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From Non-monotonic Logics to Abstract Argumentation

Results and Perspectives

Antrittsvorlesung
Professur für Formale Argumentation
und Logisches Schließen

4th December 2023
Leipzig

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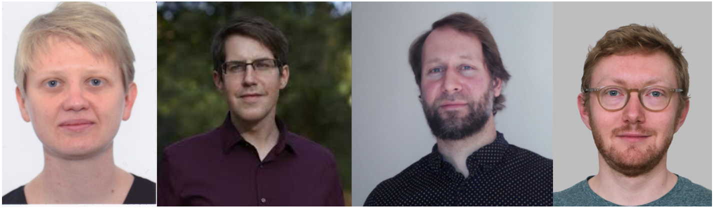

Monotonic vs. Non-monotonic Logics

Monotonic vs. Non-monotonic **Logics**

- do not allow for a retraction of inferences, i.e.

$$\text{If } S \subseteq T, \text{ then } Cn(S) \subseteq Cn(T).$$

$$\text{If } S \subseteq T \text{ and } S \vDash \phi, \text{ then } T \vDash \phi.$$

- propositional logic, first-order logic, intuitionistic logic, ...

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- propositional logic, first-order logic, intuitionistic logic, ...
- monotonic reasoning is good for mathematics
- Example: group axioms, uniqueness of the neutral element

Monotonic vs. **Non-monotonic Logics**

- represent defeasible inference, i.e.

$$S \subseteq T \text{ and } Cn(S) \nsubseteq Cn(T) \text{ is possible.}$$

$$S \subseteq T,\ S \models \phi \text{ and } T \not\models \phi \text{ is possible.}$$

- default logic, circumscription, autoepistemic logic, . . .

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$$S \subseteq T,\ S \models \phi \text{ and } T \not\models \phi \text{ is possible.}$$

- default logic, circumscription, autoepistemic logic, . . .
- reason: incomplete and/or uncertain information
- defeasible reasoning is the reasoning mode for “daily life”

☞ draw conclusions defeasibly

Monotonic vs. **Non-monotonic Logics**

Draw conclusions based on normality assumptions.

Monotonic vs. **Non-monotonic Logics**

Draw conclusions based on normality assumptions.

- Professors teach
- Birds fly
- Owls hunt at night
- Students don't like the 7th and 8th period
- Waiting for two hours at the doctor's office is frustrating
- The human heart is on the left side
- Kids like ice cream

Monotonic vs. **Non-monotonic Logics**

Draw conclusions based on normality assumptions.

- Professors teach . . . unless they are on sabbatical.
- Birds fly
- Owls hunt at night
- Students don't like the 7th and 8th period
- Waiting for two hours at the doctor's office is frustrating
- The human heart is on the left side
- Kids like ice cream

Monotonic vs. **Non-monotonic Logics**

Draw conclusions based on normality assumptions.

- Professors teach
- Birds fly . . . unless they are penguins.
- Owls hunt at night
- Students don't like the 7th and 8th period
- Waiting for two hours at the doctor's office is frustrating
- The human heart is on the left side
- Kids like ice cream

Monotonic vs. **Non-monotonic Logics**

Draw conclusions based on normality assumptions.

- Professors teach
- Birds fly
- Owls hunt at night . . . unless they live in a zoo.
- Students don't like the 7th and 8th period
- Waiting for two hours at the doctor's office is frustrating
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Monotonic vs. **Non-monotonic Logics**

Draw conclusions based on normality assumptions.

- Professors teach
- Birds fly
- Owls hunt at night
- Students don't like the 7th and 8th period . . . unless it's their favorite subject.
- Waiting for two hours at the doctor's office is frustrating
- The human heart is on the left side
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Monotonic vs. **Non-monotonic Logics**

Draw conclusions based on normality assumptions.

- Professors teach
- Birds fly
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- Waiting for two hours at the doctor's office is frustrating . . . unless you are close to finish a proof.
- The human heart is on the left side

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Monotonic vs. **Non-monotonic Logics**

Draw conclusions based on normality assumptions.

- Professors teach
- Birds fly
- Owls hunt at night
- Students don't like the 7th and 8th period
- Waiting for two hours at the doctor's office is frustrating
- The human heart is on the left side . . . unless one has dextrocardia.
- Kids like ice cream

Monotonic vs. **Non-monotonic Logics**

Draw conclusions based on normality assumptions.

- Professors teach
- Birds fly
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- Students don't like the 7th and 8th period
- Waiting for two hours at the doctor's office is frustrating
- The human heart is on the left side
- Kids like ice cream . . . ~~unless~~ no exceptions!

Non-monotonic Logics

Example (Rule-based Formalism)

1. Knowledge Base

$$r_1: \quad \Rightarrow a$$
$$r_2: \quad a \Rightarrow b$$
$$r_3: \quad b \rightarrow not\ a$$
$$r_4: \quad \rightarrow c$$
$$r_5: \quad c \Rightarrow not\ b$$

If a, then normally b.

If b, then definitely not a.

Non-monotonic Logics

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If a, then normally b.

If b, then definitely not a.

2. Conclusion

Conc = {a,c}

Towards Abstract Argumentation - The Paradigm Shift

Seminal Paper by Phan Minh Dung, *On the acceptability of arguments and its fundamental role in nonmonotonic reasoning, logic programming and n- person games*, AIJ, 1995.

Towards Abstract Argumentation - The Paradigm Shift

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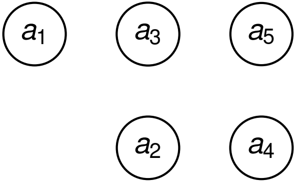
On the acceptability of arguments and its fundamental role in nonmonotonic reasoning, logic programming and n- person games, AIJ, 1995.

Two main ideas:

1. non-monotonic reasoning can be modelled as a kind of argumentation
2. determining the acceptability of arguments can be done on an abstract level

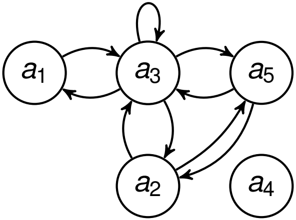
Abstract away from

- the internal structure of arguments, and (nodes)



Abstract away from

- the internal structure of arguments, and (nodes)
- the reason why an argument attacks an other (edges)



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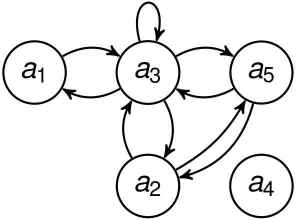
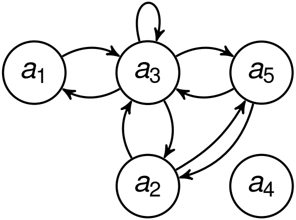


☞ an argumentation scenario is simply a directed graphs

How to select reasonable positions?



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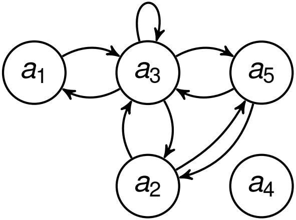


Definition

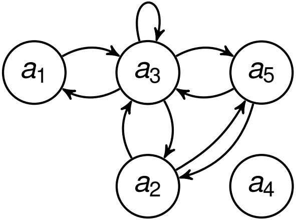
A semantics is a total function

$$\sigma : \mathcal{F} \to 2^{2^{\mathcal{U}}} \quad F = (A, R) \mapsto \sigma(F) \subseteq 2^A.$$

($\mathcal{F}$ - set of all AFs)

($\mathcal{U}$ - set of all arguments)

Semantics (select reasonable positions)



$ad(F) = \{\emptyset, \{a_1\}, \{a_2\}, \{a_4\}, \{a_5\}, \{a_1, a_2\}, \{a_1, a_4\}, \{a_1, a_5\}, \{a_2, a_4\}, \{a_4, a_5\}, \{a_1, a_2, a_4\}, \{a_1, a_4, a_5\}\}$

Definition

Admissible semantics is a total function

$$ad : \mathcal{F} \rightarrow 2^{2^{\mathcal{U}}} \quad F = (A, R) \mapsto ad(F) \subseteq 2^A.$$

$E \in ad(F)$ iff

1. $\forall a, b \in E : (a, b) \notin R$ (conflict-freeness)
2. $\forall a, b \, ((a, b) \in R \wedge b \in E \rightarrow \exists c \in E : (c, a) \in R)$ (defense)

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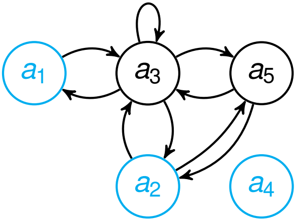

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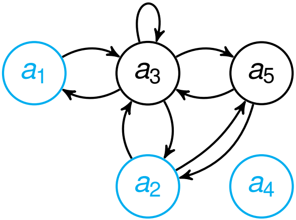

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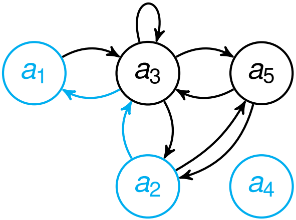

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Reconstruction via Argumentation

Example (Rule-based Formalism)

1. Knowledge Base

$$
\begin{aligned}
r_1 &: \quad \Rightarrow a \\
r_2 &: \quad a \Rightarrow b \\
r_3 &: \quad b \rightarrow not\ a \\
r_4 &: \quad \rightarrow c \\
r_5 &: \quad c \Rightarrow not\ b
\end{aligned}
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Reconstruction via Argumentation

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1. Knowledge Base

r_1 :	$\Rightarrow a$
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r_4 :	$\rightarrow c$
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$\Longrightarrow$

2. Arguments

a_1 :	$[r_1 \mid a]$
a_2 :	$[a_1, r_2 \mid b]$
a_3 :	$[a_2, r_3 \mid not\ a]$
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Reconstruction via Argumentation

Example (Rule-based Formalism)

1. Knowledge Base		**2. Arguments**	
r_1 :	$\Rightarrow a$	a_1 :	$[r_1 \mid a]$
r_2 :	$a \Rightarrow b$	a_2 :	$[a_1, r_2 \mid b]$
r_3 :	$b \rightarrow not\ a$	a_3 :	$[a_2, r_3 \mid not\ a]$
r_4 :	$\rightarrow c$	a_4 :	$[r_4 \mid c]$
r_5 :	$c \Rightarrow not\ b$	a_5 :	$[a_4, r_5 \mid not\ b]$

a_1 claims a justified by r_1

a_2 claims b justified by a_1 and r_2

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⟹

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⟹

3. Conflicts

c_1 :	a_1 attacks a_3
c_2, c_3 :	a_2 attacks a_3, a_5
c_4, c_5, c_6, c_7 :	a_3 attacks a_1, a_2, a_3, a_5
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Reconstruction via Argumentation

Example (Rule-based Formalism)

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$r_3 : b \rightarrow \mathit{not}\ a$

$r_4 : \rightarrow c$

$r_5 : c \Rightarrow \mathit{not}\ b$

2. Arguments

$a_1 : [r_1 \mid a]$

$a_2 : [a_1, r_2 \mid b]$

$a_3 : [a_2, r_3 \mid \mathit{not}\ a]$

$a_4 : [r_4 \mid c]$

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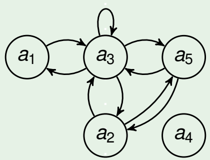
4. Instantiation



Reconstruction via Argumentation

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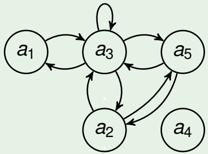

5. Resolving

$E_1 = \{a_1, a_2, a_4\}$

$E_2 = \{a_1, a_4, a_5\}$

Reconstruction via Argumentation

Example (Rule-based Formalism)

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2. Arguments

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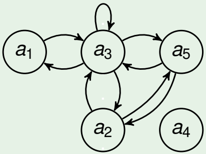

5. Resolving

$E_1 = \{a_1, a_2, a_4\}$
$E_2 = \{a_1, a_4, a_5\}$

6. Conclusion

$E_1 = \{a, b, c\}$
$E_2 = \{a, c, \textit{not } b\}$

Conc = {a,c}

Reconstruction, Explanation via Argumentation

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2. Arguments

$a_1 : \quad [r_1 \mid a]$
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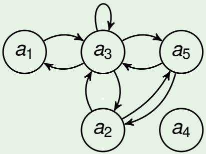

5. Resolving

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$E_1 = \{a, b, c\}$
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Conc = {a,c}

Explainability

EU's General Data Protection Regulation, 2018

" ...establishes a right for all individuals to obtain meaningful explanations of the logic involved when automated (algorithmic) decision making takes place."

Explainability

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German AI strategy, 2020

"...making AI explainable, accountable, and transparent is the key to winning over the public's trust. There are, however, a larger number of applications where the technology is still a black box..."

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3. Conflicts

$c_1 :$ a_1 attacks a_3
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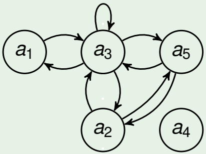
4. Instantiation



5. Resolving

$E_1 = \{a_1, a_2, a_4\}$
$E_2 = \{a_1, a_4, a_5\}$

6. Conclusion

$E_1 = \{a, b, c\}$
$E_2 = \{a, c, not\ b\}$

Conc = {a,c}

Some Contributions:

Some Contributions: Simplification

Example (Propositional Logic)

$$S = \{a, a \to b, \neg b \vee c, e \wedge f \to d, d \leftrightarrow e\}$$

Some Contributions: Simplification

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$$\equiv \{a, \top \to b, \neg b \vee c, e \wedge f \to d, d \leftrightarrow e\}$$

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Example (Propositional Logic)

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&\equiv \{a, b \qquad , \neg b \vee c, e \wedge f \rightarrow d, d \leftrightarrow e\}
\end{aligned}
$$

Some Contributions: Simplification

Example (Propositional Logic)

$$
\begin{aligned}
S &= \{a, a \to b, \neg b \lor c, e \land f \to d, d \leftrightarrow e\} \\
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&\equiv \{a, b, c, \top, d \leftrightarrow e\} \\
&\equiv \{a, b, c, d \leftrightarrow e\} = T
\end{aligned}
$$

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Example (Propositional Logic)

$$S = \{a, a \rightarrow b, \neg b \vee c, e \wedge f \rightarrow d, d \leftrightarrow e\} \text{ and}$$
$$T = \{a, b, c, d \leftrightarrow e\}$$

are equivalent, i.e. $Mod(S) = Mod(T)$.

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Example (Propositional Logic)

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$$T = \{a, b, c, d \leftrightarrow e\}$$

are equivalent, i.e. $Mod(S) = Mod(T)$.
Moreover, they are even strongly equivalent, i.e.

$$\text{For each } H, \text{ we have: } Mod(S \cup H) = Mod(T \cup H).$$

Proof:

$$\begin{aligned} Mod(S \cup H) &= Mod(S) \cap Mod(H) \\ &= Mod(T) \cap Mod(H) \\ &= Mod(T \cup H) \end{aligned}$$

Some Contributions: Simplification

- Argumentation semantics σ does not possess the intersection property, i.e.

$$\sigma\,(F \sqcup H) \neq \sigma(F) \cap \sigma(H) \text{ is possible.}$$

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$$\sigma(F \sqcup H) \neq \sigma(F) \cap \sigma(H) \text{ is possible.}$$

- but, so-called kernels guarantee strong equivalence
- admissible kernel deletes an attack $(a, b) \in R$ if

$$a \neq b, \ (a, a) \in R, \ \{(b, a), (b, b)\} \cap R \neq \emptyset$$

Some Contributions: Simplification

Example (Rule-based Formalism,)

1. Knowledge Base

r_1 :	$\Rightarrow a$
r_2 :	$a \Rightarrow b$
r_3 :	$b \rightarrow \textit{not } a$
r_4 :	$\rightarrow c$
r_5 :	$c \Rightarrow \textit{not } b$

2. Arguments

a_1 :	$[r_1 \mid a]$
a_2 :	$[a_1, r_2 \mid b]$
a_3 :	$[a_2, r_3 \mid \textit{not } a]$
a_4 :	$[r_4 \mid c]$
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3. Conflicts

c_1 :	a_1 attacks a_3
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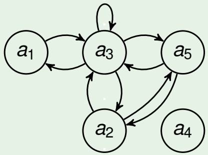

Some Contributions: Simplification

Example (Rule-based Formalism, strong equivalence)

1. Knowledge Base

$r_1 : \quad \Rightarrow a$

$r_2 : \quad a \Rightarrow b$

$r_3 : \quad b \rightarrow \textit{not } a$

$r_4 : \quad \rightarrow c$

$r_5 : \quad c \Rightarrow \textit{not } b$

2. Arguments

$a_1 : \quad [r_1 \mid a]$

$a_2 : \quad [a_1, r_2 \mid b]$

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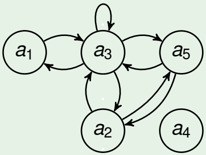
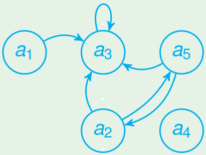


5. Simplification



Some Contributions: Simplification

Example (strong expansion equivalence)

1. Knowledge Base

$r_1: \quad \Rightarrow a$

$r_2: \quad a \Rightarrow b$

$r_3: \quad b \rightarrow \textit{not } a$

$r_4: \quad \rightarrow c$

$r_5: \quad c \Rightarrow \textit{not } b$

2. Arguments

$a_1: \quad [r_1 \mid a]$

$a_2: \quad [a_1, r_2 \mid b]$

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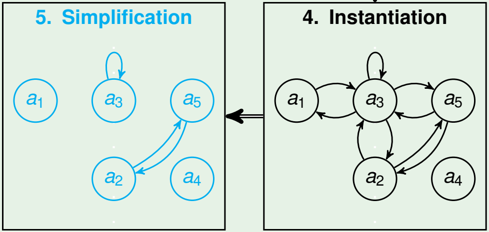
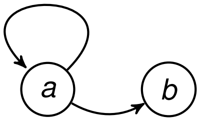


Some Contributions: Odd-cycles

- A 25 year old problem

"An interesting topic of research is the problem of self-defeating arguments as illustrated in the following example.



The only admissible extension here is empty though one can argue that since a defeats itself, b should be acceptable."

[Dung, 1995]

Some Contributions: Odd-cycles

Definition

Weak Admissibility semantics is a total function

$$ad^w : \mathcal{F} \to 2^{2^{\mathcal{U}}} \quad F = (A, R) \mapsto ad^w(F) \subseteq 2^A.$$

$E \in ad^w(F)$ iff

Some Contributions: Odd-cycles

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$E \in ad^w(F)$ iff

1. E is conflict-free, and
2. for any attacker y of E we have $y \notin \bigcup ad^w(F^E)$.

F^E is the AF F restricted to $A \setminus (E \cup E^+)$ (E-reduct)

KR 2020, *Comparing Weak Admissibility Semantics to their Dung-style Counterparts - Reduct, Modularization and Strong Equivalence*, R. Baumann, G. Brewka and M. Ulbricht (**Ray Reiter Best Paper Award**)

Some Contributions: Odd-cycles

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☞ recursive definition

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☞ main idea

KR 2020, *Comparing Weak Admissibility Semantics to their Dung-style Counterparts - Reduct, Modularization and Strong Equivalence*, R. Baumann, G. Brewka and M. Ulbricht (**Ray Reiter Best Paper Award**)

Recursiveness in action

Definition

1. E is conflict-free, and
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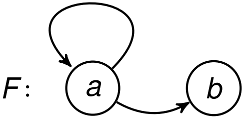
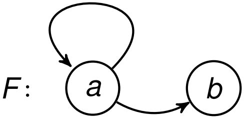


Is $E = \{b\}$ weakly admissible in F?

Recursiveness in action

Definition

1. E is conflict-free, and ✔
2. any attacker y is counter-attacked or itself not acceptable.



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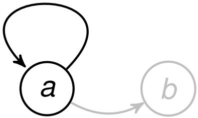
F^E:



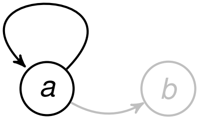
Yes, if a is not contained in a weakly admissible set of F^E.

Recursiveness in action

Definition

1. E is conflict-free, and ✔
2. any attacker y is counter-attacked or itself not acceptable. ✔

F^E:



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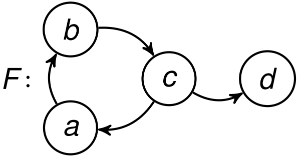


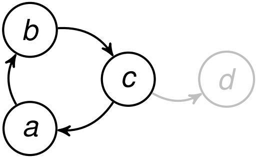
Is $E = \{d\}$ weakly admissible in F?

Recursiveness in action

Definition

1. E is conflict-free, and ✔
2. any attacker y is counter-attacked or itself not acceptable. ✍

F^E:



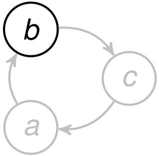
Yes, if c is not contained in a w-admissible set of F^E.

Recursiveness in action

Definition

1. E is conflict-free, and ✔
2. any attacker y is counter-attacked or itself not acceptable. ✍

$(F^E)^{\{c\}}$:



Yes, if b is contained in a w-admissible set of $(F^E)^{\{c\}}$.

Recursiveness in action

Definition

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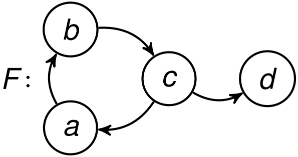

Yes, $E = \{d\}$ is weakly admissible in F.

For interested students

Two lectures dealing with the presented topics.

1. lecture “Nichtmonotones Schließen”
   2+1, winter term
2. lecture “Formale Argumentation”
   2+1, summer term

Beyond Reconstruction

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Argumentation, a phenomenon we are all familiar with, arises in response to, or in anticipation of, a real or imagined difference of opinion.

[van Eemeren and Verheij, 2017]

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dialogues, persuasion, negotiation, decision making ...

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[van Eemeren and Verheij, 2017]

dialogues, persuasion, negotiation, decision making . . .

Computational argumentation deals with formal models of an argument as well as approaches and techniques formalizing inference on the basis of arguments.

Limitations of Dung AFs

They cannot express:

- support between arguments
- collective attacks
- attacks on attacks
- values
- preferences
- …

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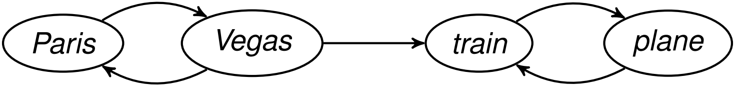
$\Rightarrow$ need for more expressive frameworks

Abstract Dialectical Frameworks

- most powerful generalization of Dung AFs
- use acceptance conditions instead of attack arcs



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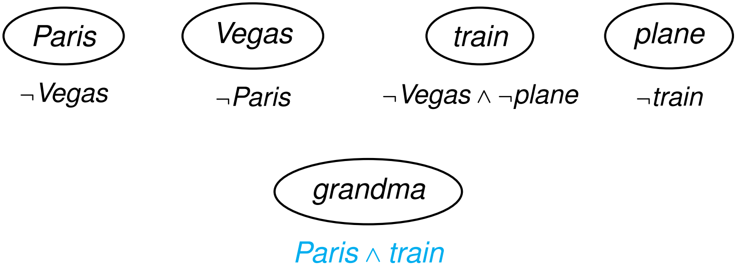

"Grandma lives in a suburb of Paris, which would be a stop on the train route."

Abstract Dialectical Frameworks

- semantics rely on the $\mathcal{C}_D$-operator

Definition

For an ADF $D = (S, P)$ we define $\mathcal{C}_D : \mathcal{V}_3^D \mapsto \mathcal{V}_3^D$ as

$$\mathcal{C}_D(v) : S \mapsto \{t, f, u\} \text{ with } s \mapsto \sqcap_i \left\{ w(\phi_s) \mid w \in [v]_2^D \right\}.$$

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- $\mathcal{V}_3^D = \{v \mid v : S \to \{t, f, u\}\}$ (three-valued interpretation)
- the information order $<_i$ is defined as: $u <_i t$ and $u <_i f$
- $\leq_i$ is the reflexive closure and $\sqcap_i$ is the consensus, i.e.

$$t \sqcap_i t = t, \quad f \sqcap_i f = f, \quad \text{and } u \text{ otherwise}$$

- $[v]_2^D = \{w \mid w : S \to \{t, f\}, v \leq_i w\}$ (two-valued completions)

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$$\mathcal{C}_D(v) : S \mapsto \{t, f, u\} \text{ with } s \mapsto \sqcap_i \left\{ w(\phi_s) \mid w \in [v]_2^D \right\}.$$

Definition

Given an ADF $D = (S, P)$ and $v \in \mathcal{V}_3^D$.

1. $v \in ad(D)$ iff $v \leq_i \mathcal{C}_D(v)$,
2. $v \in co(D)$ iff $v = \mathcal{C}_D(v)$,
3. $v \in pr(D)$ iff v is $\leq_i$-maximal in $co(D)$, and
4. $v \in gr(D)$ iff v is $\leq_i$-least in $co(D)$.

Expressive Argumentation - Planned Research Topics

1. New Semantics and Functionalities
   weak admissibility, weak defense, time, modality
2. Foundations
   realizability, replaceability, intertranslatability, modularity
3. Dynamics
   revision, contraction, expansion, enforcing, forgetting
4. Algorithms
   algorithm design and implementation of prototype systems

Computation of the Consensus

Q: Is there a three-valued logic $\mathcal{L}_3$, s.t. for any formula ϕ, any three-valued v: $v^{\mathcal{L}_3}(\phi) = \sqcap_i \left\{ w(\phi) \mid w \in [v]_2^D \right\}$?

2010, *Abstract Dialectical Frameworks*, G. Brewka and S. Woltran

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A1: There is *no truth-functional* *three-valued logic $\mathcal{L}_3$.*

2010, *Abstract Dialectical Frameworks*, G. Brewka and S. Woltran
2020, *Timed Abstract Dialectical Frameworks*, R. Baumann and M. Heinrich

Computation of the Consensus

Q: Is there a three-valued logic $\mathcal{L}_3$*, s.t. for any formula* ϕ*, any three-valued* v*:* $v^{\mathcal{L}_3}(\phi) = \sqcap_i \{w(\phi) \mid w \in [v]_2^D\}$?

A1: There is no truth-functional three-valued logic $\mathcal{L}_3$*.*

A2: There is a non-truth-functional three-valued logic, so-called Possibilistic Logic.

2010, *Abstract Dialectical Frameworks*, G. Brewka and S. Woltran
2020, *Timed Abstract Dialectical Frameworks*, R. Baumann and M. Heinrich
2022, *Possibilistic Logic Underlies Abstract Dialectical Frameworks*, J. Heynick, G. Kern-Isberner and M. Thimm

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A1: There is no truth-functional three-valued logic $\mathcal{L}_3$.

A2: There is a non-truth-functional three-valued logic, so-called Possibilistic Logic.

A3: There is a truth-functional three-valued logic, so-called Kleene's Strong Logic, if considering bipolar formulae only.

2010, *Abstract Dialectical Frameworks*, G. Brewka and S. Woltran
2020, *Timed Abstract Dialectical Frameworks*, R. Baumann and M. Heinrich
2022, *Possibilistic Logic Underlies Abstract Dialectical Frameworks*, J. Heynick, G. Kern-Isberner and M. Thimm
2023, *Bipolar Abstract Dialectical Frameworks are covered by Kleene's 3-valued Logic*, R. Baumann and M. Heinrich

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Results and Perspectives

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