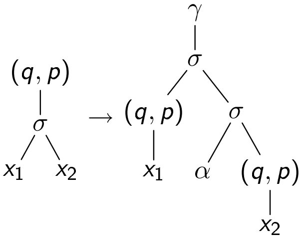
Composition construction (cont'd)

Example





Resulting rule of R':



Composition

Theorem (cf. [Engelfriet '75] & [Baker '79])

Let M and N be εtdtts. Then $M\,;N$ computes $\tau_M\,;\tau_N$ if

- *N is linear,*
- *M is total or N is nondeleting, and*
- *M is in one-symbol form.*

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Consequences

- The class of transformations computed by linear nondeleting εtdtt in one-symbol form is closed under composition.

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Consequences

- The class of transformations computed by linear nondeleting εtdtt in one-symbol form is closed under composition.
- The class of transformations computed by synchronous context-free grammars (incl. chain rules) is closed under composition.

Composition (cont'd)

Theorem

Any composition of linear nondeleting εtdtts can be simulated by an εtdtt.

$$\text{ln-eTOP} ; \cdots ; \text{ln-eTOP} \subseteq \text{eTOP}$$

Proof.

1. Take first εtdtt into one-symbol form (losing linearity and nondeletion)
2. Compose with next linear and nondeleting εtdtt
3. Result computes composition of first 2 εtdtts
4. Repeat □

References

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- Rounds: Mappings and grammars on trees. *Math. Systems Theory* 4(3), 1970
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Thank you for your attention!