Selected Papers of MFCS 2017

Decidability Results for Max-Plus Tree Automata

and

Monitor Logics for Quantitative Monitor Automata

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The Equivalence, Unambiguity and Sequentiality Problems of Finitely Ambiguous Max-Plus Tree Automata are Decidable

Max-Plus Automata

$$q_0 \xrightarrow{a} q_1 \xrightarrow{a} q_2 \xrightarrow{b} q_3 \xrightarrow{a} q_4 \xrightarrow{b} q_5 \xrightarrow{a} q_6$$

Weights in $\mathbb{R} \cup \{-\infty\}$

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Weight of run:

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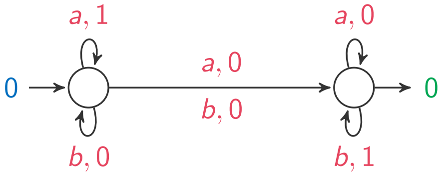

one "initial state"

sequential / deterministic — no two valid $p \xrightarrow{a} q_1,\ p \xrightarrow{a} q_2$

$$\text{Run}(w) = \{\text{Runs } r \text{ on } w \text{ with weight}(r) \neq -\infty\}$$

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Three Decision Problems

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Equivalence problem

Given $\mathcal{A}_1, \mathcal{A}_2$ Is $[\![\mathcal{A}_1]\!](w) = [\![\mathcal{A}_2]\!](w)$ for all w?

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Given $\mathcal{A}$ — Is there unamb $\mathcal{A}'$ with $[\![\mathcal{A}]\!] = [\![\mathcal{A}']\!]$?

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Three Decision Problems

Decidability for max-plus automata on words

	Equivalence	Unambiguity	Sequentiality
fin-amb			
poly-amb			
general			

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fin-amb			
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Krob

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... on trees up to now

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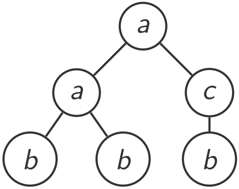
fin-amb	yes	yes	yes
poly-amb	no	?	?
general	no	?	?



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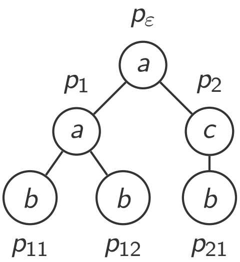

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fin-amb	**yes**	**yes**	**yes**
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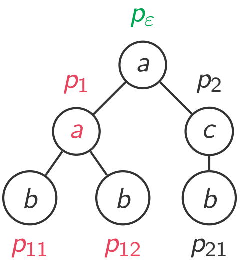
weight of run =

transition weights + final weight

(p_{11}, p_{12}, a, p_1)



We show: $\mathcal{A}_1, \mathcal{A}_2$ max-plus word automata, $\mathcal{A}_1$ fin-amb

$\implies \mathcal{A}_1 \geq \mathcal{A}_2$ decidable [Hashiguchi et al.]

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all runs of $\mathcal{A}_1$, one of $\mathcal{A}_2$ in parallel

p_1		p_2		p_2		p_1		p_3		p_2		p_2
p_1	a	p_3	b	p_3	a	p_1	b	p_2	b	p_3	a	p_4
p_2		p_4		p_3		p_1		p_1		p_3		p_3
q_1		q_2		q_1		q_3		q_2		q_1		q_1

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never $\mathrm{wt}_1, \mathrm{wt}_2, \mathrm{wt}_3 < \mathrm{wt}_4 \quad \Longleftrightarrow \quad \mathcal{A}_1 \geq \mathcal{A}_2$

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Cycle Decomposition

$$\vec{P}_1 \quad x_1 \quad \vec{P}_2 \quad y_2 \quad \vec{P}_2 \quad x_3 \quad \vec{P}_3 \quad y_4 \quad \vec{P}_3 \quad x_5 \quad \vec{P}_4 \quad y_6 \quad \vec{P}_4 \quad x_7 \quad \vec{P}_5$$

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x_i, y_i short: $|x_i|, |y_i| \leq |\text{states}(\mathcal{A}_1)|^3 \cdot |\text{states}(\mathcal{A}_2)|$

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Vectors of weights

$$\begin{matrix} \mathrm{wt}_1 \\ \mathrm{wt}_2 \\ \mathrm{wt}_3 \\ \mathrm{wt}_4 \end{matrix} = \begin{matrix} 7 \\ 11 \\ 4 \\ 8 \end{matrix} + \begin{matrix} 13 \\ 8 \\ 6 \\ 19 \end{matrix} + \begin{matrix} 7 \\ 3 \\ 1 \\ 9 \end{matrix} + \begin{matrix} 2 \\ 3 \\ 15 \\ 4 \end{matrix} + \begin{matrix} 12 \\ 10 \\ 9 \\ 4 \end{matrix} + \begin{matrix} 3 \\ 7 \\ 5 \\ 14 \end{matrix} + \begin{matrix} 18 \\ 2 \\ 5 \\ 1 \end{matrix}$$

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$\mathrm{wt}_1, \mathrm{wt}_2, \mathrm{wt}_3 < \mathrm{wt}_4$? for some choice of $X_1, X_2, X_3 \in \mathbb{N}$?

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for no cycle decomposition satisfiable $\Longleftrightarrow$ $\mathcal{A}_1 \geq \mathcal{A}_2$

satisfiability decidable! (linear Diophantine inequalities)

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1. Check satisfiability for all cycle decompositions of “short” words

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1. Check satisfiability for all cycle decompositions of "short" words

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We show: $\mathcal{A}_1$ fin-amb $\Longrightarrow$ $\mathcal{A}_1 \geq \mathcal{A}_2$ decidable

Cycle Decomposition

$\vec{P_1}\quad x_1\quad \vec{P_2}\quad y_2\quad \vec{P_2}\quad x_3\quad \vec{P_3}\quad y_4\quad \vec{P_3}\quad x_5\quad \vec{P_2}\quad y_2\quad \vec{P_2}\quad x_7\quad \vec{P_5}$

Vectors of weights

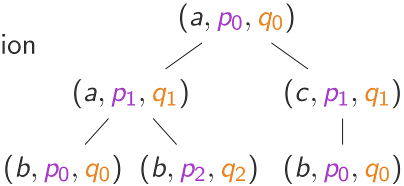
$$\begin{matrix}\mathrm{wt}_1\\ \mathrm{wt}_2\\ \mathrm{wt}_3\\ \mathrm{wt}_4\end{matrix} = \begin{matrix}7\\11\\4\\8\end{matrix} + \begin{matrix}13X_1\\8X_1\\6X_1\\19X_1\end{matrix} + \begin{matrix}7\\3\\1\\9\end{matrix} + \begin{matrix}2X_2\\3X_2\\15X_2\\4X_2\end{matrix} + \begin{matrix}12\\10\\9\\4\end{matrix} + \begin{matrix}13X_3\\8X_3\\6X_3\\19X_3\end{matrix} + \begin{matrix}18\\2\\5\\1\end{matrix}$$

$\mathrm{wt}_1, \mathrm{wt}_2, \mathrm{wt}_3 < \mathrm{wt}_4$? for some choice of $X_1, X_2, X_3 \in \mathbb{N}$?

1. Check satisfiability for all cycle decompositions of "short" words

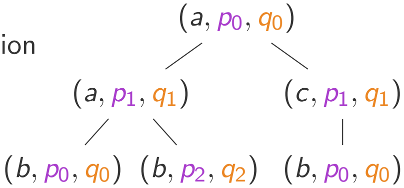
2. "Long words": one cycle two times $\Longrightarrow$ cut

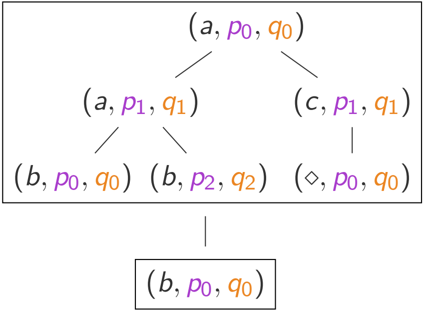
Cycle Decomposition



The Equivalence Problem on Trees

Cycle Decomposition





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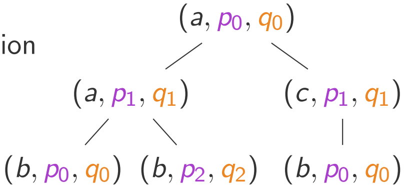
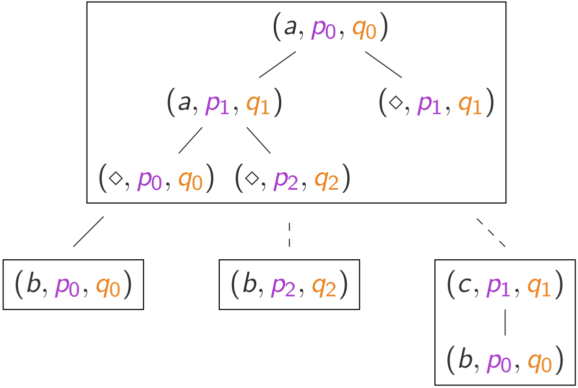
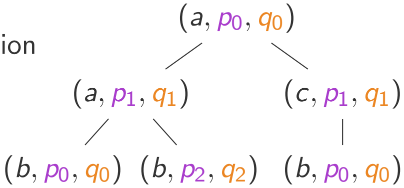
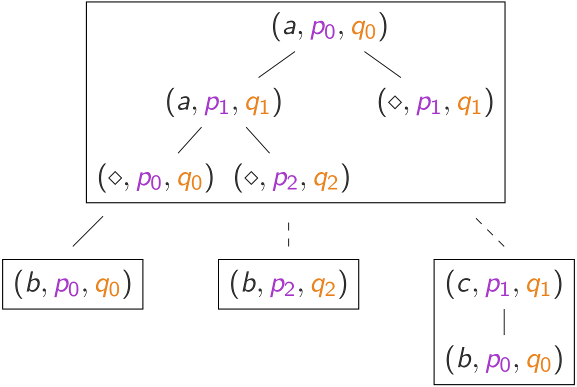




Cycle Decomposition



Removing Cycles?



Monitor Logics
for
Quantitative Monitor Automata

Motivation

item x restocked (in a shop) every Monday

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$\Rightarrow$ Quantitative Monitor Automata [Chatterjee, Henzinger, Otop '16]

Quantitative Monitor Automata

$\mathcal{A} = (\Sigma, Q, I, F, n, \delta, \mathrm{Val})$ Quantitative Monitor Automaton

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e.g. minimum, maximum, long-term average

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$\delta \subseteq Q \times \Sigma \times Q \times (\mathbb{Z} \cup \{s, t\})^n$ transitions

$$q_0 \xrightarrow[]{\substack{a \\ \binom{s}{0}}} q_1 \xrightarrow[]{\substack{a \\ \binom{1}{0}}} q_2 \xrightarrow[]{\substack{b \\ \binom{1}{s}}} q_3 \xrightarrow[]{\substack{a \\ \binom{t}{2}}} q_4 \xrightarrow[]{\substack{b \\ \binom{0}{3}}} q_5 \xrightarrow[]{\substack{a \\ \binom{s}{t}}} \dots$$

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$\delta \subseteq Q \times \Sigma \times Q \times (\mathbb{Z} \cup \{s, t\})^n$ transitions

$$q_0 \xrightarrow[2]{a\,\binom{s}{0}} q_1 \xrightarrow{a\,\binom{1}{0}} q_2 \xrightarrow{b\,\binom{1}{s}} q_3 \xrightarrow{a\,\binom{t}{2}} q_4 \xrightarrow{b\,\binom{0}{3}} q_5 \xrightarrow{a\,\binom{s}{t}} \dots$$

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$$q_0 \xrightarrow[2]{a \, \binom{s}{0}} q_1 \xrightarrow{a \, \binom{1}{0}} q_2 \xrightarrow[5]{b \, \binom{1}{s}} q_3 \xrightarrow{a \, \binom{t}{2}} q_4 \xrightarrow{b \, \binom{0}{3}} q_5 \xrightarrow{a \, \binom{s}{t}} \dots$$

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$\delta \subseteq Q \times \Sigma \times Q \times (\mathbb{Z} \cup \{s, t\})^n$ transitions

$$q_0 \xrightarrow[2]{a \begin{pmatrix} s \\ 0 \end{pmatrix}} q_1 \xrightarrow[\bot]{a \begin{pmatrix} 1 \\ 0 \end{pmatrix}} q_2 \xrightarrow[5]{b \begin{pmatrix} 1 \\ s \end{pmatrix}} q_3 \xrightarrow[\bot]{a \begin{pmatrix} t \\ 2 \end{pmatrix}} q_4 \xrightarrow[\bot]{b \begin{pmatrix} 0 \\ 3 \end{pmatrix}} q_5 \xrightarrow[3]{a \begin{pmatrix} s \\ t \end{pmatrix}} \cdots$$

$\mathcal{A} = (\Sigma, Q, I, F, n, \delta, \mathrm{Val})$ — Quantitative Monitor Automaton

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$\delta \subseteq Q \times \Sigma \times Q \times (\mathbb{Z} \cup \{s, t\})^n$ — transitions

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Weight of run: $\mathrm{Val}((z_i)_{i \geq 1})$

$\mathcal{A} = (\Sigma, Q, I, F, n, \delta, \mathrm{Val})$ Quantitative Monitor Automaton

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$\delta \subseteq Q \times \Sigma \times Q \times (\mathbb{Z} \cup \{s, t\})^n$ transitions

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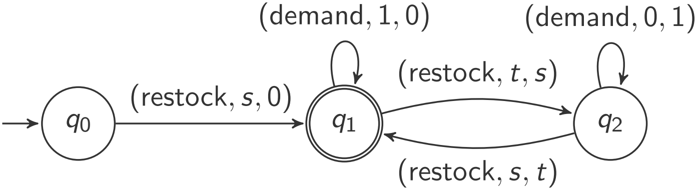
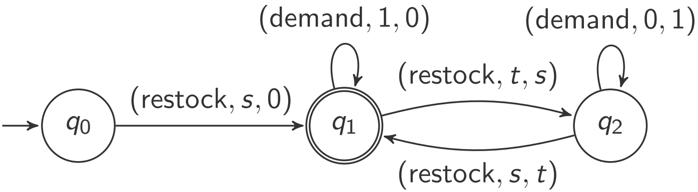
Weight of run: $\mathrm{Val}((z_i)_{i \geq 1})$

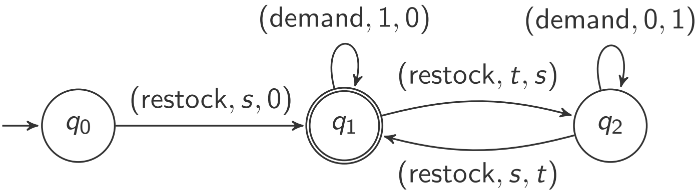
Weight of ω-word: infimum over all runs

Example





⇒ sequence 5, 3, 7, 4, ... of demands per week



⇒ sequence 5, 3, 7, 4, . . . of demands per week

valuation function to compute long-term average, minimum, ...

$$\beta ::= P_a(x) \mid x \leq y \mid x \in X \mid \neg\beta \mid \beta \vee \beta \mid \exists x.\beta \mid \exists X.\beta$$

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$$\psi ::= k \mid \beta \,?\, \psi : \psi$$

$$[\![\beta \,?\, \psi_1 : \psi_2]\!](w) = \begin{cases} [\![\psi_1]\!](w) & \text{if } w \models \beta \\ [\![\psi_2]\!](w) & \text{otherwise} \end{cases}$$

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$$\varphi = \inf Z.\left(\forall z.(z \in Z \leftrightarrow P_{\text{restock}}(z))\,?\,\mathrm{Val}\, x.\left(\bigoplus^{x,Z} y.1\right) : \infty\right)$$

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Muller automata: $\mathrm{Val}\, x.\psi$

$z_i = [\![\psi]\!](w[x \to i])$

$[\![\mathrm{Val}\, x.\psi]\!](w) = \mathrm{Val}(z_0, z_1, z_2, \ldots)$

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$w = a_0 a_1 a_2 a_3 a_4 \ldots \longrightarrow \binom{a_0}{z_0} \binom{a_1}{z_1} \binom{a_2}{z_2} \binom{a_3}{z_3} \binom{a_4}{z_4} \ldots$

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"correct weights" is a recognizable property

Monitor Logics for Quantitative Monitor Automata

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QMA: $\mathrm{Val}\, x.\zeta_x$

$$\varphi = \mathrm{Val}\, x. \left(\bigoplus^{x,Z} y.1\right)$$

$$\begin{pmatrix} \text{restock} \\ s \\ \bot \\ 1 \end{pmatrix} \begin{pmatrix} \text{demand} \\ 1 \\ \bot \\ 0 \end{pmatrix} \begin{pmatrix} \text{demand} \\ 1 \\ \bot \\ 0 \end{pmatrix} \begin{pmatrix} \text{restock} \\ t \\ s \\ 1 \end{pmatrix} \begin{pmatrix} \text{demand} \\ \bot \\ 1 \\ 0 \end{pmatrix} \cdots$$