Geometric Analogue of Holographic Reduced Representations

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Outline

- Background
  - Distributed representations
  - Previous architectures
- GA model
  - Binary parametrization of the geometric product
  - Cartan representation
  - Example
  - Signatures
- Test results
  - Recognition tests
  - Blade linearity
- Future work
- Computer program *CartanGA*

Distributed representations

Example

Distributed representations

Example

"green color"

+ "triangular shape"

= "green triangle"

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    - in most distributed representations only the overall pattern of activated units has a meaning (resemblance to a multi-superimposed photograph)

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Atomic objects and complex statements

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- roles: $bite_{agent}$, $bite_{object}$
- fillers: $Fido$, Pat

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$$see_{agent} \circledast John + see_{object} \circledast \text{“}Fido\ bit\ Pat\text{”} = \text{“John saw Fido bit Pat”}$$

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Binding and chunking

$\circledast$ binding
$+$ superposition (also called *chunking*)

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"What did Fido do?"

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Clean-up memory

- An auto-associative collection of elements and statements (excluding single chunks) produced by the system.
- Given a noisy extracted vector such structure must be able to:
  - either recall the most similar item stored ...
  - ... or indicate, that no matching object had been found.
- We need a measure of similarity (in most models: scalar product, Hamming/Euclidean distance).

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Advantages

- Noise tolerance, good error correction properties.
- Storage efficiency.
- The number of superimposed patterns is not fixed.

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- Similarity measure: dot product

Binary parametrization of the geometric product

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Let $\{x_1, \ldots, x_n\}$ be a string of bits and $\{e_1, \ldots, e_n\}$ be base vectors. Then

$$e_{x_1 \ldots x_n} = e_1^{x_1} \ldots e_n^{x_n}$$

where $e_k^0 = \mathbf{1}$.

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$$1 = e_{0\ldots0} \qquad e_1 = e_{10\ldots0} \qquad e_{235} = e_{011010\ldots0}$$

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More formally, for two arbitrary strings of bits we have:

$$e_{A_1 \dots A_n} e_{B_1 \dots B_n} = (-1)^{\sum\limits_{k<l} B_k A_l} e_{(A_1 \dots A_n)\oplus(B_1 \dots B_n)}$$

Matrix representation

Pauli's matrices

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

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Cartan representation

$$\begin{aligned} b_{2k} &= \underbrace{\sigma_1 \otimes \cdots \otimes \sigma_1}_{n-k} \otimes \sigma_2 \otimes \underbrace{1 \otimes \cdots \otimes 1}_{k-1}, \\ b_{2k-1} &= \underbrace{\sigma_1 \otimes \cdots \otimes \sigma_1}_{n-k} \otimes \sigma_3 \otimes \underbrace{1 \otimes \cdots \otimes 1}_{k-1}. \end{aligned}$$

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PSmith's name $= PSmith \sharp name^{-1} = PSmith \circledast name$

Example

Encoding and Decoding

PSmith

$$\begin{aligned} PSmith &= name \circledast Pat + sex \circledast male + age \circledast 66 \\ &= c_{00010}c_{00100} + c_{11100}c_{00111} + c_{10001}c_{11000} \\ &= -c_{00110} + c_{11011} + c_{01001} \end{aligned}$$

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Example

Clean-up

Scalar product

Scalar (inner) product is performed by the means of matrix trace.

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$$\begin{aligned}\langle Pat' | e_{Pat} \rangle &= Tr\Big((-c_{00100} - c_{11001} - c_{01011})c_{00100}\Big) \\ &= Tr\Big(-\mathbf{1} + c_{11101} - c_{01111}\Big) \\ &= -32 \neq 0\end{aligned}$$

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\begin{aligned}
\langle Pat' | e_{Pat} \rangle &= Tr\Big((-c_{00100} - c_{11001} - c_{01011}) c_{00100} \Big) \\
&= Tr\Big(-\mathbf{1} + c_{11101} - c_{01111} \Big) \\
&= -32 \neq 0 \\
\langle Pat' | e_{male} \rangle &= 0 \\
\langle Pat' | e_{66} \rangle &= 0 \\
\langle Pat' | e_{name} \rangle &= 0 \\
\langle Pat' | e_{sex} \rangle &= 0 \\
\langle Pat' | e_{age} \rangle &= 0 \\
\langle Pat' | PSmith \rangle &= 0
\end{aligned}
$$

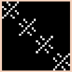
Signatures

Examples

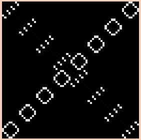
Signatures

Examples

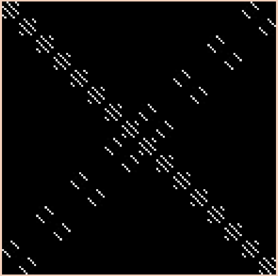
PSmith, $n = 5$



PSmith, $n = 6$



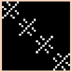
PSmith, $n = 7$



Signatures

Examples

PSmith, $n = 5$



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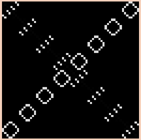

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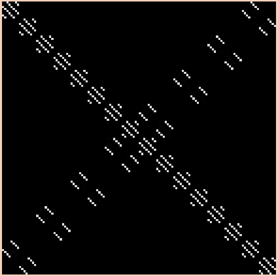

Signatures:

 LHS-signature

 RHS-signature

Agnieszka Patyk, patyk@mif.pg.gda.pl

Signatures

Dimensions

Signatures

Dimensions

- $2^{\lfloor \frac{n}{2} \rfloor}$ signatures on one of the diagonals.
- Each signature is of dimensions $2^{\lceil \frac{n}{2} \rceil} \times 2^{\lceil \frac{n}{2} \rceil}$.

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Cartan representation

$$
\begin{aligned}
b_{2k} &= \underbrace{\sigma_1 \otimes \cdots \otimes \sigma_1}_{at\ least\ \lfloor \frac{n}{2} \rfloor} \otimes \sigma_2 \otimes \underbrace{1 \otimes \cdots \otimes 1}_{k-1}, \\
b_{2k-1} &= \underbrace{\sigma_1 \otimes \cdots \otimes \sigma_1}_{at\ least\ \lfloor \frac{n}{2} \rfloor} \otimes \sigma_3 \otimes \underbrace{1 \otimes \cdots \otimes 1}_{k-1}.
\end{aligned}
$$

Recognition tests

Construction

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Vocabulary/Sentence set

Similar sentences, e.g:

- Fido bit Pat.
- Fido bit PSmith.
- Pat fled from Fido.
- PSmith fled from Fido.
- Fido bit PSmith causing PSmith to flee from Fido.
- Fido bit Pat causing Pat to flee from Fido.
- John saw that Fido bit PSmith causing PSmith to flee from Fido.
- John saw that Fido bit Pat causing Pat to flee from Fido.
- etc...

Altogether 42 atomic objects and 19 sentences.

Recognition tests

Agent-Object construction

Works better for:

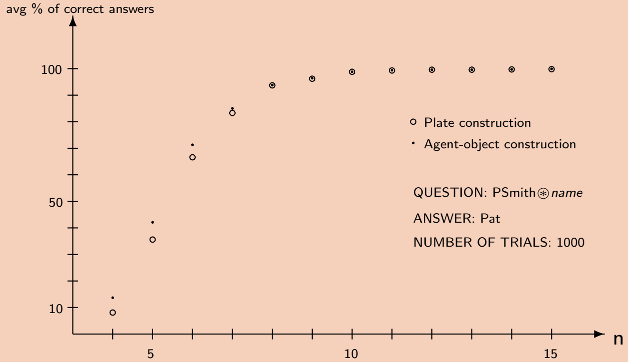
- simple sentences and simple answers

Recognition tests

Agent-Object construction

Works better for:

- simple sentences and simple answers



Recognition tests

Agent-Object construction

Works better for:

- simple sentences and simple answers,
- nested sentences, from which a rather unique information is to be derived.

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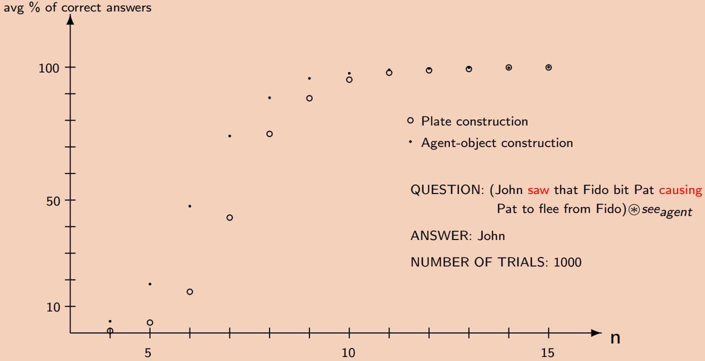
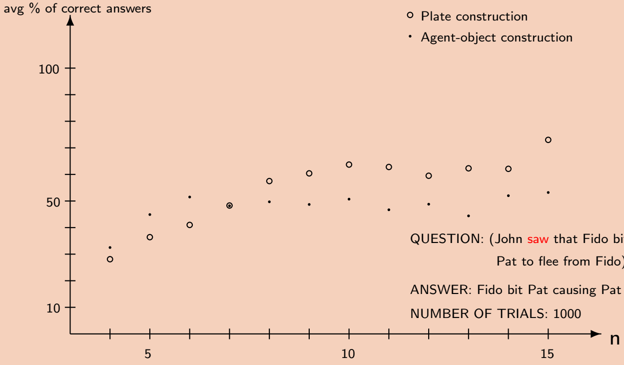


Recognition tests

Plate (HRR) construction

Works better for nested sentences, from which a complex information needs to be derived.



QUESTION: (John saw that Fido bit Pat causing Pat to flee from Fido)$\circledast$ see_{object}

ANSWER: Fido bit Pat causing Pat to flee from Fido

NUMBER OF TRIALS: 1000

Blade linearity

Blade linearity

Problems

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Problems

- How often does the system produce identical blades representing atomic objects?

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- How do the two above problems affect the number of (pseudo-) correct answers?

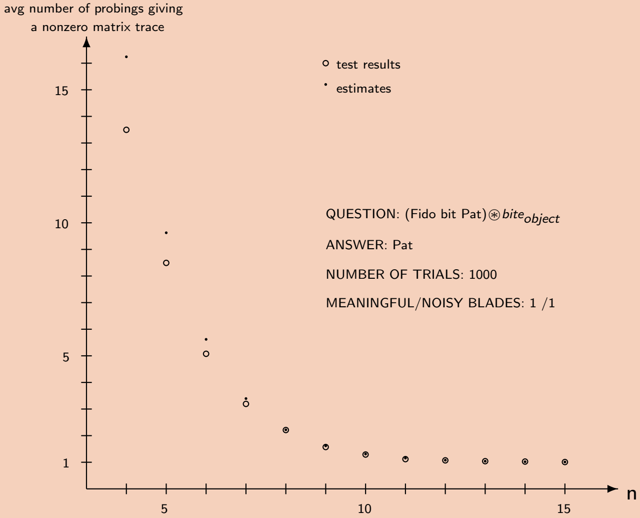
Assumption: ideal conditions

No two chunks of a sentence at any time are identical (up to a constant).

$$\langle A|C\rangle \neq 0 \equiv A \text{ and } C \text{ share a common blade}$$

Blade linearity

One meaningful blade, $L > 0$ noisy blades



QUESTION: (Fido bit Pat)$\circledast bite_{object}$

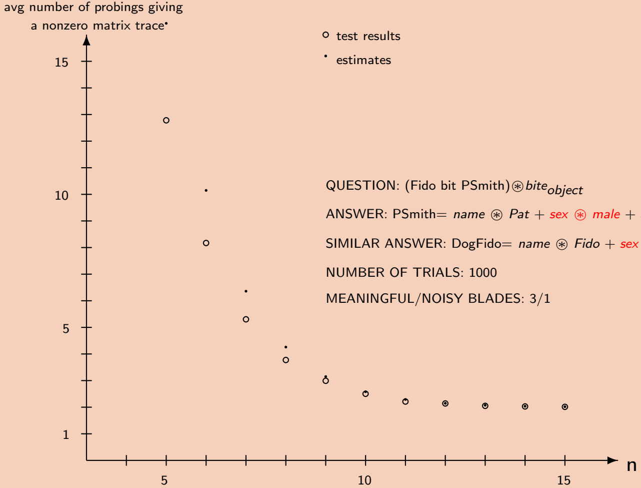
ANSWER: Pat

NUMBER OF TRIALS: 1000

MEANINGFUL/NOISY BLADES: 1 /1

Blade linearity

$K > 1$ meaningful blades, $L > 0$ noisy blades



QUESTION: (Fido bit PSmith)$\circledast bite_{object}$

ANSWER: PSmith= $name \circledast Pat + sex \circledast male + age \circledast 66$

SIMILAR ANSWER: DogFido= $name \circledast Fido + sex \circledast male + \ldots$

NUMBER OF TRIALS: 1000

MEANINGFUL/NOISY BLADES: 3/1

What's next?

Issues

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- Decide on construction (something between Plate and Agent-Object?).

What's next?

Issues

- Decide on construction (something between Plate and Agent-Object?).
- Tests for greater n.

What's next?

Issues

- Decide on construction (something between Plate and Agent-Object?).
- Tests for greater n.
- Scaling.

References

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