Hierarchies of Tree Series Transducers—Revisited[1]

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Machine Translation

Problem

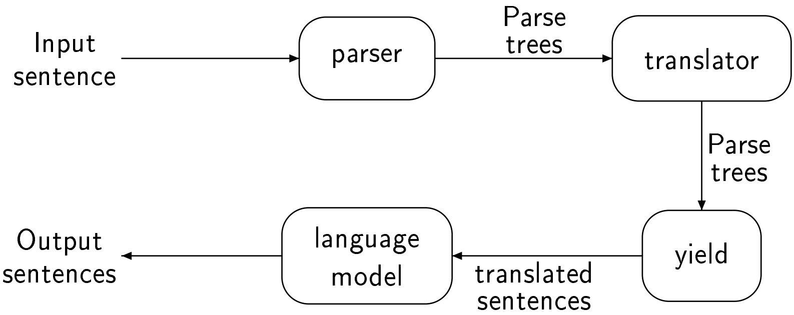
Translate text of language X into grammatical text of language Y.

1. Preserve meaning
2. Preserve connotation
3. Preserve style

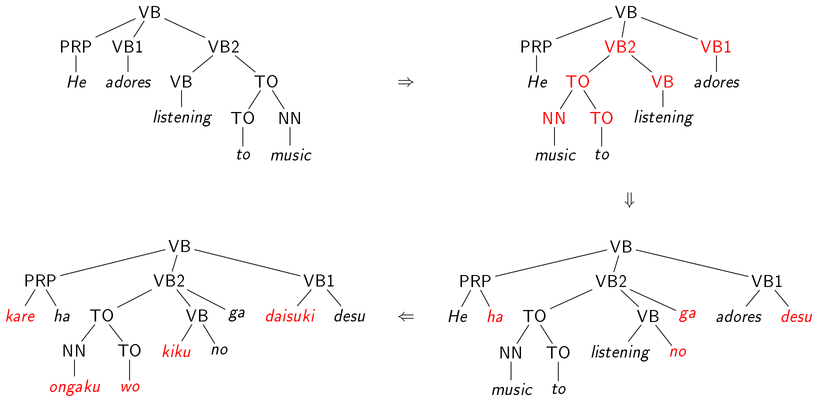
Approaches

- traditional phrase-based systems (e.g. CARMEL)
- upcoming syntax-based systems (e.g. Tiburon)

Syntax-based Machine Translation System



Syntax-based Model [Yamada, Knight 01]

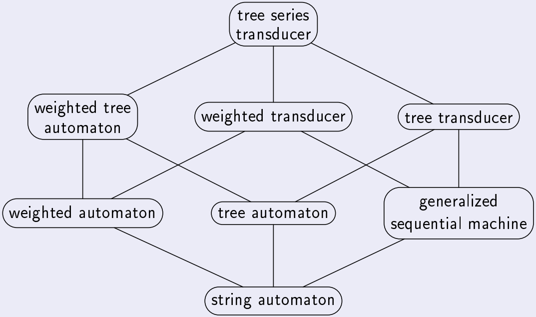


kare ha ongaku wo kiku no ga daisuki desu

3 phases: (i) Reorder, (ii) Insert, (iii) Translate

Tree Series Transducers

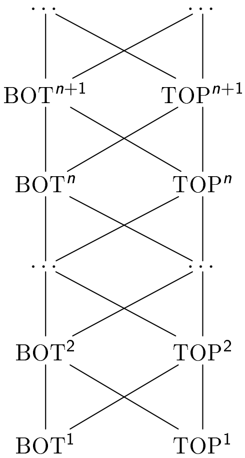
Overview



History

- Introduced in [Kuich 99]
- Extended to full generality in [Engelfriet, Fülöp, Vogler 02]

Hasse Diagram for Compositions [Engelfriet 82]



Semiring

Definition

$(A, +, \cdot, 0, 1)$ semiring, if

- $(A, +, 0)$ commutative monoid
- $(A, \cdot, 1)$ monoid
- $\cdot$ distributes (both sided) over $+$
- 0 is absorbing for $\cdot$ $(a \cdot 0 = 0 = 0 \cdot a)$

Example

- Reals $(\mathbb{R}, +, \cdot, 0, 1)$
- Probabilities $([0, 1], \max, \cdot, 0, 1)$
- Subsets $(\mathfrak{P}(A), \cup, \cap, \emptyset, A)$

Tree Series

Definition

- Tree series is mapping $\psi\colon T_\Sigma(X) \to A$
- Set of tree series $A\langle\!\langle T_\Sigma(X) \rangle\!\rangle$

Conventions

- A usually endowed with semiring structure
- $\widetilde{0}$ is tree series that maps every tree to 0
- $\psi(t)$ written as (ψ, t)

Tree Series

Definition

Let $\psi \in A\langle\langle T_\Sigma(X)\rangle\rangle$

$$\mathrm{supp}(\psi) = \{t \in T_\Sigma(X) \mid (\psi, t) \neq 0\}$$

i.e. set of nonzero-weighted trees

Convention

- tree series ψ written as $\sum_{t \in T_\Sigma(X)} (\psi, t)\ t$
- $(\psi + \varphi, t) = (\psi, t) + (\varphi, t)$
- $(a \cdot \psi, t) = a \cdot (\psi, t)$

Tree Series Transducer [Engelfriet et al 02]

Definition

Top-down tree series transducer $(Q, \Sigma, \Delta, \mathcal{A}, I, R)$ where

- Q finite set of states
- Σ and Δ input and output ranked alphabet
- $\mathcal{A} = (A, +, \cdot, 0, 1)$ semiring
- $I \subseteq Q$ set of initial states
- R finite set of rules of the form

$$q(\sigma(x_1, \ldots, x_k)) \xrightarrow{a} t$$

where $t \in T_\Delta(Q(X_k))$

Tree Series Transducer [Engelfriet et al 02]

Definition

Bottom-up tree series transducer $(Q, \Sigma, \Delta, \mathcal{A}, F, R)$ where

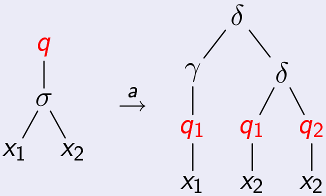
- Q finite set of states
- Σ and Δ input and output ranked alphabet
- $\mathcal{A} = (A, +, \cdot, 0, 1)$ semiring
- $F \subseteq Q$ set of final states
- R finite set of rules of the form

$$\sigma(q_1(x_1), \ldots, q_k(x_k)) \xrightarrow{a} q(t)$$

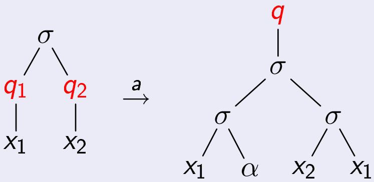
where $t \in T_\Delta(X_k)$

Tree Series Transducer

Top-down Rules

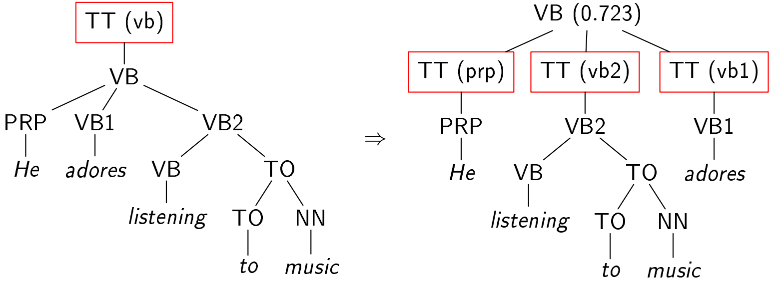


Bottom-up Rules



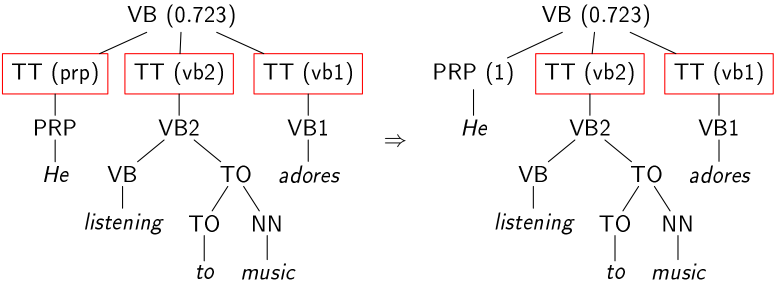
Implementation of Phase (i): Reorder

Implementation by top-down tree series transducer



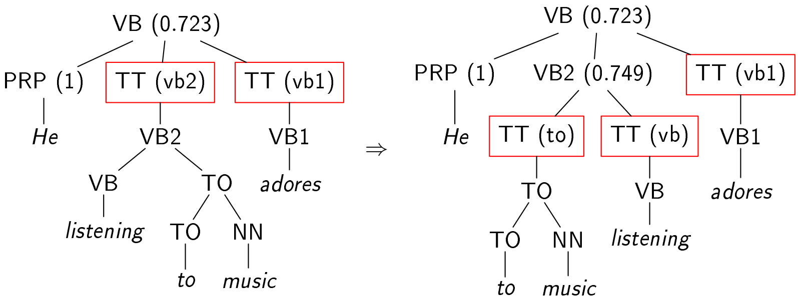
$$\mathrm{vb}(\mathrm{VB}(x_1, x_2, x_3)) \stackrel{0.723}{\longrightarrow} \mathrm{VB}(\mathrm{prp}(x_1), \mathrm{vb2}(x_3), \mathrm{vb1}(x_2))$$

Implementation of Phase (i): Reorder



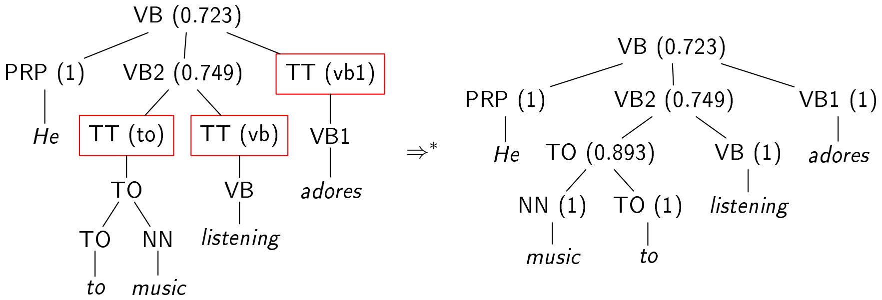
$$\text{prp}(\text{PRP}(x_1)) \xrightarrow{1} \text{PRP}(x_1)$$

Implementation of Phase (i): Reorder

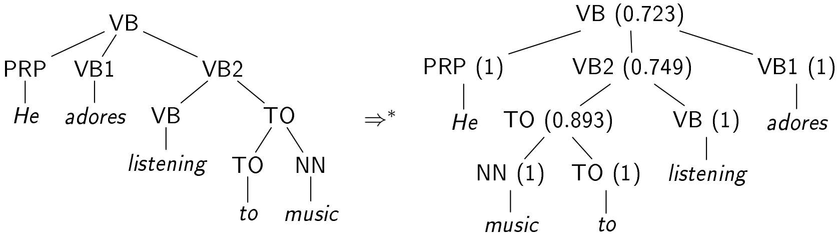


$$\text{vb2}(\text{VB2}(x_1, x_2)) \xrightarrow{0.749} \text{VB2}(\text{to}(x_2), \text{vb}(x_1))$$

Implementation of Phase (i): Reorder



Implementation of Phase (i): Reorder



The above reordering has probability:

$$0.723 \cdot 0.749 \cdot 0.893 = 0.484$$

Classes of Transformations

Definition

denotation	class of transformations computed by
$\mathrm{TOP}(\mathcal{A})$	top-down tree series transducers
$\mathrm{BOT}(\mathcal{A})$	bottom-up tree series transducers

Composition of Transformations

Definition

Let

- $\varphi\colon T_\Sigma \times T_\Delta \to A$
- $\psi\colon T_\Delta \times T_\Gamma \to A$

Composition of φ and ψ

$$(\varphi\,;\psi)\colon T_\Sigma \times T_\Gamma \to A$$

$$(t, v) \mapsto \sum_{u \in T_\Delta} \varphi(t, u) \cdot \psi(u, v)$$

Required Semiring Properties

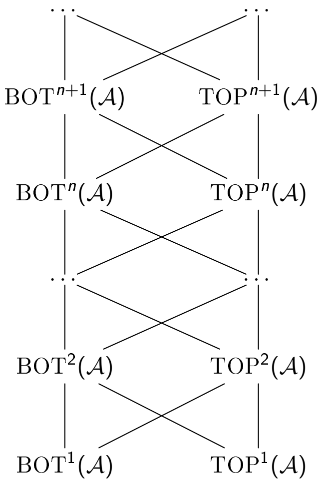
Definition

$\mathcal{A} = (A, +, \cdot, 0, 1)$

- commutative, if $a \cdot b = b \cdot a$
- idempotent, if $1 + 1 = 1$
- zero-divisor free, if $a \cdot b = 0$ implies $a = 0$ or $b = 0$
- zero-sum free, if $a + b = 0$ implies $a = 0$
- positive, if zero-divisor free and zero-sum free

Hasse Diagram for Compositions [Fülöp et al 05]

$\mathcal{A}$ commutative, positive idempotent semiring



Homomorphic Images

Definition

$\chi\colon A \to \{0,1\}$

$$\chi(a) = \begin{cases} 0 & \text{if } a = 0 \\ 1 & \text{otherwise} \end{cases}$$

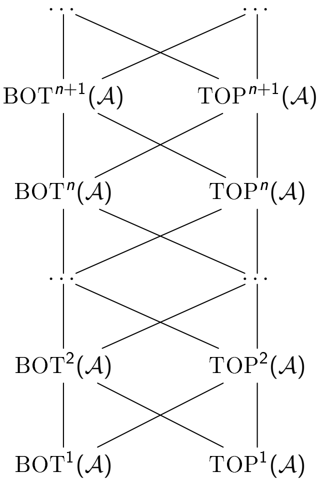
Theorem

$\mathcal{A}$ positive semiring

- $\chi(\mathrm{TOP}(\mathcal{A})) = \mathrm{TOP}(\mathbb{B})$ *and* $\chi(\mathrm{BOT}(\mathcal{A})) = \mathrm{BOT}(\mathbb{B})$
- $\chi(\tau_1 \, ; \tau_2) = \chi(\tau_1) \, ; \chi(\tau_2)$

Hasse Diagram for Compositions

$\mathcal{A}$ commutative, positive semiring (e.g. $\mathbb{R}_+$)



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