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	Equivalence	Unambiguity	Sequentiality	Fin Seq
fin-amb	**yes**	**yes**	**yes**	**unamb**
poly-amb	**no**	**?**	**?**	**?**
general	**no**	**?**	**?**	**?**

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