Finite Sequentiality of Unambiguous Max-Plus Tree Automata

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QuantLA

Max-Plus Automata

$$q_0 \xrightarrow{a} q_1 \xrightarrow{a} q_2 \xrightarrow{b} q_3 \xrightarrow{a} q_4 \xrightarrow{b} q_5 \xrightarrow{a} q_6$$

Max-Plus Automata

Weights in $\mathbb{R} \cup \{-\infty\}$

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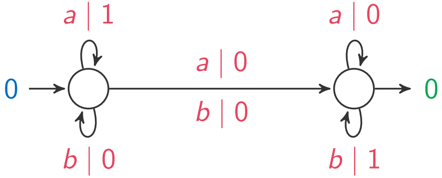

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sequential / deterministic	no two valid $p \xrightarrow{a} q_1,\ p \xrightarrow{a} q_2$

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Given $\mathcal{A}$ — Is there determ $\mathcal{A}'$ with $[\![\mathcal{A}]\!] = [\![\mathcal{A}']\!]$?

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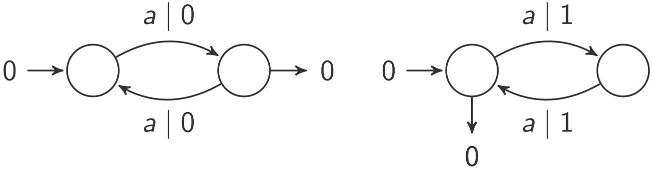
decidable on words for unamb $\mathcal{A}$ — [Mohri]


a | 0
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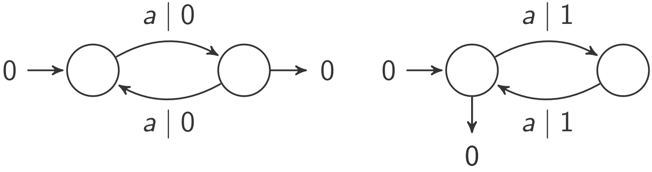
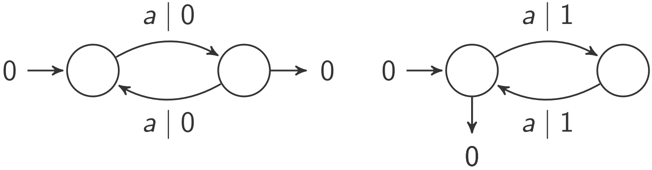


$[\![\mathcal{A}]\!](w) =$ $|w|$ uneven $\rightsquigarrow 0$ $|w|$ even $\rightsquigarrow |w|$

Sequentiality Problem: $\mathcal{A}$ determinizable?

$\mathcal{A} = (Q, \lambda, \mu, \nu)$ unamb $\qquad$ $p, q \in Q$ states



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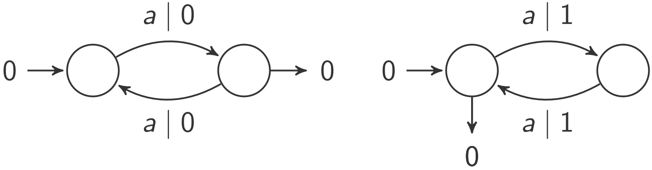

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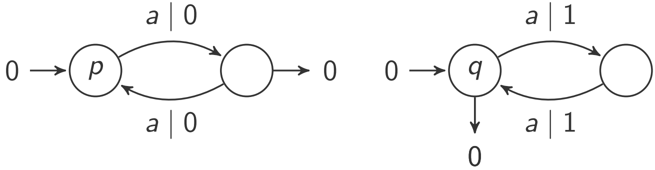

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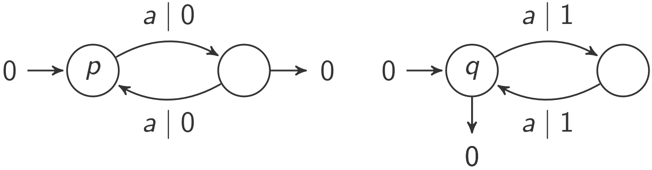

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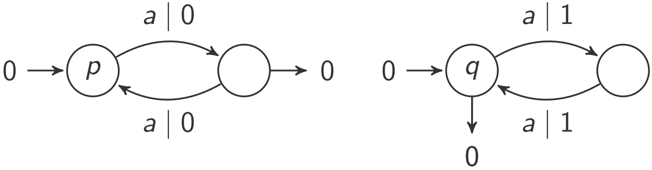

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assume rivals p,q exist

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$$[\![\mathcal{A}]\!](uv^n w_p) = x_p + n \cdot y_p + z_p$$
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$\Rightarrow [\![\mathcal{A}]\!]$ not sequential

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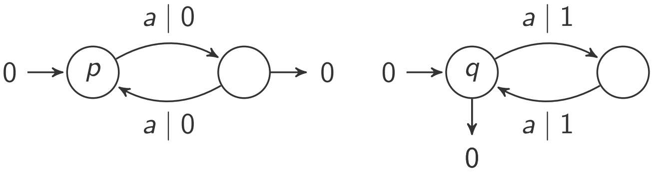

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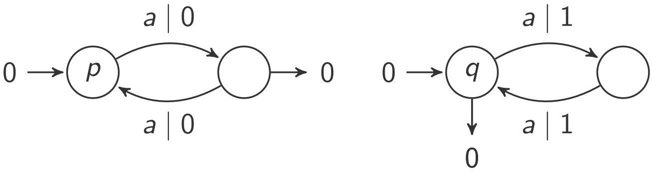

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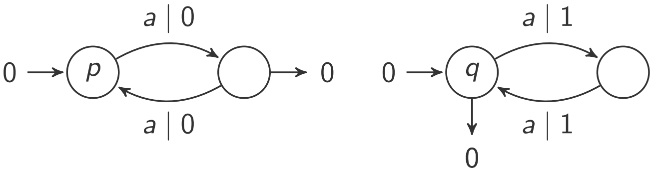

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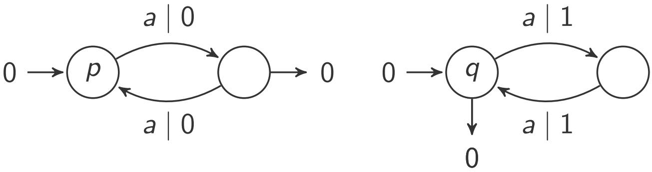

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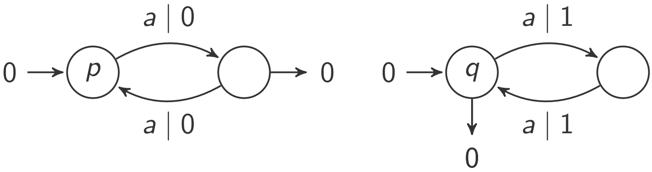

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Proof "$\rightarrow$" elementary "$\leftarrow$" interesting

Show $\mathcal{A}$ unamb and $\nexists$ rivals p, q, fork w: $p \xrightarrow{w} p \quad p \xrightarrow{w} q$

$\Rightarrow [\![\mathcal{A}]\!] = \max_{i=1}^{n} [\![\mathcal{A}_i]\!]$ $\qquad$ $\mathcal{A}_i$ deterministic

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p_i, q_j earliest visits of $\{p, q\}$ $\qquad$ assume $i = j$ and $p_i = q_j$

$\Rightarrow p_i \xrightarrow{u_{i+1} \cdots u_n} p$ and $p_i \xrightarrow{u_{i+1} \cdots u_n} q$

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$\xrightarrow{u} p \xrightarrow{v|y_p} p$ $\qquad$ $p_0 \xrightarrow{u_1} p_1 \xrightarrow{u_2} \ldots \xrightarrow{u_{n-1}} p_{n-1} \xrightarrow{u_n} p$

$\xrightarrow{u} q \xrightarrow{v|y_q} q$ $\qquad$ $q_0 \xrightarrow{u_1} q_1 \xrightarrow{u_2} \ldots \xrightarrow{u_{n-1}} q_{n-1} \xrightarrow{u_n} q$

p_i, q_j earliest visits of $\{p, q\}$ $\qquad$ assume $i = j$ and $p_i = q_j$

$\Rightarrow q \xrightarrow{u_{i+1} \cdots u_n} p$ and $q \xrightarrow{u_{i+1} \cdots u_n} q$

$\Rightarrow u_{i+1} \cdots u_n$ fork $\lightning$

Show $\mathcal{A}$ unamb and $\nexists$ rivals p, q, fork w: $p \xrightarrow{w} p \quad p \xrightarrow{w} q$

$\Rightarrow [\![\mathcal{A}]\!] = \max_{i=1}^{n} [\![\mathcal{A}_i]\!]$ $\mathcal{A}_i$ deterministic

---

Proof construct unamb $\mathcal{A}_1, \ldots, \mathcal{A}_n$ with

- $[\![\mathcal{A}]\!] = \max_{i=1}^{n} [\![\mathcal{A}_i]\!]$
- no rivals in $\mathcal{A}_i$ $\Rightarrow$ determinizable

$\xrightarrow{u} p \xrightarrow{v|y_p} p$ $p_0 \xrightarrow{u_1} p_1 \xrightarrow{u_2} \ldots \xrightarrow{u_{n-1}} p_{n-1} \xrightarrow{u_n} p$

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Idea record first visit of rival

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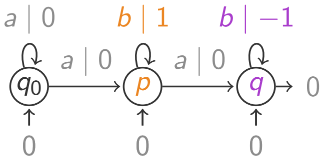

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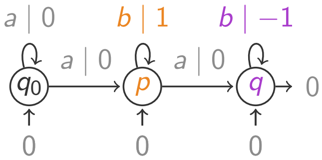
Idea record first visit of rival word aaa



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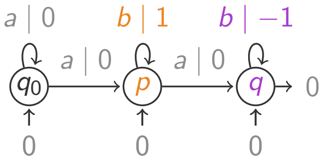
Idea record first visit of rival $\qquad$ word aaa



$\rightarrow \quad q_0$

$\rightarrow \quad p, 1_p$

$\rightarrow \quad q, 1_q$

Show $\mathcal{A}$ unamb and $\nexists$ rivals p, q, fork w: $p \xrightarrow{w} p \quad p \xrightarrow{w} q$

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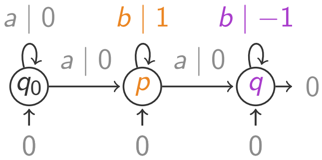

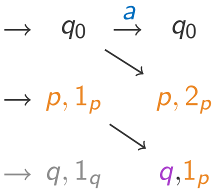

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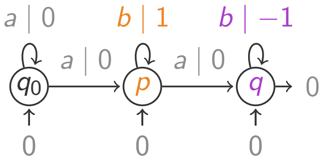
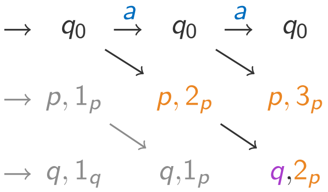
Idea record first visit of rival $\qquad$ word aaa





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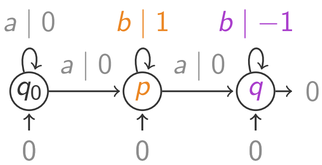

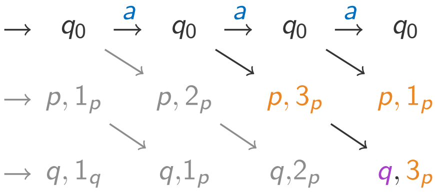

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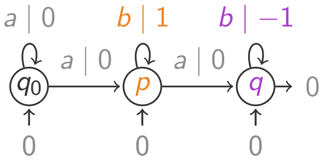
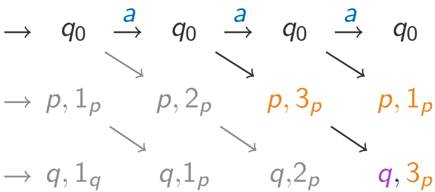
Idea record first visit of rival — word aaa





$\Rightarrow$ separate rivals by separating markers into different automata

Show $\mathcal{A}$ unamb and $\nexists$ rivals p, q, fork w: $p \xrightarrow{w} p \quad p \xrightarrow{w} q$

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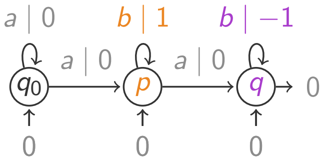
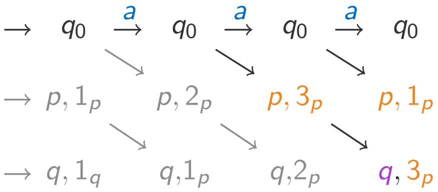
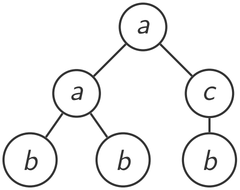
Idea record first visit of rival word aaa

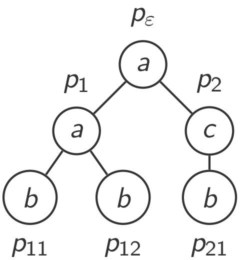




$\Rightarrow$ separate rivals by separating markers into different automata

$\Rightarrow$ unamb automata without rivals

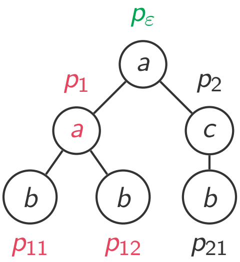

a
a
c
b
b
b


p_ε
a
p_1
p_2
a
c
b
b
b
p_{11}
p_{12}
p_{21}

weight of run =

transition weights + final weight

(p_{11}, p_{12}, a, p_1)

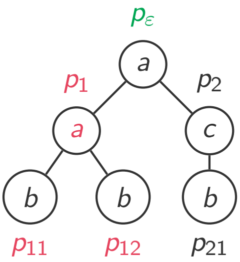


weight of run =

transition weights + final weight

(p_{11}, p_{12}, a, p_1)

determinism: bottom-up



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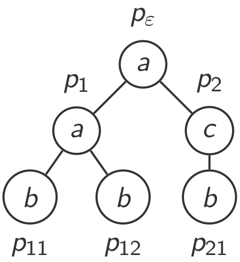


Finite Sequentiality: $[\![\mathcal{A}]\!] = \max_{i=1}^{n} [\![\mathcal{A}_i]\!]$ for some determ $\mathcal{A}_i$?

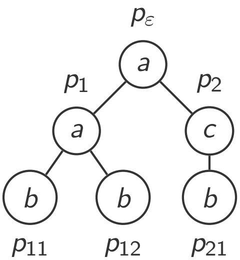
68%

weight of run =

transition weights + final weight

(p_{11}, p_{12}, a, p_1)

determinism: bottom-up



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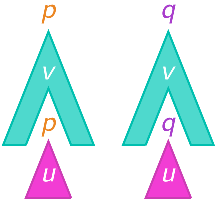
rivals



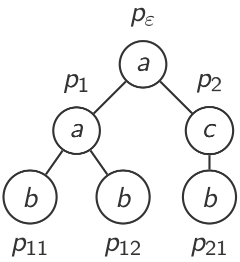
$y_p \neq y_q$

weight of run =

transition weights + final weight

(p_{11}, p_{12}, a, p_1)

determinism: bottom-up



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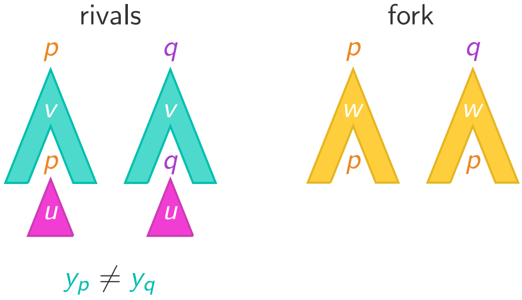

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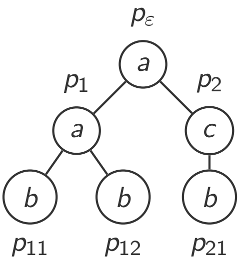

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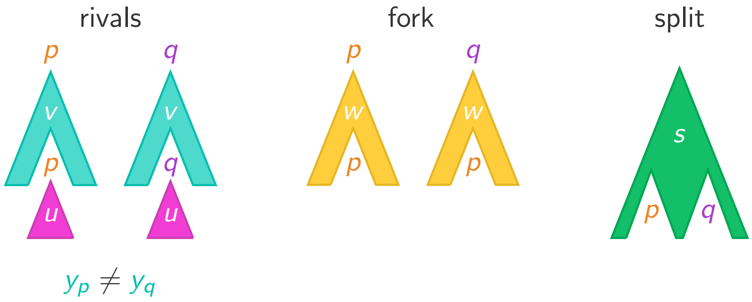

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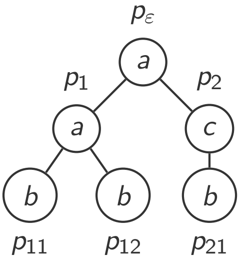

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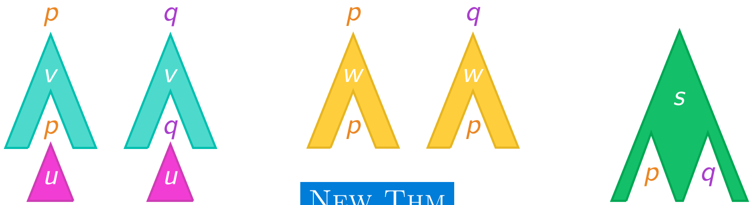
rivals | fork | split



$y_p \neq y_q$

NEW THM

$\mathcal{A}$ unamb $\Rightarrow$ $\mathcal{A}$ fin seq $\leftrightarrow$ no forks, no splits

Show $\mathcal{A}$ unamb and no splits, no forks $p \xrightarrow{w} p$ $p \xrightarrow{w} q$

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Idea record first visit of rival bottom-up

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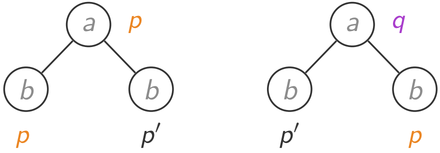

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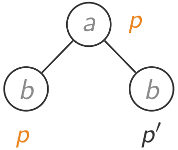

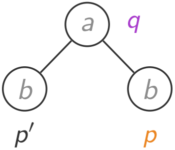

non-linearity $\rightarrow$ problems

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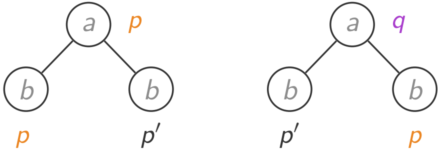

non-linearity $\rightarrow$ problems

Solution

Schützenberger-covering

Show $\mathcal{A}$ unamb and no splits, no forks $p \xrightarrow{w} p$ $p \xrightarrow{w} q$

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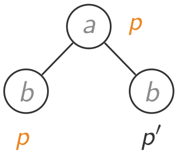

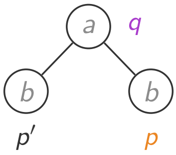

non-linearity → problems

Solution

Schützenberger-covering

Powerset construction

Show $\mathcal{A}$ unamb and no splits, no forks $p \xrightarrow{w} p$ $p \xrightarrow{w} q$

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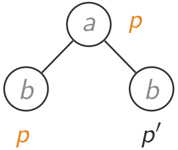
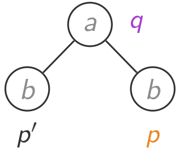
Idea record first visit of rival bottom-up





non-linearity → problems

Solution

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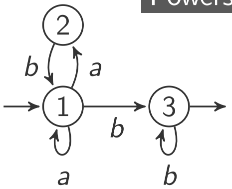
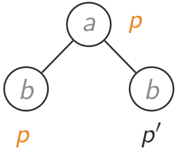
Powerset construction

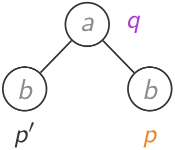


Show $\mathcal{A}$ unamb and no splits, no forks $p \xrightarrow{w} p \quad p \xrightarrow{w} q$

$\Rightarrow [\![\mathcal{A}]\!] = \max_{i=1}^{n} [\![\mathcal{A}_i]\!]$ $\qquad$ $\mathcal{A}_i$ unamb, no rivals in $\mathcal{A}_i$

Idea record first visit of rival $\qquad$ bottom-up



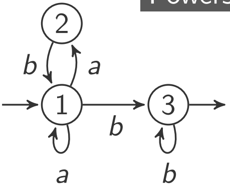


non-linearity → problems

Solution

Schützenberger-covering

Powerset construction



$\rightarrow \quad \{1\} \xrightarrow{a} \quad \{1,2\} \xrightarrow{b} \quad \{1,3\} \rightarrow$

Show $\mathcal{A}$ unamb and no splits, no forks $p \xrightarrow{w} p$ $p \xrightarrow{w} q$

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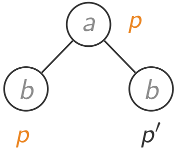
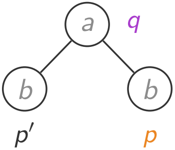


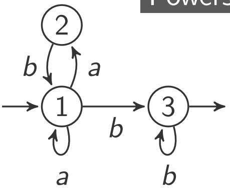


non-linearity $\rightarrow$ problems

Solution

Schützenberger-covering

Powerset construction



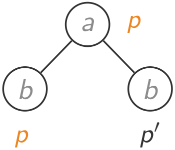
$\rightarrow \{1\} \xrightarrow{a} \{1,2\} \xrightarrow{b} \{1,3\} \rightarrow$

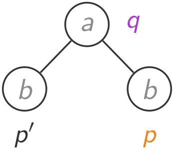
Product automaton

Show $\mathcal{A}$ unamb and no splits, no forks $p \xrightarrow{w} p \quad p \xrightarrow{w} q$

$\Rightarrow [\![\mathcal{A}]\!] = \max_{i=1}^{n} [\![\mathcal{A}_i]\!]$ $\qquad$ $\mathcal{A}_i$ unamb, no rivals in $\mathcal{A}_i$

Idea record first visit of rival $\qquad$ bottom-up



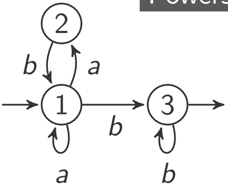


non-linearity $\rightarrow$ problems

Solution

Schützenberger-covering

Powerset construction



$\rightarrow \quad \{1\} \xrightarrow{a} \quad \{1,2\} \xrightarrow{b} \quad \{1,3\} \rightarrow$

Product automaton

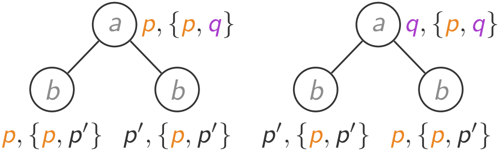
$\rightarrow 1, \{1\} \xrightarrow{a} 1, \{1,2\} \xrightarrow{b} 3, \{1,3\} \rightarrow$

Show $\mathcal{A}$ unamb and no splits, no forks $p \xrightarrow{w} p$ $\quad p \xrightarrow{w} q$

$\Rightarrow [\![\mathcal{A}]\!] = \max_{i=1}^{n} [\![\mathcal{A}_i]\!]$ $\qquad \mathcal{A}_i$ unamb, no rivals in $\mathcal{A}_i$

Idea record first visit of rival

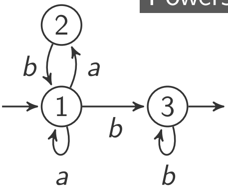
bottom-up



non-linearity $\rightarrow$ problems

Solution

Schützenberger-covering



Powerset construction

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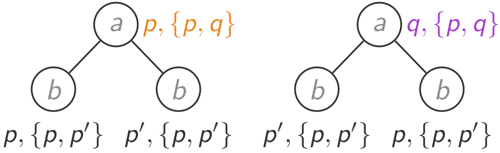

non-linearity → problems

Solution

Schützenberger-covering

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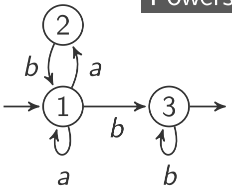

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$Q =$ states of $\mathcal{A}$

$\Rightarrow$ states of Schützenberger-covering $\mathcal{S}$ from $Q \times \mathcal{P}(Q)$

Show $\mathcal{A}$ unamb and no splits, no forks $p \xrightarrow{w} p \quad p \xrightarrow{w} q$

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$[\![\mathcal{S}]\!] = [\![\mathcal{A}]\!]$ $\mathcal{A}$ unamb $\Rightarrow \mathcal{S}$ unamb

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$[\![\mathcal{S}]\!] = [\![\mathcal{A}]\!]$ $\mathcal{A}$ unamb $\Rightarrow \mathcal{S}$ unamb

rivals of $\mathcal{S}$: $(p, P), (q, P)$ for rivals p, q of $\mathcal{A}$

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$[\![\mathcal{S}]\!] = [\![\mathcal{A}]\!]$ $\mathcal{A}$ unamb $\Rightarrow \mathcal{S}$ unamb

rivals of $\mathcal{S}$: $(p, P), (q, P)$ for rivals p, q of $\mathcal{A}$

Dichotomy $(p, P), (q, P)$ rivals $\Rightarrow$ for all runs of $\mathcal{S}$:

Show $\mathcal{A}$ unamb and no splits, no forks $p \xrightarrow{w} p$ $p \xrightarrow{w} q$

$\Rightarrow [\![\mathcal{S}]\!] = \max_{i=1}^{n} [\![\mathcal{A}_i]\!]$ $\mathcal{A}_i$ unamb, no rivals in $\mathcal{A}_i$

$Q =$ states of $\mathcal{A}$

$\Rightarrow$ states of Schützenberger-covering $\mathcal{S}$ from $Q \times \mathcal{P}(Q)$

$[\![\mathcal{S}]\!] = [\![\mathcal{A}]\!]$ $\mathcal{A}$ unamb $\Rightarrow$ $\mathcal{S}$ unamb

rivals of $\mathcal{S}$: $(p, P), (q, P)$ for rivals p, q of $\mathcal{A}$

Dichotomy $(p, P), (q, P)$ rivals $\Rightarrow$ for all runs of $\mathcal{S}$:

- either one of the rivals does not occur

Show $\mathcal{A}$ unamb and no splits, no forks $p \xrightarrow{w} p \quad p \xrightarrow{w} q$

$\Rightarrow [\![\mathcal{S}]\!] = \max_{i=1}^{n} [\![\mathcal{A}_i]\!]$ $\mathcal{A}_i$ unamb, no rivals in $\mathcal{A}_i$

$Q =$ states of $\mathcal{A}$

$\Rightarrow$ states of Schützenberger-covering $\mathcal{S}$ from $Q \times \mathcal{P}(Q)$

$[\![\mathcal{S}]\!] = [\![\mathcal{A}]\!]$ $\mathcal{A}$ unamb $\Rightarrow \mathcal{S}$ unamb

rivals of $\mathcal{S}$: $(p, P), (q, P)$ for rivals p, q of $\mathcal{A}$

Dichotomy $(p, P), (q, P)$ rivals $\Rightarrow$ for all runs of $\mathcal{S}$:

- either one of the rivals does not occur

$\Rightarrow$ unamb $\mathcal{A}_1, \mathcal{A}_2$

Show $\mathcal{A}$ unamb and no splits, no forks $p \xrightarrow{w} p \quad p \xrightarrow{w} q$

$\Rightarrow [\![\mathcal{S}]\!] = \max_{i=1}^{n} [\![\mathcal{A}_i]\!]$ $\mathcal{A}_i$ unamb, no rivals in $\mathcal{A}_i$

$Q =$ states of $\mathcal{A}$

$\Rightarrow$ states of Schützenberger-covering $\mathcal{S}$ from $Q \times \mathcal{P}(Q)$

$[\![\mathcal{S}]\!] = [\![\mathcal{A}]\!]$ $\mathcal{A}$ unamb $\Rightarrow \mathcal{S}$ unamb

rivals of $\mathcal{S}$: $(p, P), (q, P)$ for rivals p, q of $\mathcal{A}$

Dichotomy $(p, P), (q, P)$ rivals $\Rightarrow$ for all runs of $\mathcal{S}$:

- either one of the rivals does not occur
- or all states with second entry P occur linearly

$\Rightarrow$ unamb $\mathcal{A}_1, \mathcal{A}_2$

Show $\mathcal{A}$ unamb and no splits, no forks $p \xrightarrow{w} p \quad p \xrightarrow{w} q$

$\Rightarrow [\![\mathcal{S}]\!] = \max_{i=1}^{n} [\![\mathcal{A}_i]\!]$ $\qquad$ $\mathcal{A}_i$ unamb, no rivals in $\mathcal{A}_i$

Q = states of $\mathcal{A}$

$\Rightarrow$ states of Schützenberger-covering $\mathcal{S}$ from $Q \times \mathcal{P}(Q)$

$[\![\mathcal{S}]\!] = [\![\mathcal{A}]\!]$ $\qquad$ $\mathcal{A}$ unamb $\Rightarrow \mathcal{S}$ unamb

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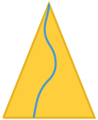

$\Rightarrow$ unamb $\mathcal{A}_1, \mathcal{A}_2$

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$Q =$ states of $\mathcal{A}$

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$[\![\mathcal{S}]\!] = [\![\mathcal{A}]\!]$ $\qquad$ $\mathcal{A}$ unamb $\Rightarrow \mathcal{S}$ unamb

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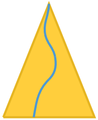

$\Rightarrow$ unamb $\mathcal{A}_1, \mathcal{A}_2$ and $\mathcal{A}_3, \ldots, \mathcal{A}_n$ through markers