Monitor Logics for Quantitative Monitor Automata

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Motivation

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$\Rightarrow$ Quantitative Monitor Automata [Chatterjee, Henzinger, Otop '16]

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e.g. minimum, maximum, long-term average

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$$q_0 \xrightarrow[]{a \; \binom{s}{0}} q_1 \xrightarrow[]{a \; \binom{1}{0}} q_2 \xrightarrow[]{b \; \binom{1}{s}} q_3 \xrightarrow[]{a \; \binom{t}{2}} q_4 \xrightarrow[]{b \; \binom{0}{3}} q_5 \xrightarrow[]{a \; \binom{s}{t}} \dots$$

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Weight of run: $\mathrm{Val}((z_i)_{i \geq 1})$

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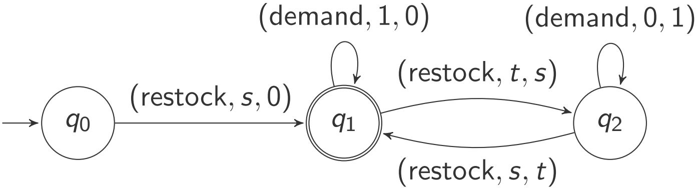
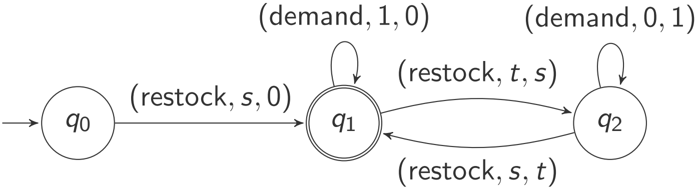
$$q_0 \xrightarrow[2]{a \begin{pmatrix} s \\ 0 \end{pmatrix}} q_1 \xrightarrow[\bot]{a \begin{pmatrix} 1 \\ 0 \end{pmatrix}} q_2 \xrightarrow[5]{b \begin{pmatrix} 1 \\ s \end{pmatrix}} q_3 \xrightarrow[\bot]{a \begin{pmatrix} t \\ 2 \end{pmatrix}} q_4 \xrightarrow[\bot]{b \begin{pmatrix} 0 \\ 3 \end{pmatrix}} q_5 \xrightarrow[3]{a \begin{pmatrix} s \\ t \end{pmatrix}} \cdots$$

Weight of run: $\mathrm{Val}((z_i)_{i \geq 1})$

Weight of ω-word: infimum over all runs

Example





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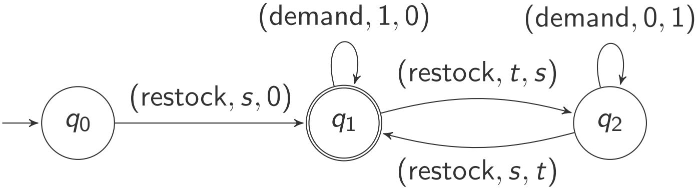

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valuation function to compute long-time average, minimum, ...

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$$\psi ::= k \mid \beta \,?\, \psi : \psi$$

$$[\![\beta \,?\, \psi_1 : \psi_2]\!](w) = \begin{cases} [\![\psi_1]\!](w) & \text{if } w \models \beta \\ [\![\psi_2]\!](w) & \text{otherwise} \end{cases}$$

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$$\begin{cases} \sum_{i=x+1}^{\bar{x}-1} [\![\psi]\!](w[y \to i]) & \text{if } x \in Z \text{ and } \exists \bar{x} \in Z : \bar{x} > x \\ \bot & \text{otherwise} \end{cases}$$

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$$\varphi = \inf Z.\left(\forall z.(z \in Z \leftrightarrow P_{\text{restock}}(z)) \,?\, \mathrm{Val}\, x.\left(\bigoplus^{x,Z} y.1\right) : \infty\right)$$

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Muller automata: $\mathsf{Val}\, x.\psi$

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"correct weights" is a recognizable property

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QMA: $\mathrm{Val}\, x.\zeta_x$

$$\varphi = \mathrm{Val}\, x. \left(\bigoplus^{x,Z} y.1\right)$$

$$\begin{pmatrix}\text{restock}\\ s\\ \bot\\ 1\end{pmatrix} \begin{pmatrix}\text{demand}\\ 1\\ \bot\\ 0\end{pmatrix} \begin{pmatrix}\text{demand}\\ 1\\ \bot\\ 0\end{pmatrix} \begin{pmatrix}\text{restock}\\ t\\ s\\ 1\end{pmatrix} \begin{pmatrix}\text{demand}\\ \bot\\ 1\\ 0\end{pmatrix} \cdots$$