Extended Multi Bottom-up Tree Transducers

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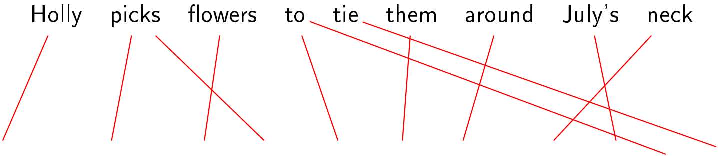
Kyoto — September 16, 2008

Classic Approach (simplified)

Source: Google Translate



Steps

1. Select scheme
2. Translate words individually
3. Rescore using word sequences (bigrams, trigrams, etc.)

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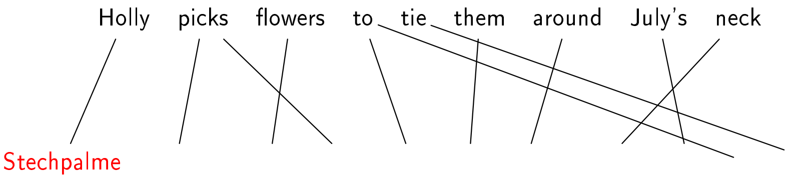

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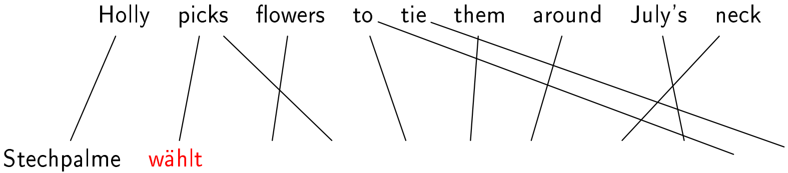

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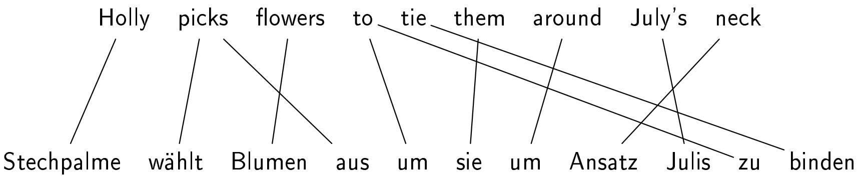

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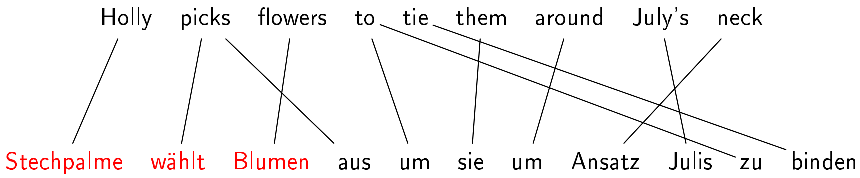

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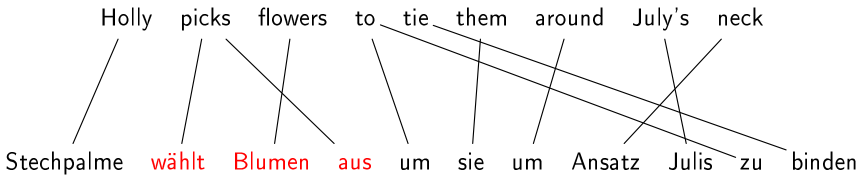

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Syntax-based Approach

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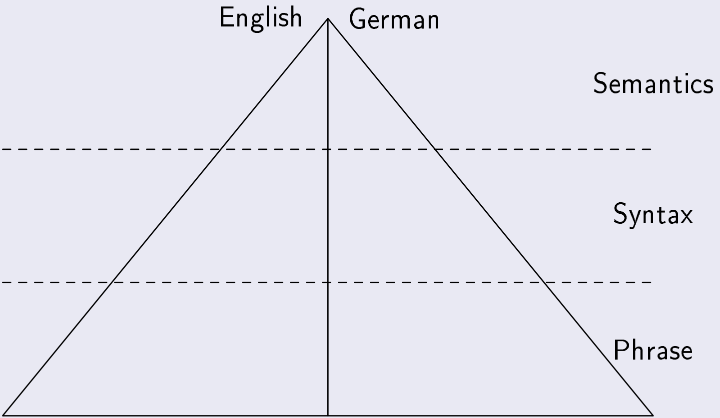

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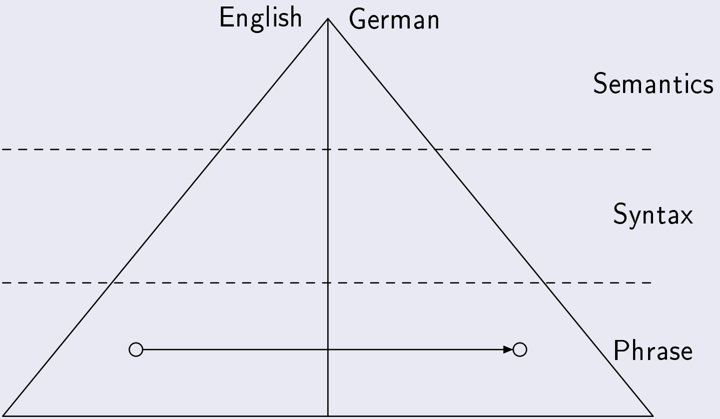

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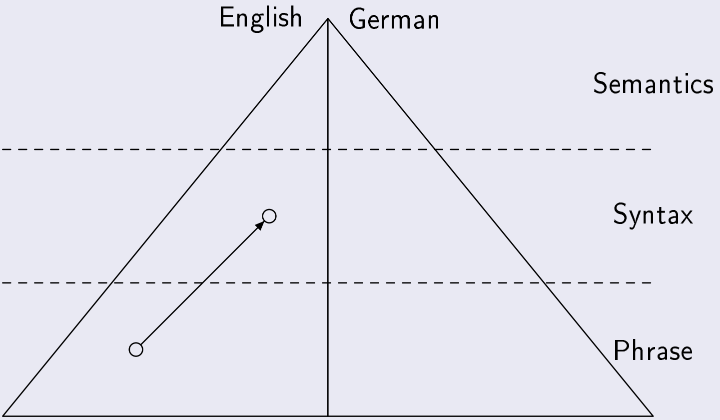

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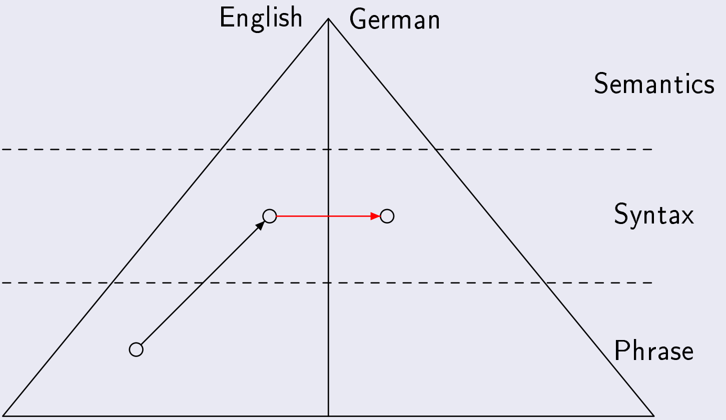

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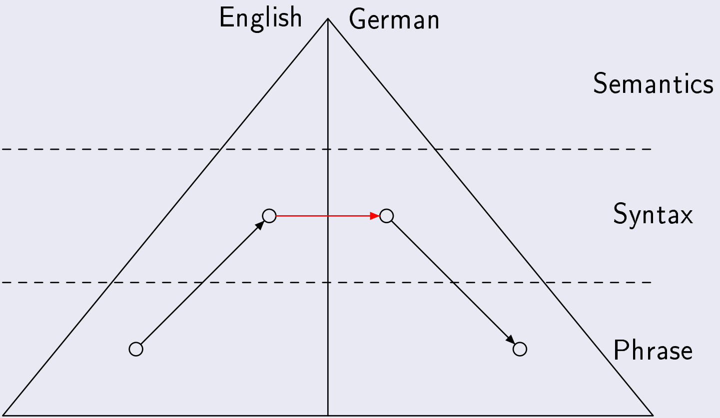

Syntax-based Approach (cont'd)

Holly picks flowers to tie them around July's neck

$\Downarrow$

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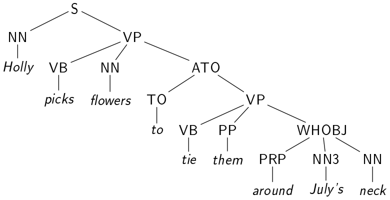

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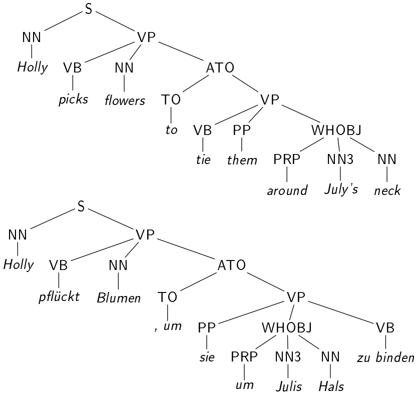

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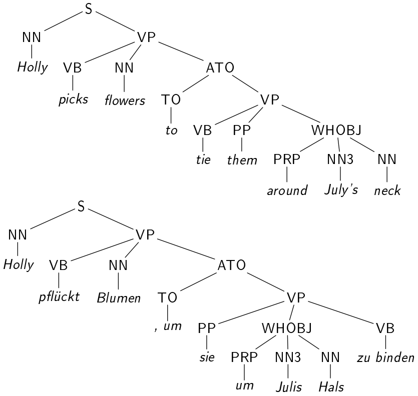

$\Downarrow$

Holly pflückt Blumen, um sie um Julis Hals zu binden

Reasonable formal models (ala Knight)

Required properties

(a) generalize finite-state transducers (the string case)
(b) efficiently trainable
(c) able to handle rotations
(d) class of transformations closed under composition

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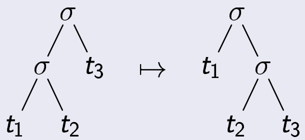
Example rotation



Models of tree transformations

Overview

Model \ Criterion	(a)	(b)	(c)	(d)
Linear nondeleting top-down tt				
Quasi-alphabetic tree bimorphism		?		
Synchronous context-free grammar				
Synchronous tree substitution grammar				
Synchronous tree adjoining grammar				
Linear complete tree bimorphism				
Linear extended top-down tt				
Linear multi bottom-up tt		?		

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Syntax

Definition

eXtended Multi Bottom-Up Tree Transducer $(Q, \Sigma, \Delta, F, R)$

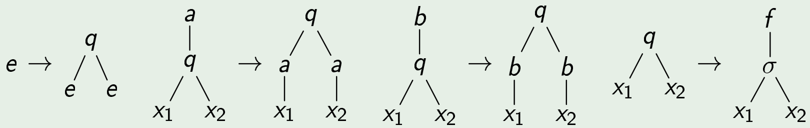
- Q ranked alphabet of *states*
- Σ and Δ ranked alphabets of input and output symbols
- $F \subseteq Q$ final states (non-zero ranked)
- R finite set of (rewrite) rules $l \to r$ with $l \in T_\Sigma(Q(X))$ and $r \in Q(T_\Delta(X))$

A full example

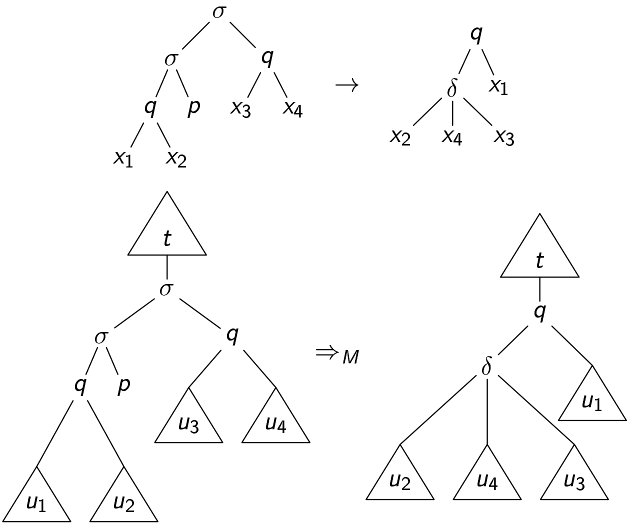
Example

xmbutt $(Q, \Sigma, \Delta, F, R)$ with

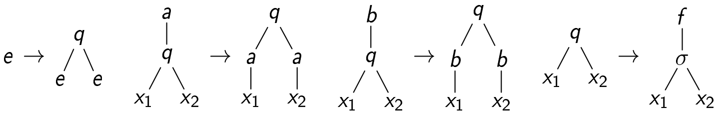
- $Q = \{f, q\}$
- $\Sigma = \{a^{(1)}, b^{(1)}, e^{(0)}\}$ and $\Delta = \Sigma \cup \{\sigma^{(2)}\}$
- $F = \{f\}$
- the following rules in R



Semantics



A full example (cont'd)



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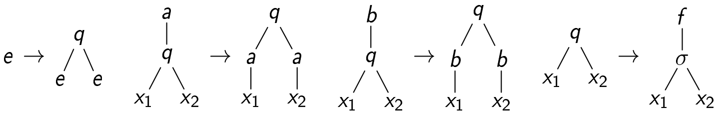

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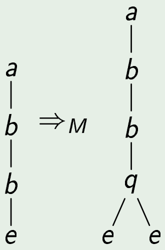

A full example (cont'd)

$$e \to q(e, e) \qquad a(q(x_1, x_2)) \to q(a(x_1), a(x_2)) \qquad b(q(x_1, x_2)) \to q(b(x_1), b(x_2)) \qquad q(x_1, x_2) \to f(\sigma(x_1, x_2))$$

Example

$$a(b(b(e))) \Rightarrow_M a(b(b(q(e, e)))) \Rightarrow_M a(b(q(b(e), b(e))))$$

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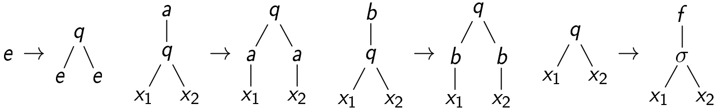

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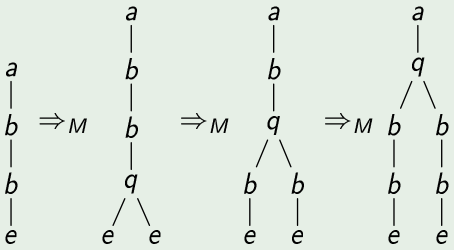

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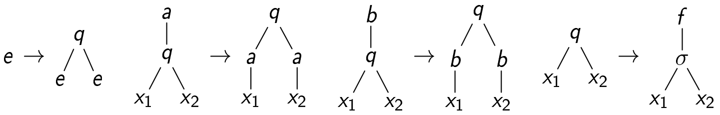

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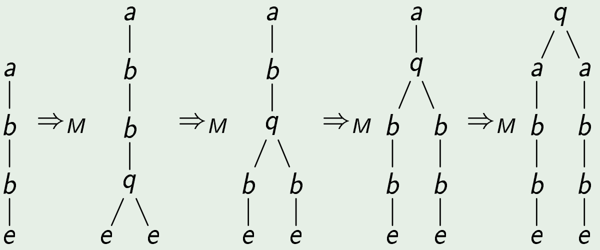

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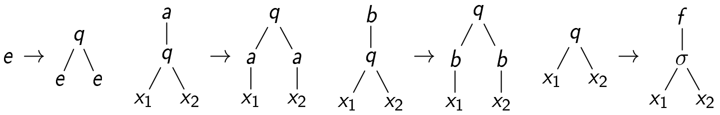

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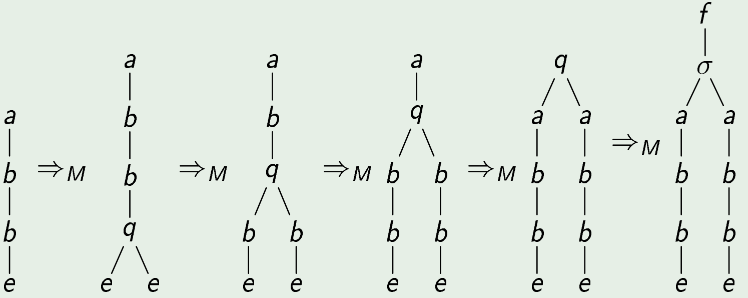

Semantics (cont'd)

Definition

$M = (Q, \Sigma, \Delta, F, R)$ xmbutt

$$\tau_M = \{(t, u) \mid \exists q \in F \colon t \Rightarrow^*_M q(u, \dots)\}$$

Example

Our example xmbutt M computes:

$$\tau_M = \{(t, \sigma(t, t)) \mid t \in T_\Sigma\}$$

Note that the image is not recognizable.

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Syntactical restrictions

Definition

xmbutt $(Q, \Sigma, \Delta, F, R)$ is

- linear if every right-hand side (of a rule) does not contain duplicate variables
- nondeleting, if every right-hand side contains all variables of its left-hand side

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Our example xmbutt is linear and nondeleting.

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Generalization of the string case

Theorem

Every fst (including epsilon-rules) can be simulated by an xmbutt.

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For every (linear) xmbutt there exists an equivalent (linear) nondeleting xmbutt.

Proof.

Nondeterministically guess which subtrees will be deleted and process those in nullary states. □

Note

Nullary states act like look-ahead.

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Theorem (Decomposition)

Every (linear) xmbutt can be simulated by a composition of a linear and nondeleting extended top-down tree transducer and a deterministic (single-use) top-down tree transducer.

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Every linear extended top-down tree transducer can be simulated by a linear xmbutt.

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Similar as the proof for the non-extended case. □

Corollary

Xmbutt can handle rotations.

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One-symbol normal form

Definition

xmbutt $(Q, \Sigma, \Delta, F, R)$ in one-symbol normal form if exactly one input or output symbol occurs in each rule.

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For every (linear, nondeleting) xmbutt there exists an equivalent (linear, nondeleting) xmbutt in one-symbol normal form.

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One-symbol normal form (cont'd)

Example

Rule not in one-symbol normal form:

$$b(q(x_1, x_2)) \to q(b(x_1), b(x_2))$$

Replacement rules for this rule:

$$b(q(x_1, x_2)) \to q_1(x_1, x_2) \qquad q_1(x_1, x_2) \to q_2(b(x_1), x_2) \qquad q_2(x_1, x_2) \to q(x_1, b(x_2))$$

Composition construction

Definition

from xmbutt $M = (Q, \Sigma, \Gamma, F, R)$ and $N = (Q', \Gamma, \Delta, G, P)$ construct

$$M \,;\, N = (Q\langle Q'\rangle, \Sigma, \Delta, F\langle G\rangle, R')$$

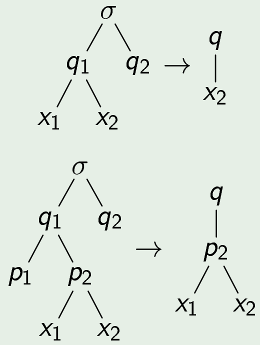
with three types of rules:

1. input-consuming rules constructed from input-consuming rules of R
2. epsilon rules constructed from epsilon-rules of P
3. epsilon rules constructed from an epsilon rule of R followed by an input consuming rule of P

Composition construction (cont'd)

Example

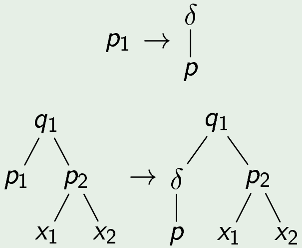
Input consuming rule of R and resulting rule:



Composition construction (cont'd)

Example

Epsilon rule of P and resulting rule:



Composition construction (cont'd)

Example

Epsilon rule of R and input consuming of P and resulting rule:

$$q_1(x_1, x_2) \to q(\gamma(x_2)) \qquad \gamma(p_2(x_1, x_2)) \to p(x_2)$$

$$q_1(p_1, p_2(x_1, x_2)) \to q(\gamma(p_2(x_1, x_2))) \to q(p(x_2))$$

Closure under composition

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For every linear xmbutt M and xmbutt N, there exists an xmbutt that computes the composition of the transformations computed by M and N.

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The class of transformations computed by linear xmbutt is closed under composition.

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Closure under composition (cont'd)

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References

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Thank you for your attention!