Weight of run: initial weight + transition weights + final weight

Weight of word: maximum over all runs

unambiguous | Run($w$) | ≤ | 1

finitely ambiguous | Run($w$) | ≤ | $M$

polynomially ambiguous | Run($w$) | ≤ | $P(|w|)$

Finite Sequentiality problem

| Given $A$ is $J$ $A$ $K$ | $= n_{\text{max}} i=1 J A i K$ for some determ $A$ |

<table>
<thead>
<tr>
<th>Equivalence</th>
<th>Unambiguity</th>
<th>Sequentiality</th>
<th>Fin Seq</th>
</tr>
</thead>
<tbody>
<tr>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
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<td>no</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>general</td>
<td>no</td>
<td>?</td>
<td>?</td>
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</tbody>
</table>
Weight of run: initial weight + transition weights + final weight

Weight of word: maximum over all runs
Weight of run: \textit{initial weight} + \textit{transition weights} + \textit{final weight}

Weight of word: \textit{maximum} over all runs

<table>
<thead>
<tr>
<th>Ambiguity</th>
<th>Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>unambiguous</td>
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<tr>
<td>finitely ambiguous</td>
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</tr>
</tbody>
</table>
Weight of run: initial weight + transition weights + final weight

Weight of word: maximum over all runs

unambiguous $|\text{Run}(w)| \leq 1$

finitely ambiguous $|\text{Run}(w)| \leq M$

polynomially ambiguous $|\text{Run}(w)| \leq P(|w|)$

Finite Sequentiality problem
Given $\mathcal{A}$
Is $[\mathcal{A}] = \max_{i=1}^{n} [\mathcal{A}_i]$ for some determ $\mathcal{A}_i$?
Weight of run: initial weight + transition weights + final weight

Weight of word: maximum over all runs

<table>
<thead>
<tr>
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<th>finitely ambiguous</th>
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<tbody>
<tr>
<td></td>
<td>$</td>
<td>\text{Run}(w)</td>
<td>\leq 1$</td>
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</table>

Finite Sequentiality problem
Given $\mathcal{A}$ Is $\mathbb{[A]} = \max_{i=1}^{n} [\mathcal{A}_i]$ for some determ $\mathcal{A}_i$?

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</thead>
<tbody>
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<td>yes</td>
<td>yes</td>
<td>unamb</td>
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<tr>
<td>poly-amb</td>
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\[
\sigma = (\text{Rel}, \text{ar}) \quad \text{signature} = (\text{relation symbols}, \text{arity})
\]

\[
\mathcal{A} = (A, \mathcal{I}) \quad \sigma\text{-structure} = (\text{finite universe}, \text{interpretation})
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\[ \sigma = (\text{Rel}, \text{ar}) \quad \text{signature} = (\text{relation symbols, arity}) \]

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\[ \beta ::= R(x_1, \ldots, x_n) \mid x \in X \mid \neg \beta \mid \beta \lor \beta \mid \exists x.\beta \mid \exists X.\beta \]

\[ P ::= x_i \mid y_i \mid P \lor P \mid P \land P \]
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\[ (S, \oplus, \otimes, 0, 1) \quad \text{semiring} \]

\[ \psi ::= \beta \mid s \mid \psi \oplus \psi \mid \psi \otimes \psi \]

\[ \varphi ::= \beta \mid s \mid \varphi \oplus \varphi \mid \varphi \otimes \varphi \mid \bigoplus x. \varphi \mid \bigotimes x. \psi \mid \bigoplus X. \varphi \]
\( \sigma = (\text{Rel, ar}) \) signature = (relation symbols, arity)

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\[
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\beta & ::= R(x_1, \ldots, x_n) \mid x \in X \mid \neg \beta \mid \beta \lor \beta \mid \exists x. \beta \mid \exists X. \beta \\
P & ::= x_i \mid y_i \mid P \lor P \mid P \land P
\end{align*}
\]

\((S, \oplus, \otimes, 0, 1)\) semiring

\[
\begin{align*}
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\varphi & ::= \beta \mid s \mid \varphi \oplus \varphi \mid \varphi \otimes \varphi \mid \bigoplus x. \varphi \mid \bigotimes x. \psi \mid \bigoplus X. \varphi \\
E & ::= x_i \mid y_i \mid E \oplus E \mid E \otimes E
\end{align*}
\]