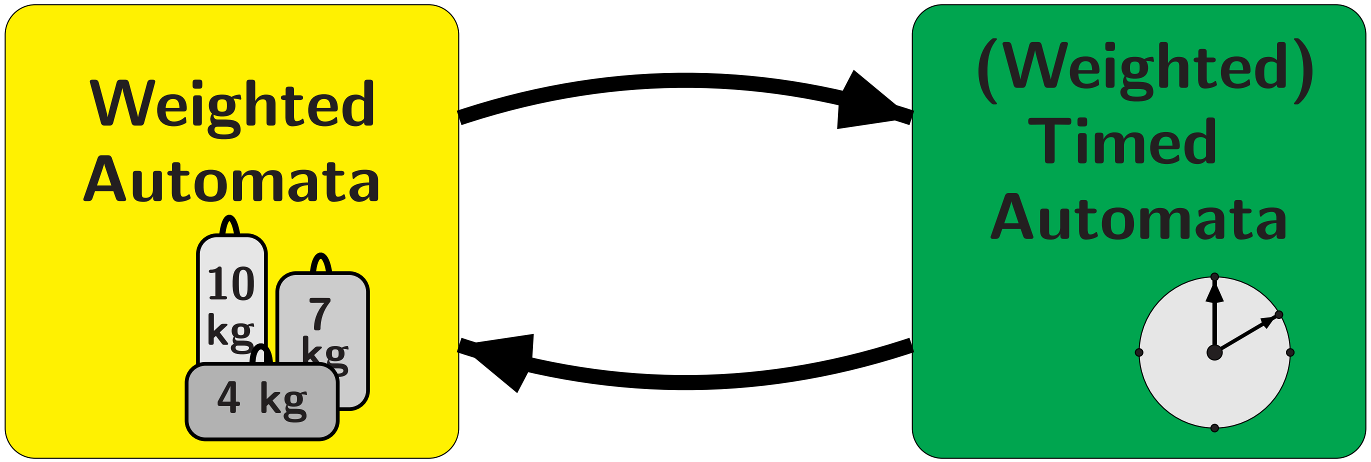
The Fruitful Interplay of Weighted Timed Automata and Weighted Automata

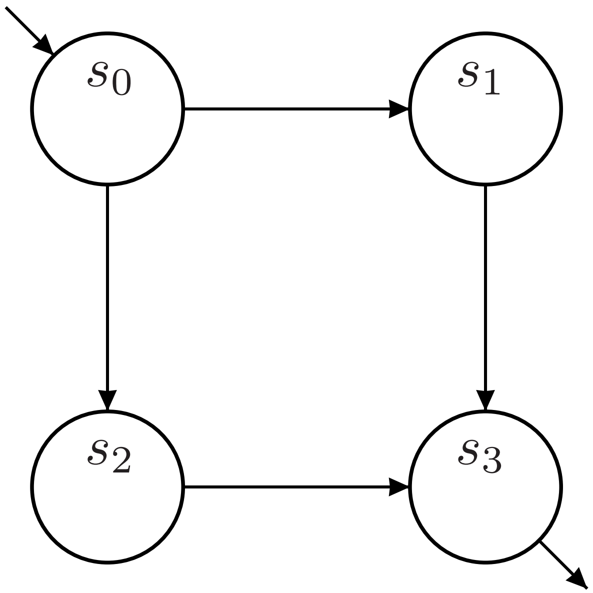
Karin Quaas

31st of May, 2012

A Fruitful Interplay?



Weighted Timed Automata (WTA)



Clock variables: x_1, x_2

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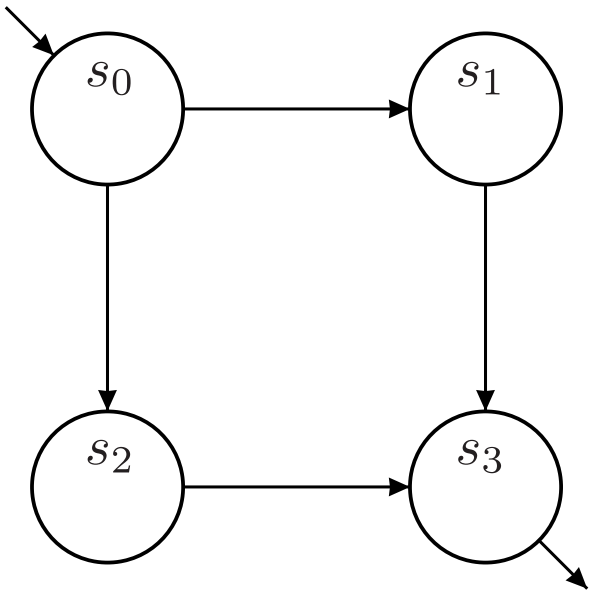

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$(s_0, 0, 0)$

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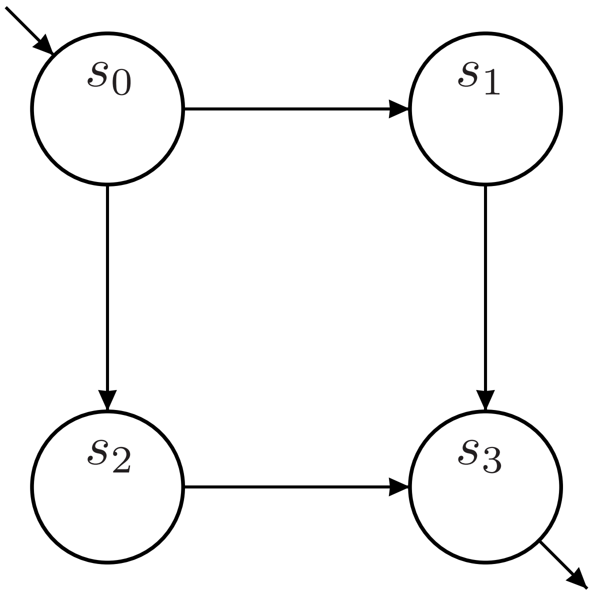

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$$(s_0, 0, 0) \xrightarrow{0.8} (s_0, 0.8, 0.8)$$

Timed transition

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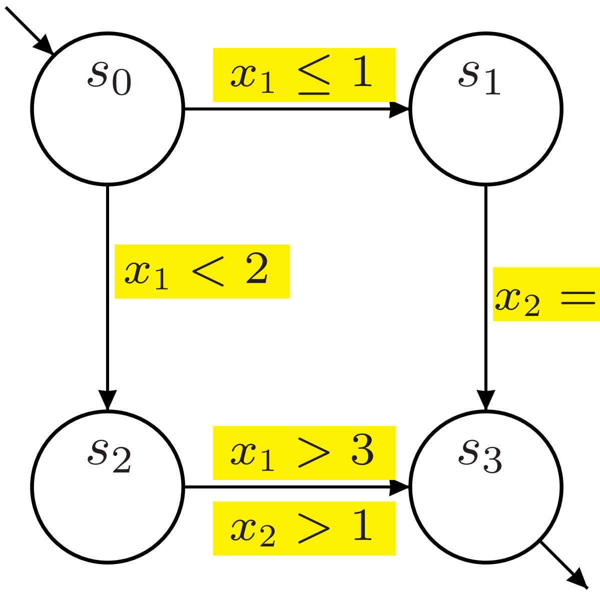

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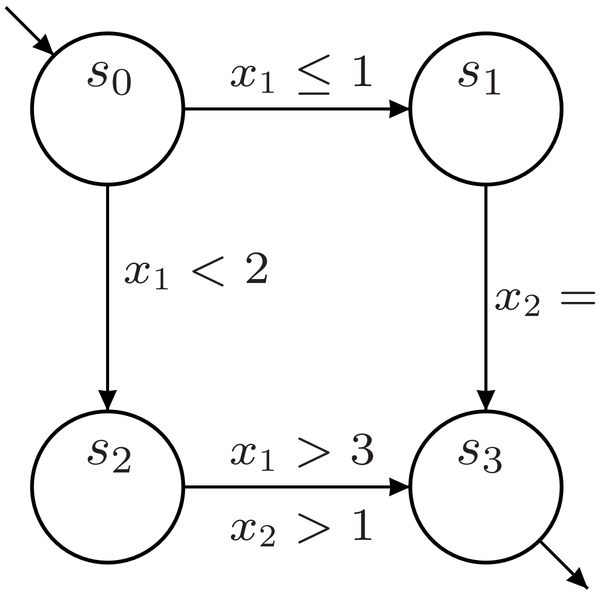

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Discrete transition

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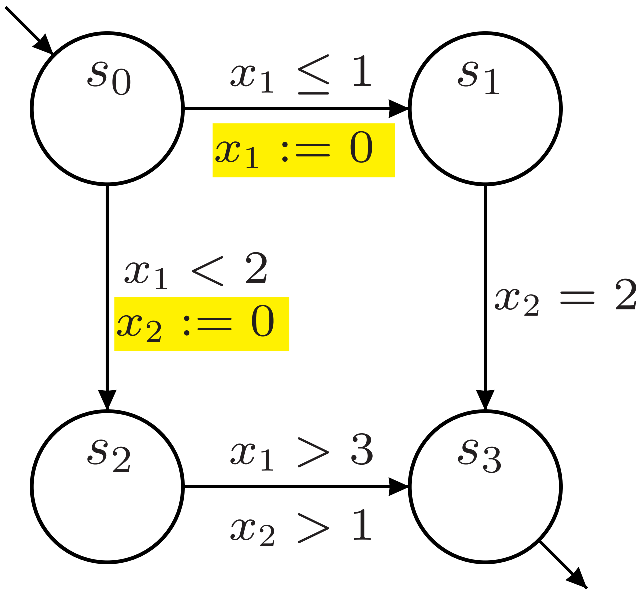

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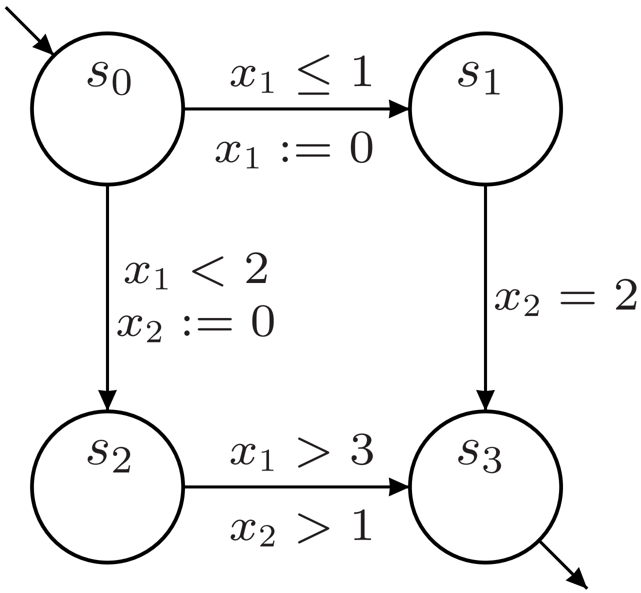

Clock variables: x_1, x_2

$$(s_0, 0, 0) \xrightarrow{0.8} (s_0, 0.8, 0.8) \longrightarrow (s_2, 0.8, 0)$$

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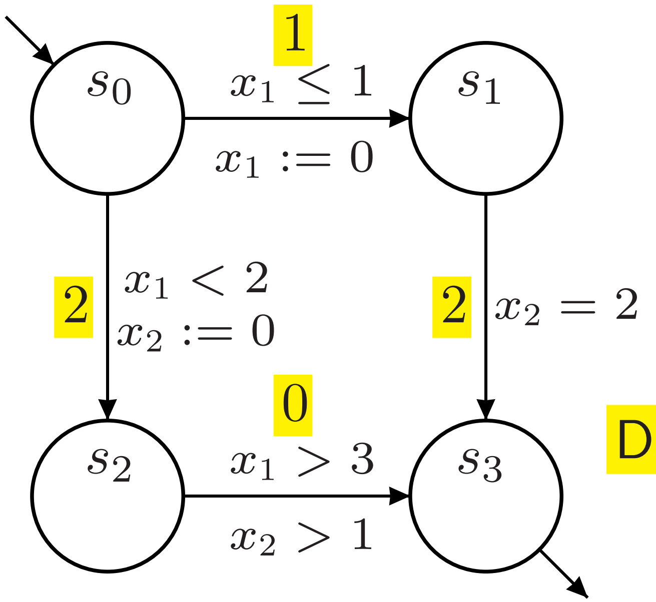
Weighted Timed Automata (WTA)



Discrete weights

Clock variables: x_1, x_2

$$(s_0, 0, 0) \xrightarrow{0.8} (s_0, 0.8, 0.8) \xrightarrow[2]{} (s_2, 0.8, 0)$$

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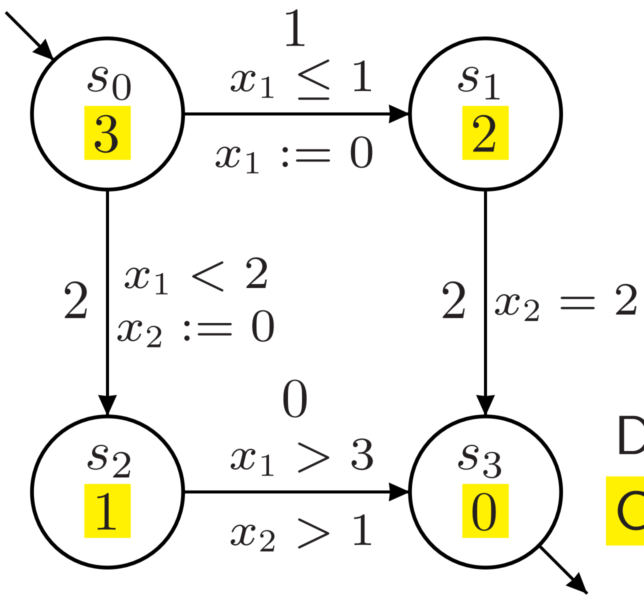
Weighted Timed Automata (WTA)



$$(s_0, 0, 0) \xrightarrow[2.4]{0.8} (s_0, 0.8, 0.8) \xrightarrow[2]{} (s_2, 0.8, 0)$$

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Discrete weights

Continuous weights

Clock variables: x_1, x_2

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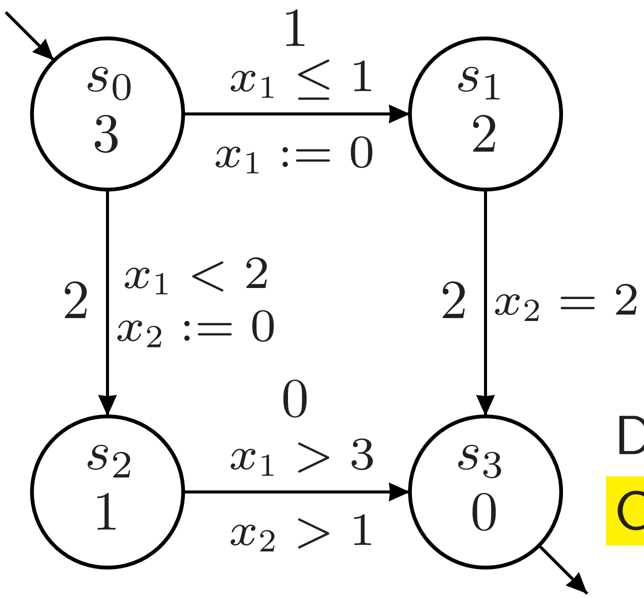

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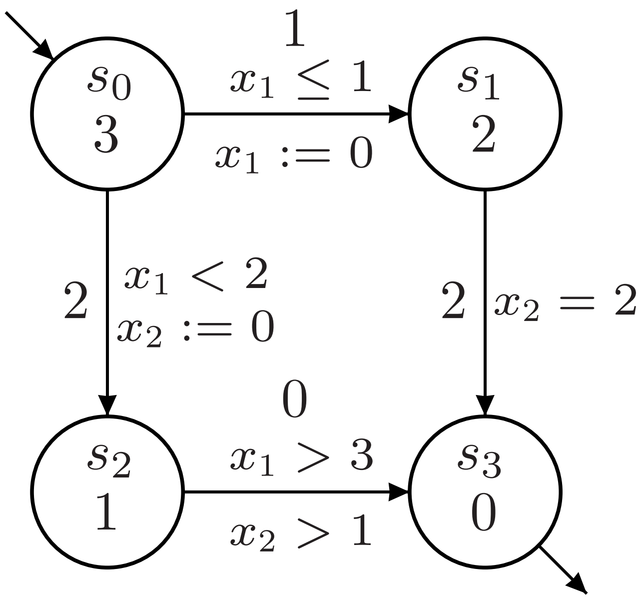
Discrete weights

Continuous weights

$\Rightarrow$ More expressive than weighted automata!

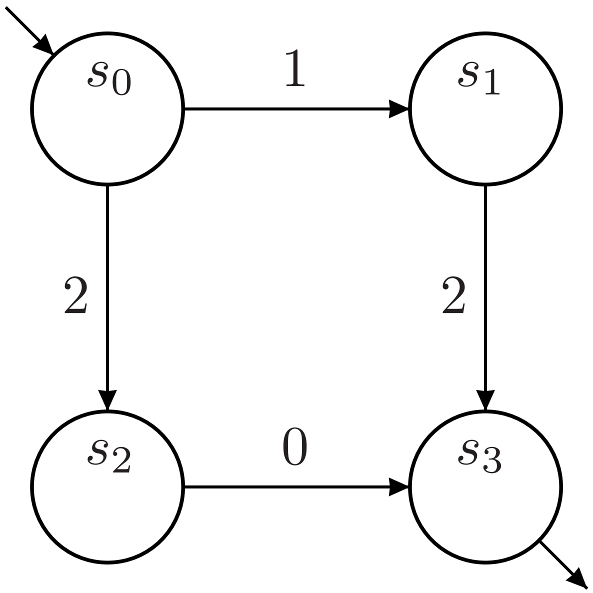
The Optimal Reachability Problem (ORP) for WTA



What is the cheapest path from one given state to some other?

- Alur, La Torre, Pappas (2001)
- Behrmann, Brinksma, Fehnker, Hune, Larsen, Pettersson, Romijn (2001)
- Bouyer, Brihaye, Bruyère, Raskin (2007)

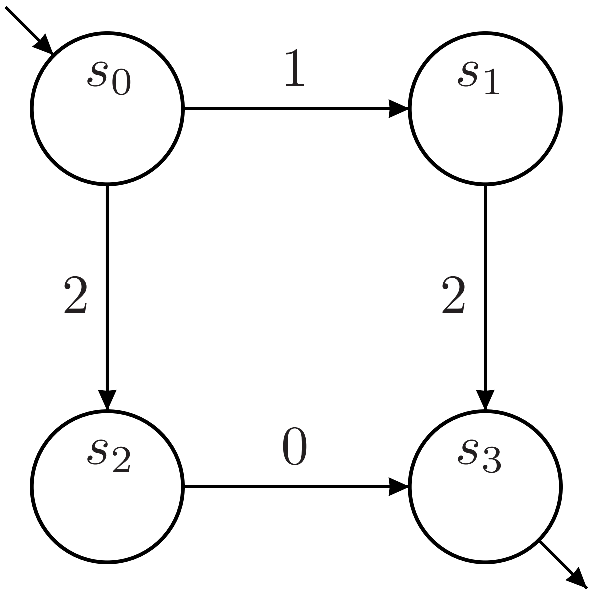
The ORP for Weighted Automata



Weighted automata with weights in $\mathbb{N}$ ($\mathbb{Z}$) summed up along the run:

- Dijkstra (Bellman-Ford) Algorithm

The ORP for Weighted Automata



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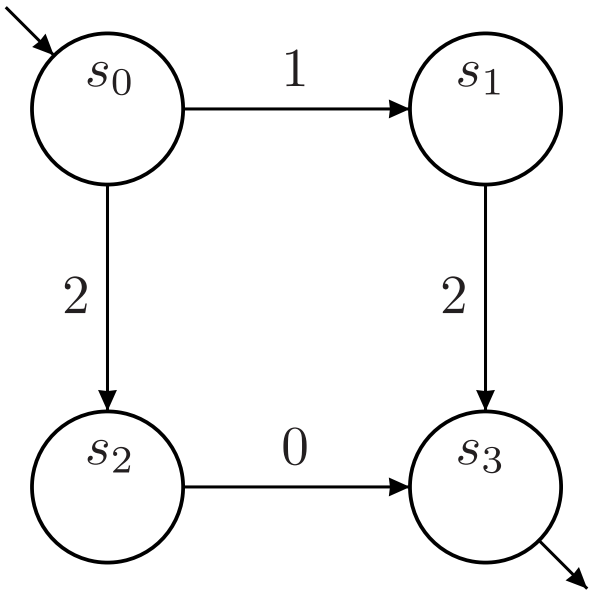
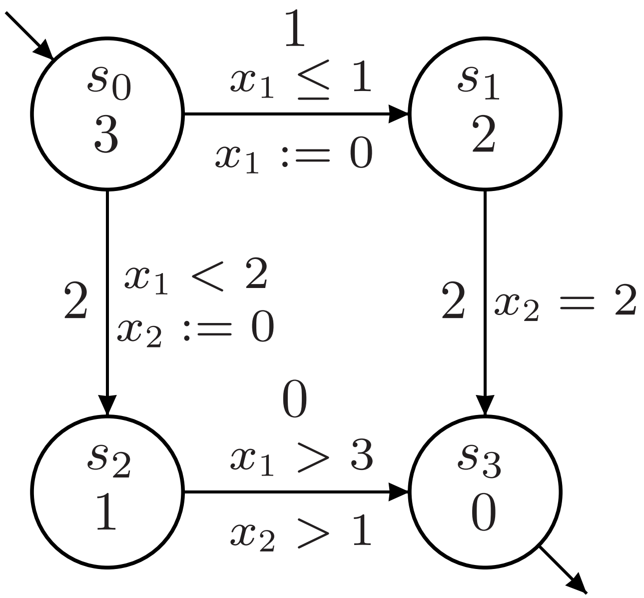
The ORP for Weighted Automata

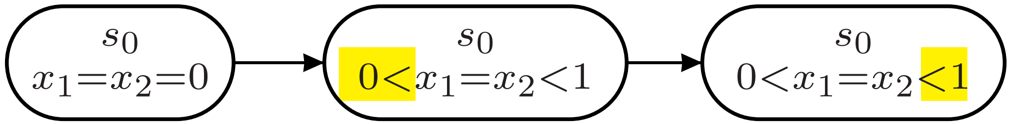


Weighted automata with weights in $\mathbb{N}$ ($\mathbb{Z}$) summed up along the run:

- Dijkstra (Bellman-Ford) Algorithm
- Not applicable to WTA: induce infinite weighted automaton
- Is there some discrete abstraction (= a weighted automaton) that is sound and complete with respect to optimal reachability?

Discrete Abstraction? - Weighted Cornerpoint-Region Graph!





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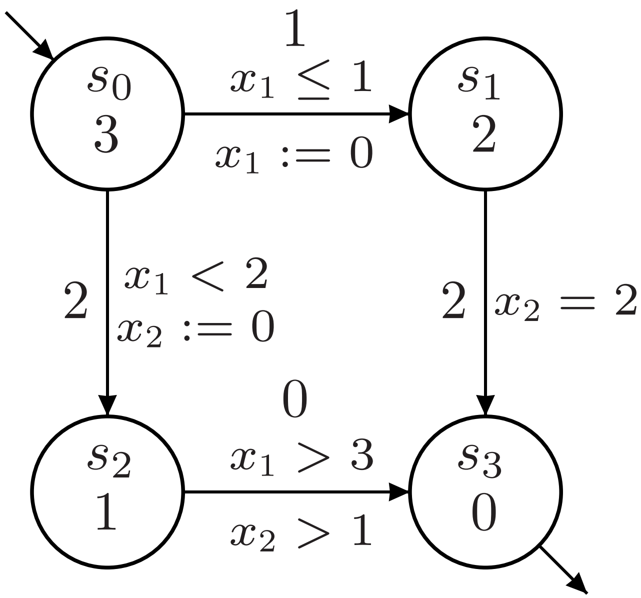

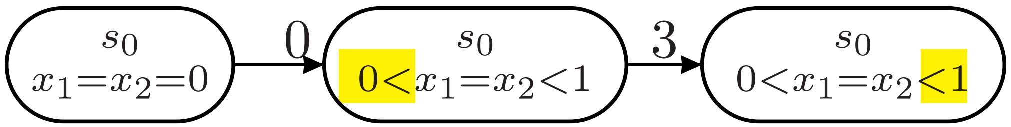

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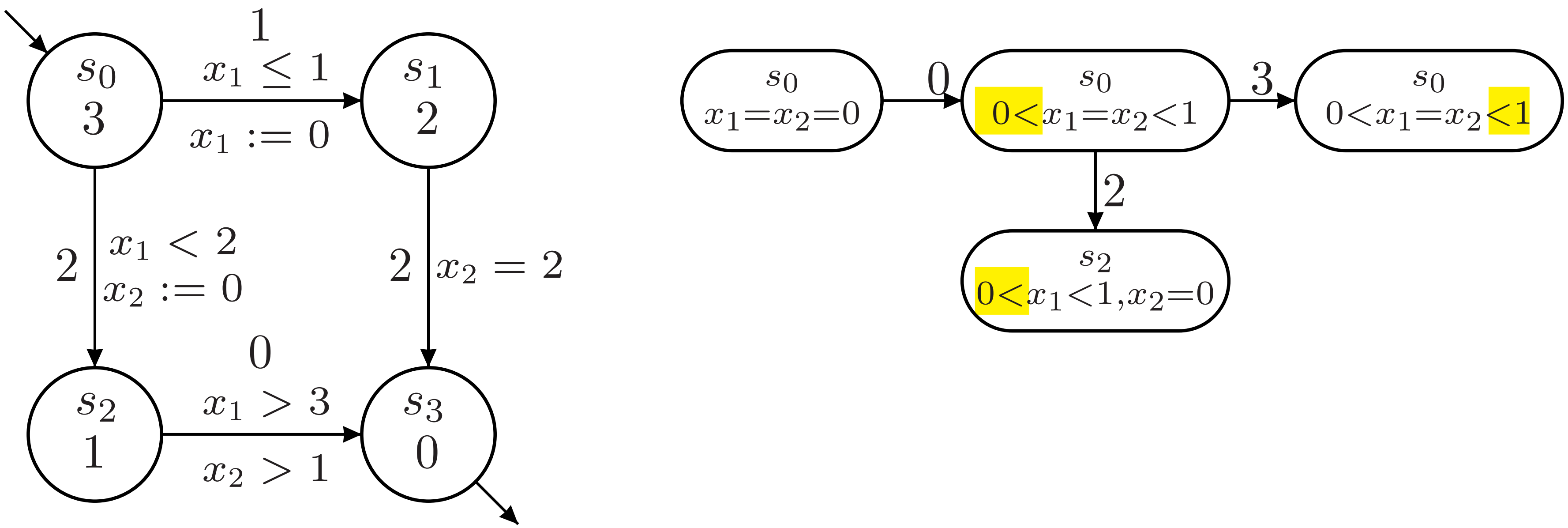

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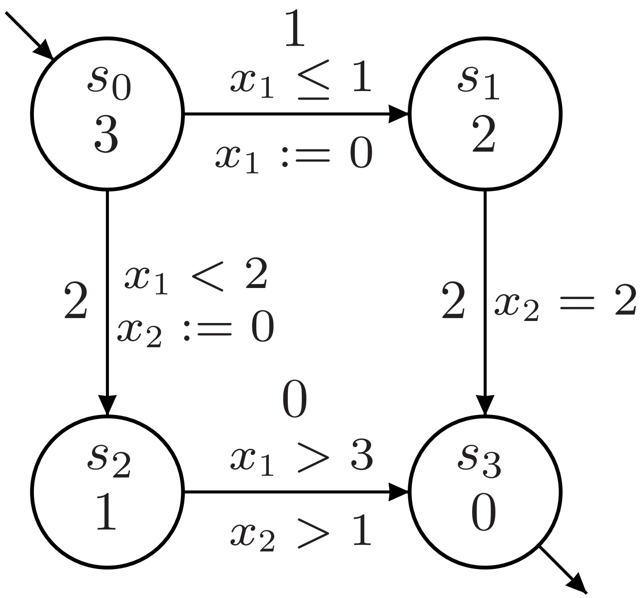

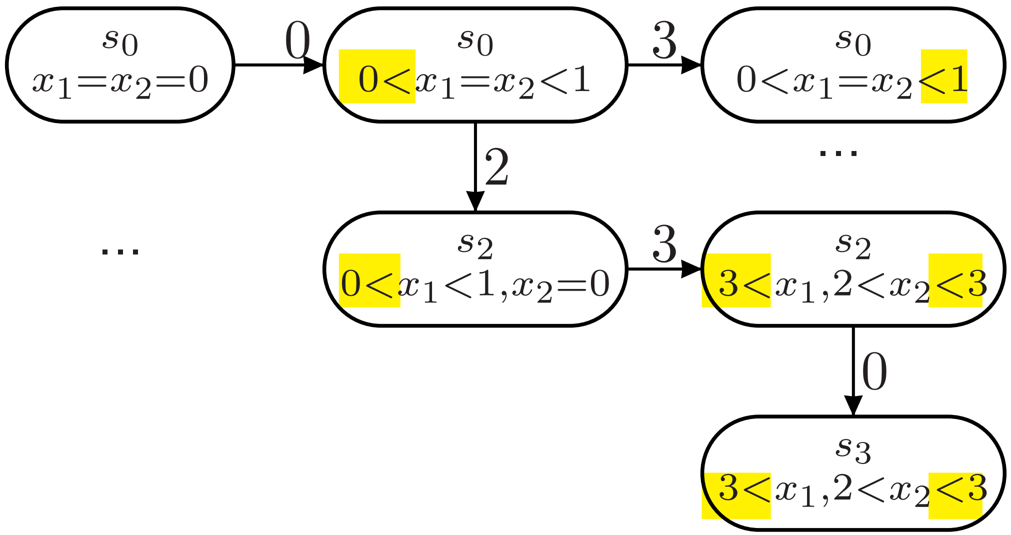

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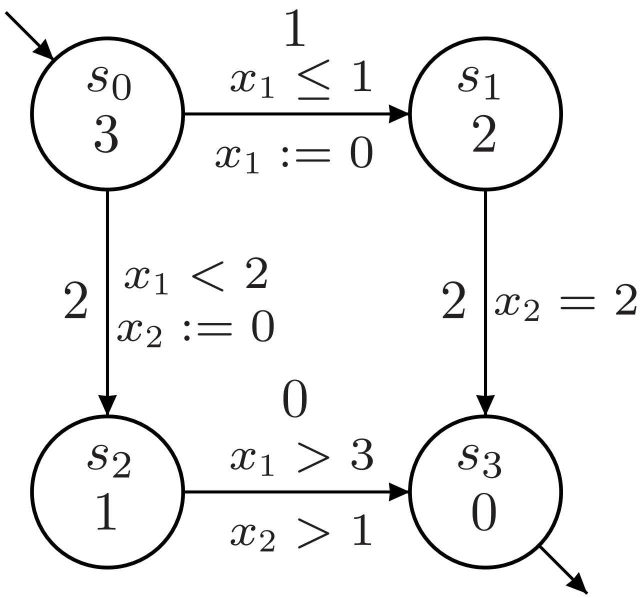
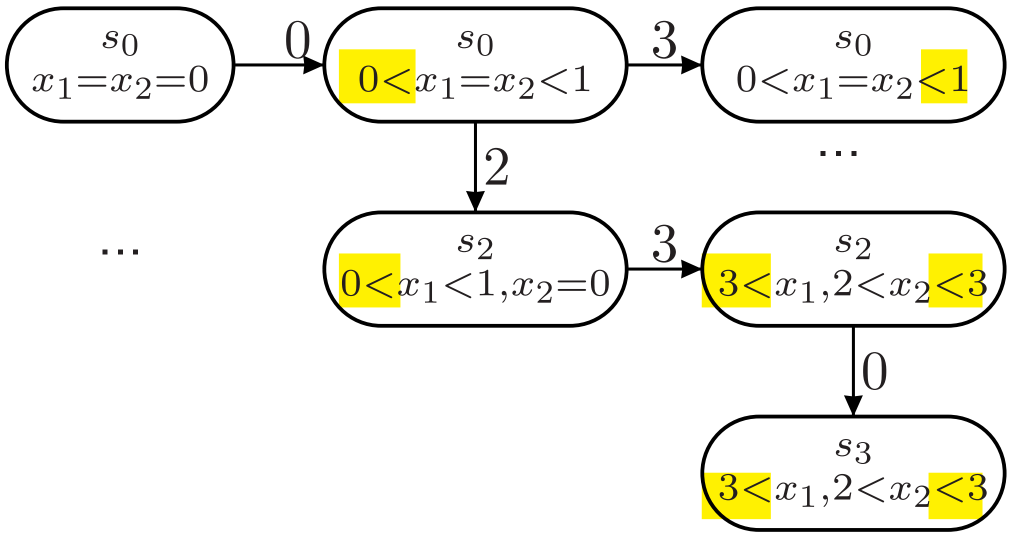
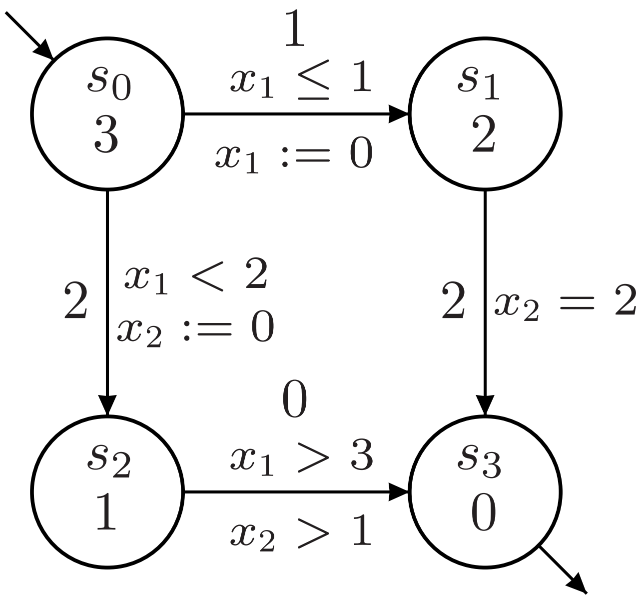




$\Rightarrow$ Sound and complete with respect to ORP
(Bouyer, Brihaye, Bruyère and Raskin, 2007)

Optimal Reachability Problem (ORP) for WTA



What is the cheapest path from one given state to some other?

The optimal reachability problem for weighted timed automata is PSPACE-complete.
(Bouyer, Brihaye, Bruyère and Raskin, 2007)

WTA - State of the Art

- WTA over different weight structures:
  - positive / negative / multiple weights
  - linear / exponential growth rates
  - along a run, weights are summed up / discounted / mean valued...

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- WTA over different weight structures:
  - positive / negative / multiple weights
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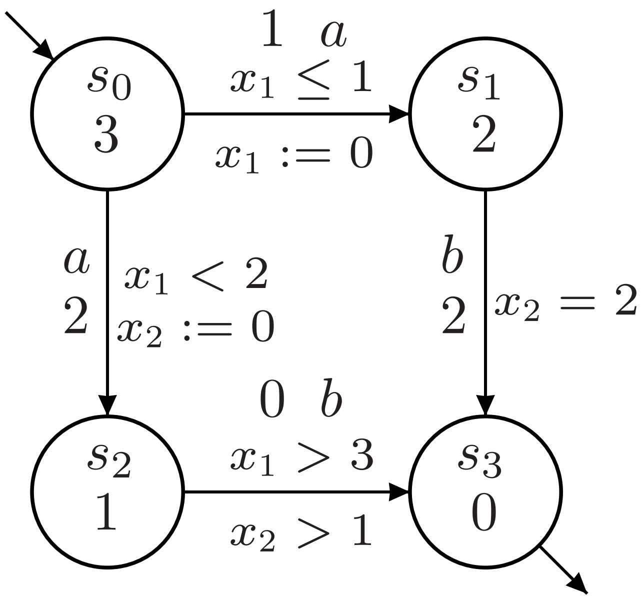
  - along a run, weights are summed up / discounted / mean valued...
- Problems considered:
  - Optimal Reachability / Scheduling
  - Model Checking weighted timed extensions of temporal logics
  - Games...

Unifying Definition for WTA



Droste, Quaas (2008)

WTA defined over

- Semiring: $(\mathbb{R}_{\geq 0} \cup \{\infty\}, \min, +, \infty, 0)$

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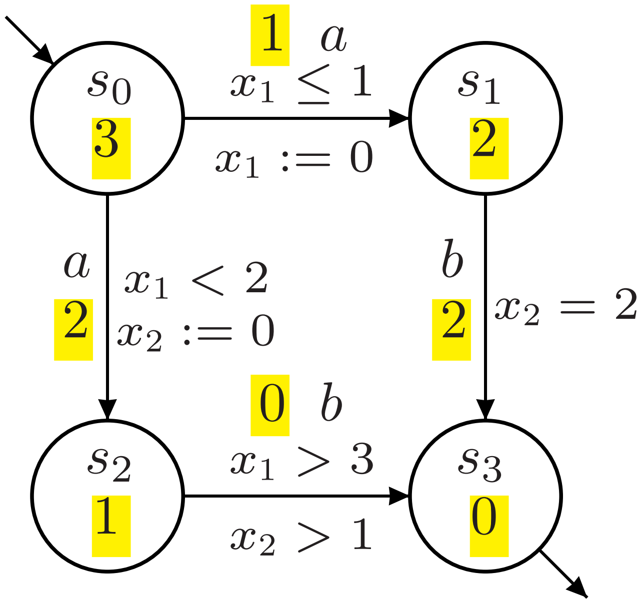

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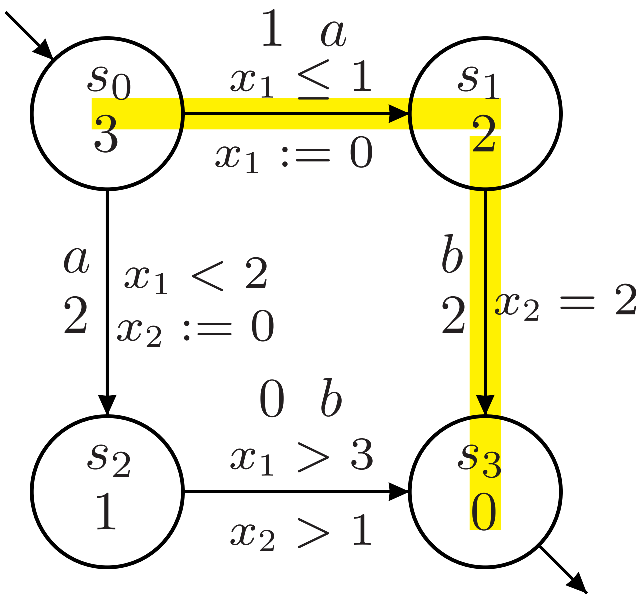

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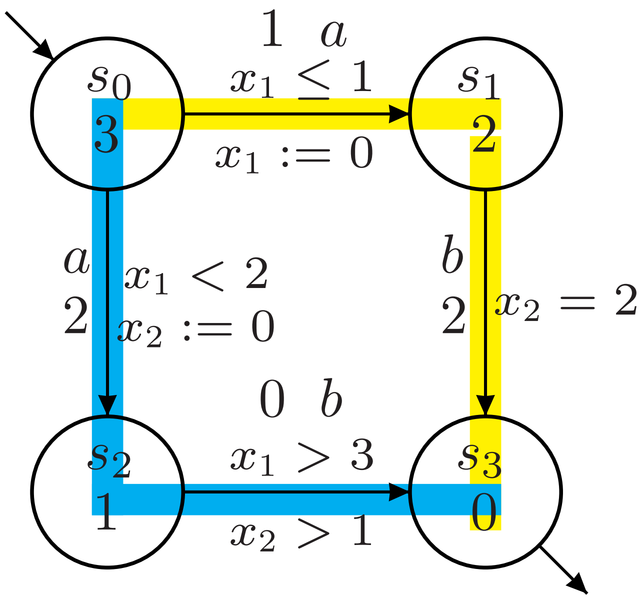

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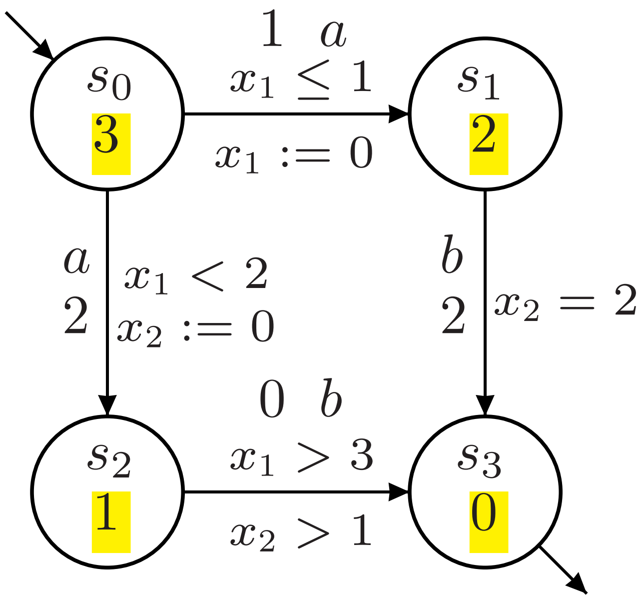

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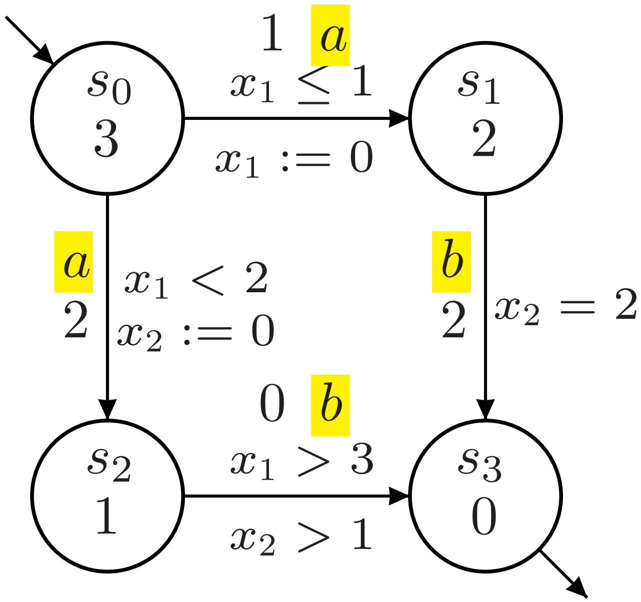

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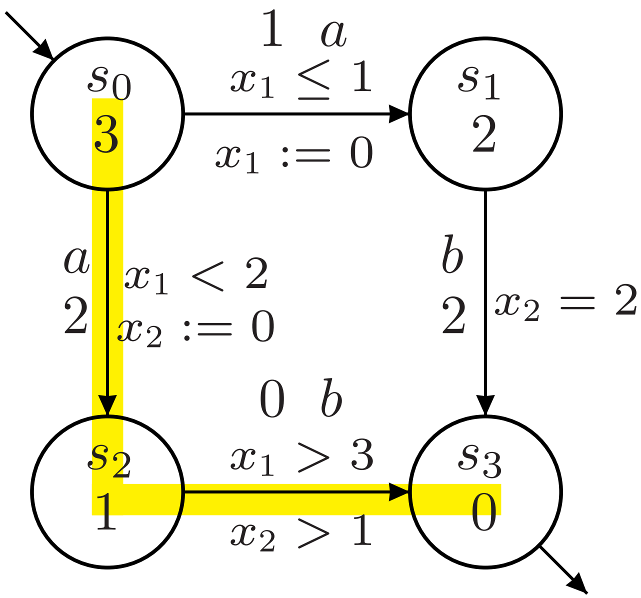

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Timed Words, e.g. $(a, 1.5)(b, 3.1)$

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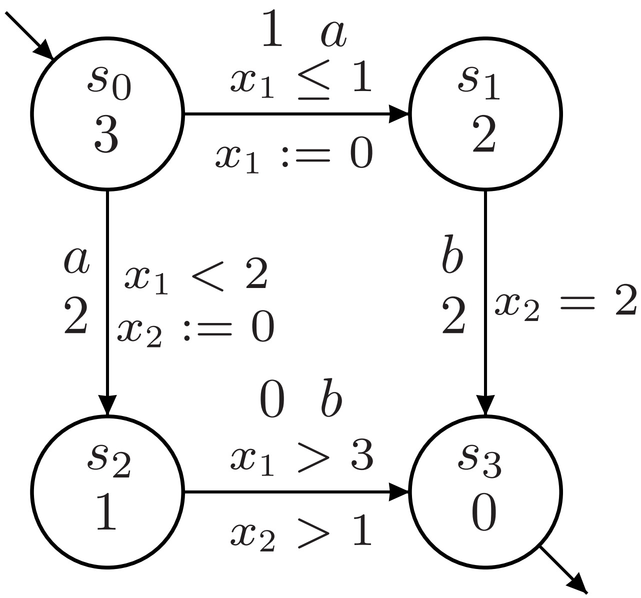

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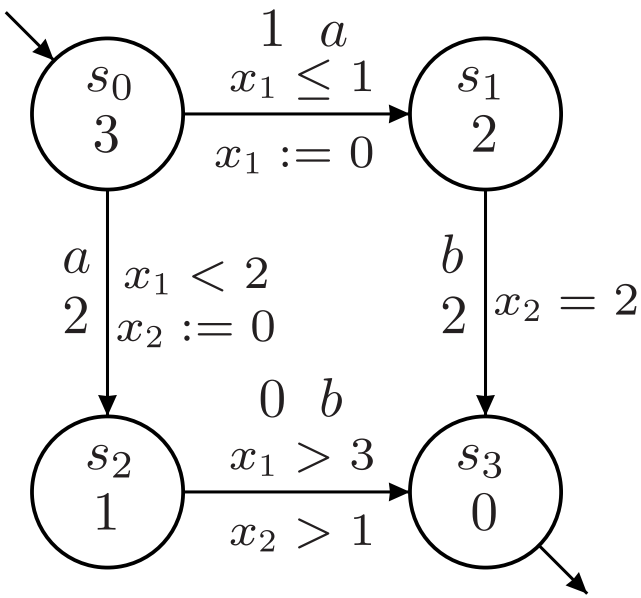

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$\Rightarrow$ WTA recognize timed series

WTA as Recognizers of Timed Series

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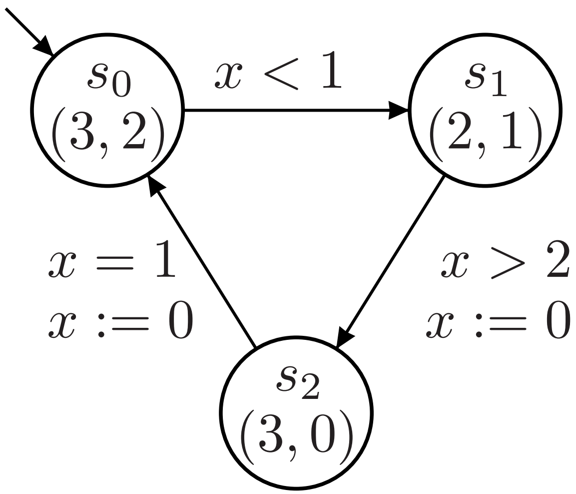
  Timed series are recognizable, if, and only if, they are definable in restricted weighted timed MSO logic (Quaas, 2009).
- Some further results, e.g., decidability of the equivalence problem of WTA over $(\mathbb{R}, +, \cdot, 0, 1)$ and LIN (Quaas, 2009) (Compare with undecidability of equivalence problem for timed automata)

A Unifying Framework for WTA

Using this unifying framework for WTA, can we generalize some of the (un)decidability results of specific WTA to WTA over certain classes of semirings?

A Unifying Framework???



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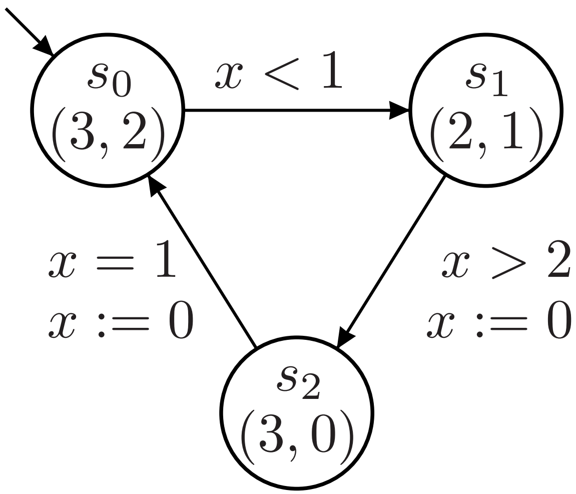

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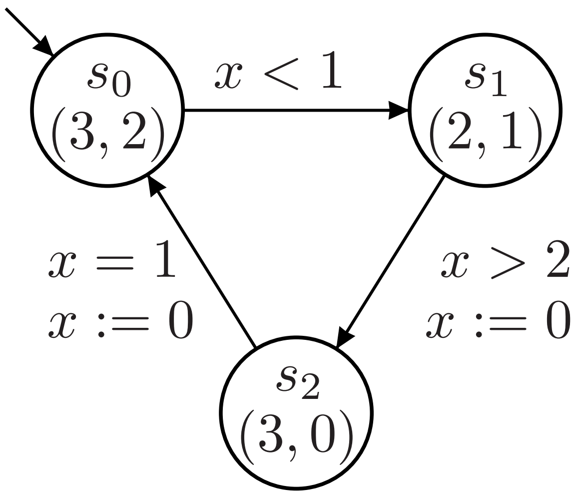

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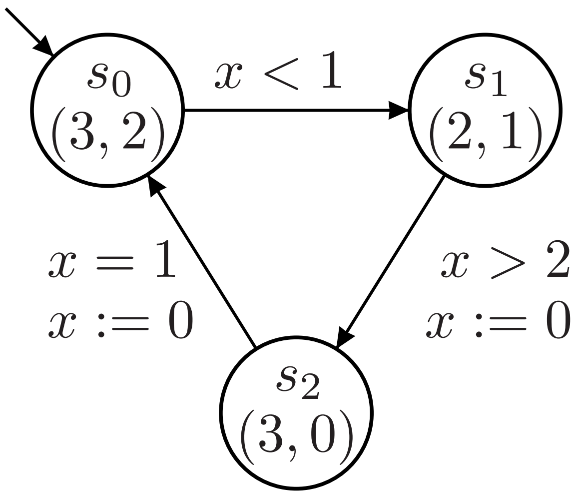

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- No semiring operation!!!

A Unifying Framework!!!

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A Unifying Framework!!!

- Similar situation: weighted automata vs. quantitative automata (Chatterjee, Doyen, Henzinger, 2008)
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- Droste and Meinecke, 2010, proposed a unifying framework: weighted automata over valuation monoids

$$(D, +, \mathsf{Val}, 0), \text{ where } (D, +, 0) \text{ is a monoid, } \mathsf{Val} : D^+ \to D$$

- Büchi-type theorem for weighted automata over valuation monoids

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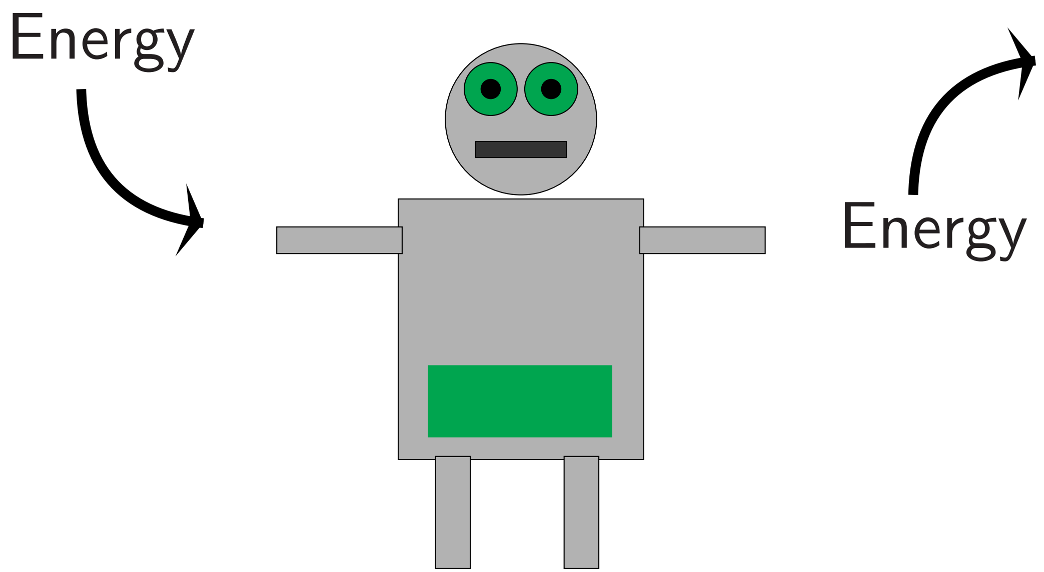
- Open problem: Is there such a unifying framework for WTA?

Energy Problems



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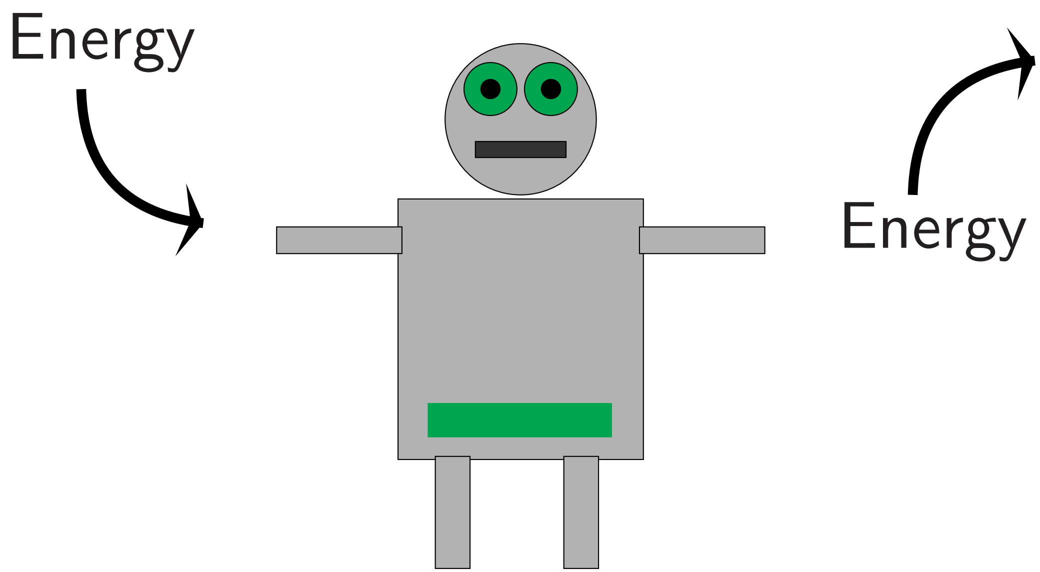

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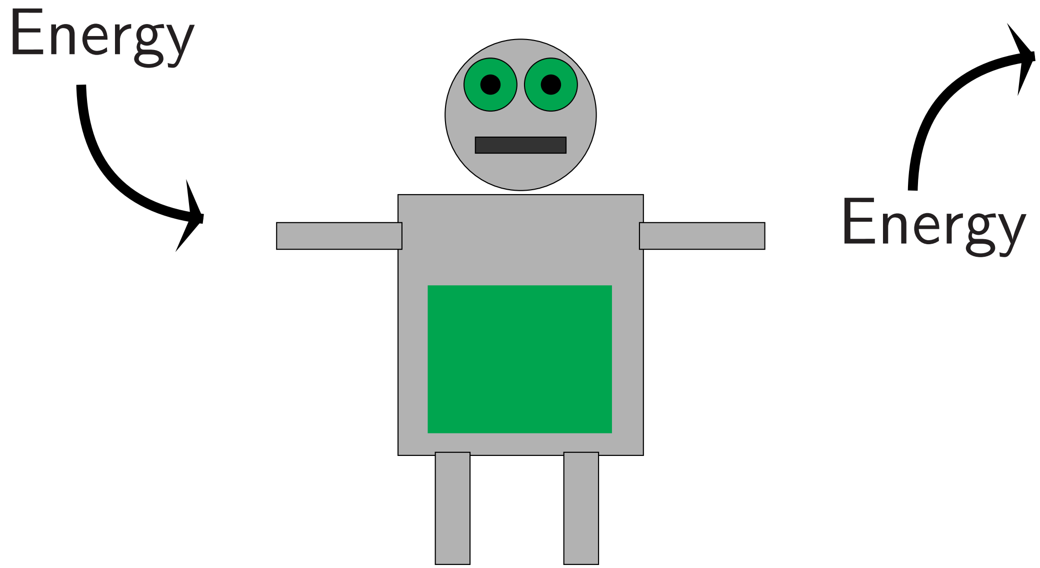

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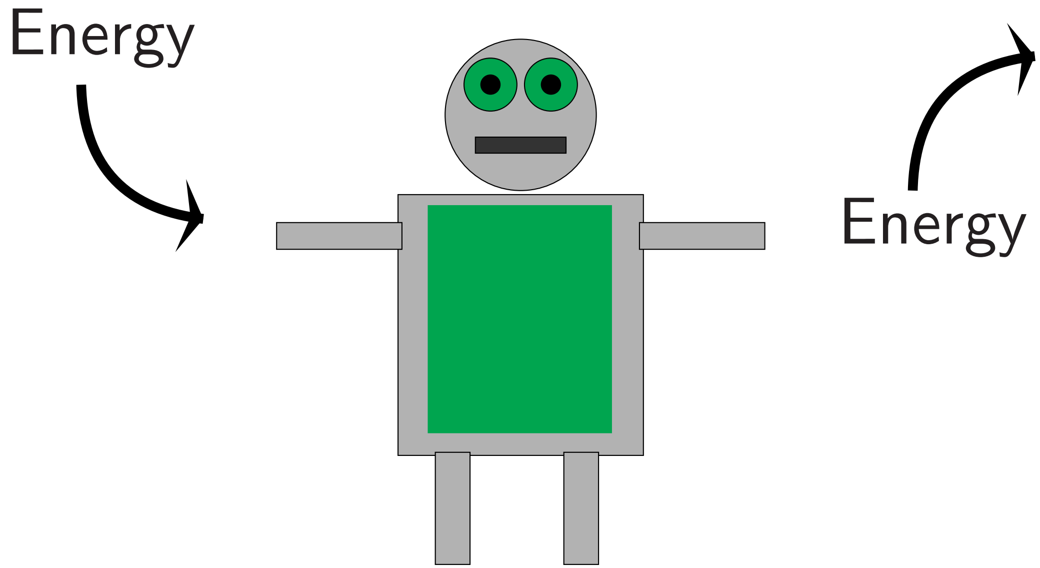
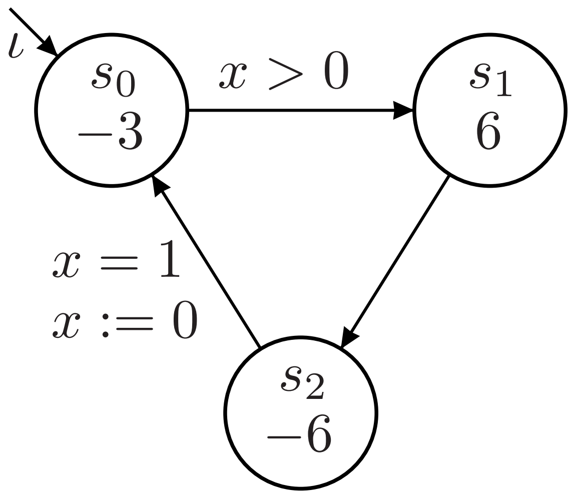


Energy Problems for WTA over $\mathbb{Z}$ and LIN



(Bouyer, Fahrenberg, Larsen, Markey, Srba, 2008)

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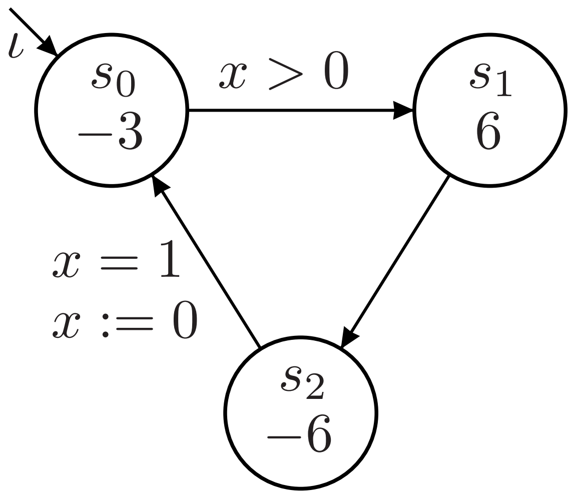
Energy Problems for WTA over $\mathbb{Z}$ and LIN



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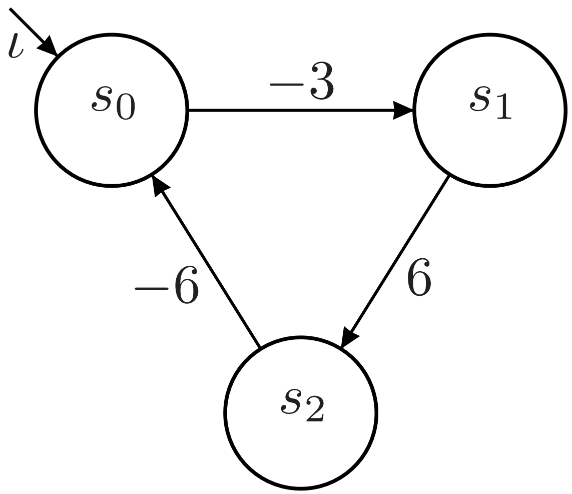
Special case: value always within $[0, \infty]$,
$\Rightarrow$ Lower bound energy problem.

Lower Bound Energy Problem for Weighted Automata over $\mathbb{Z}$



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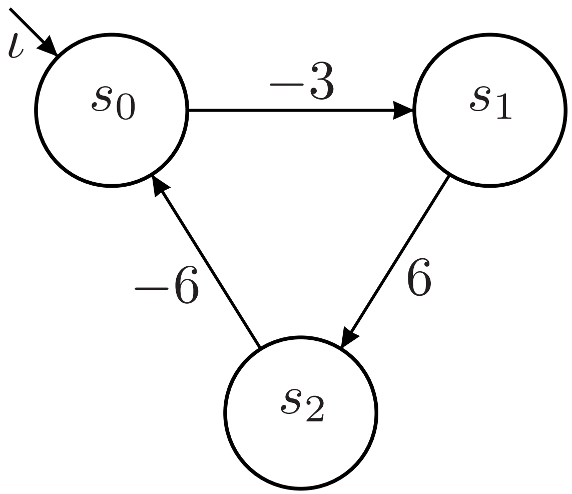

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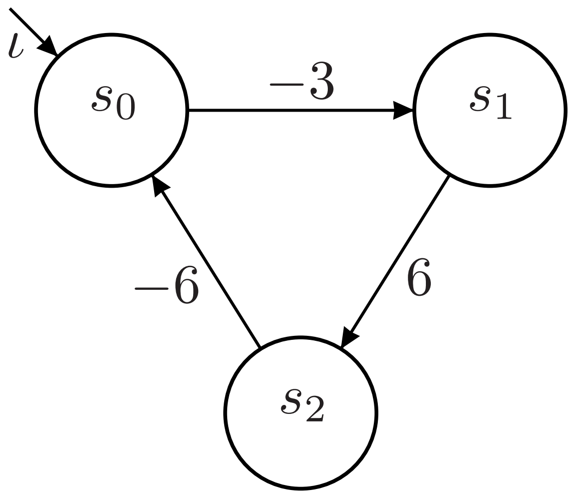

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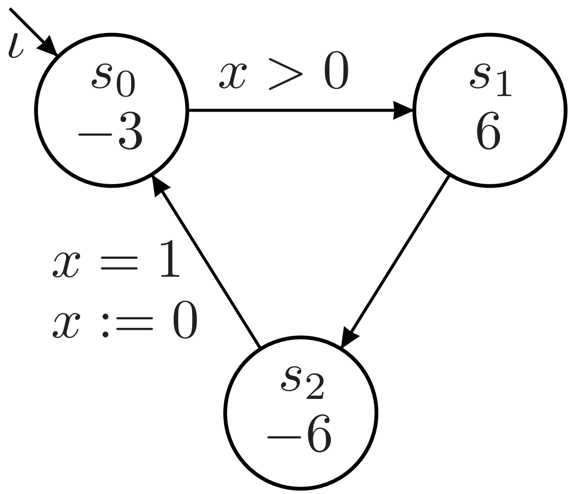
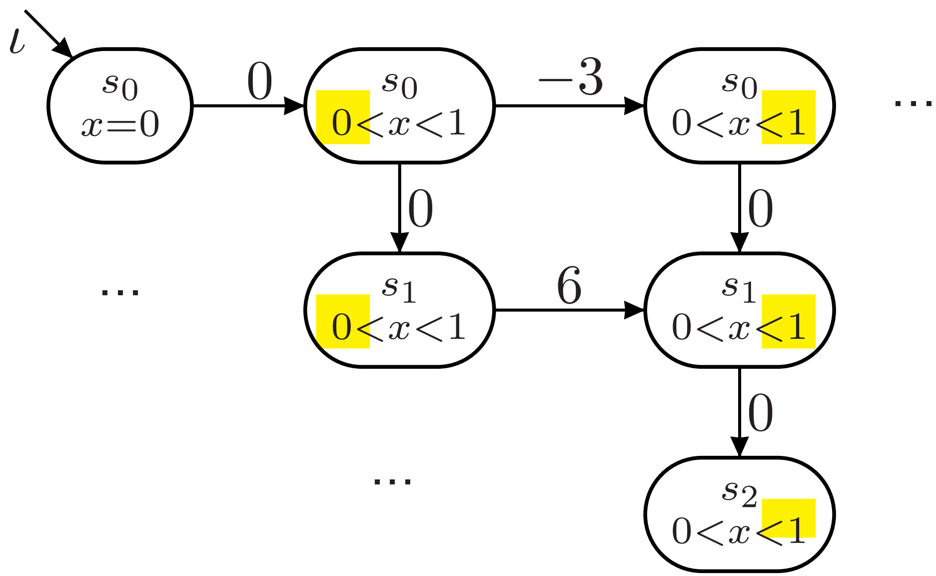
- Is there some reachable cycle that is not energy losing?
- Bellman-Ford algorithm
- Lower bound energy problem for weighted automata over $\mathbb{Z}$ is in P.

Discrete Abstraction? - Weighted Cornerpoint-Region Graph!





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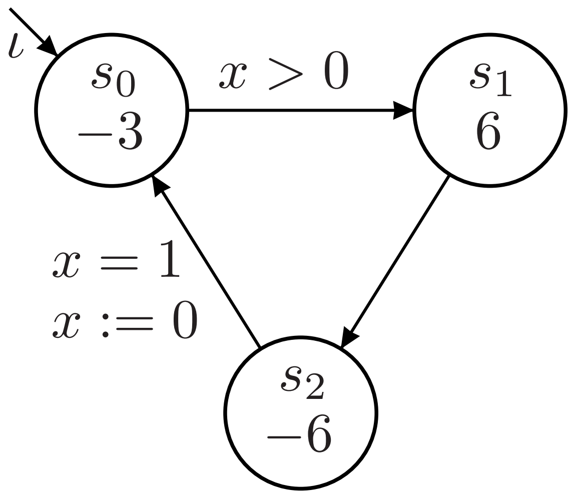
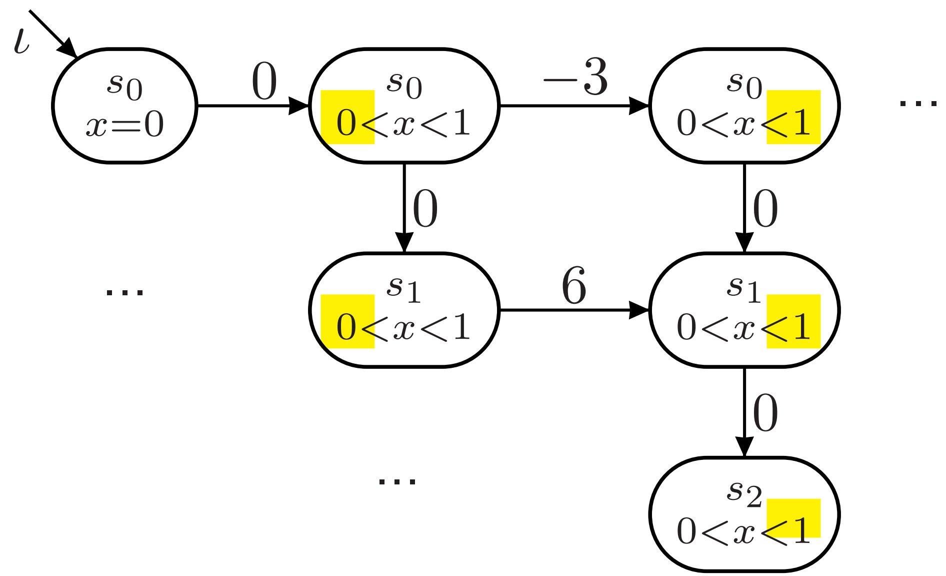
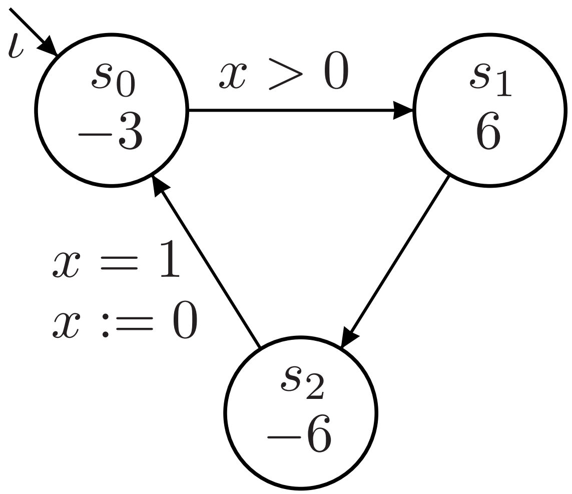




⇒ Sound and complete with respect to energy problem $[0, \infty)$ (BFLMS, 2008)

Lower Bound Energy Problem for WTA over $\mathbb{Z}$ and LIN



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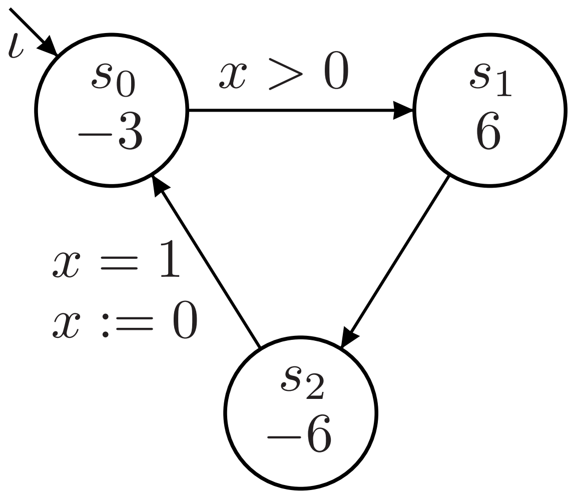
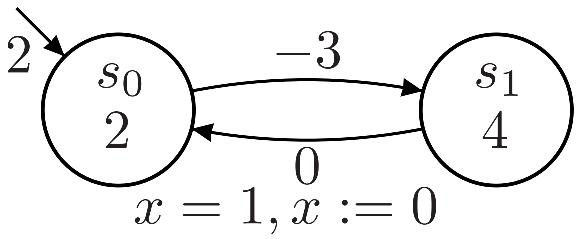


The lower bound energy problem for WTA over $\mathbb{Z}$ and LIN is in P (BFLMS, 2008) if the WTA does not have any discrete weights.

Another Discrete Abstraction: Energy Automata



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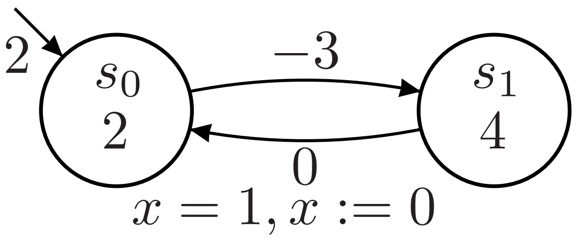

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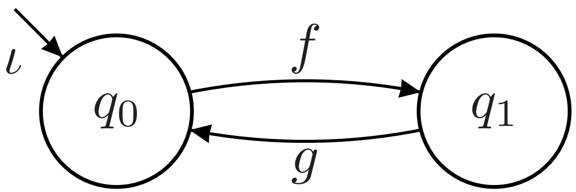

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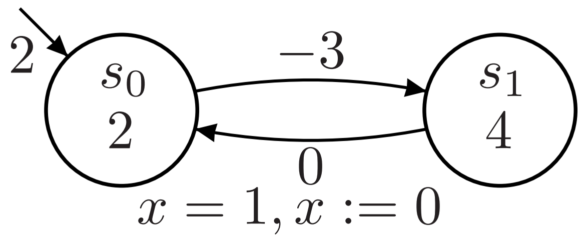

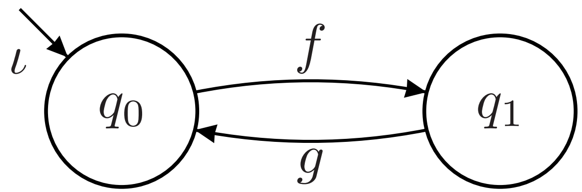

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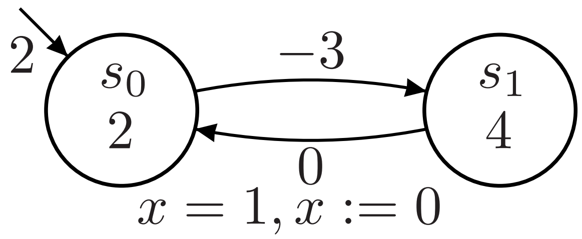

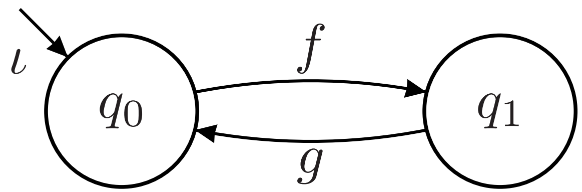

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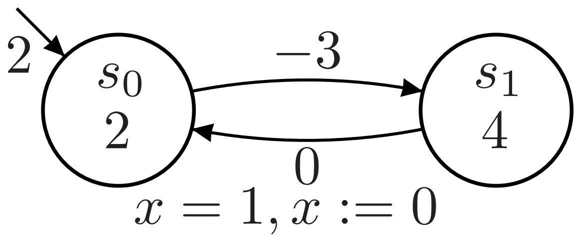

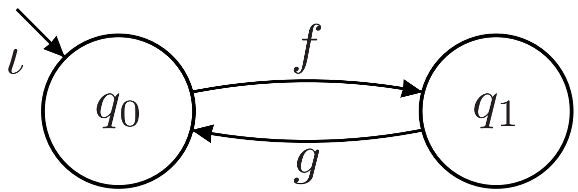

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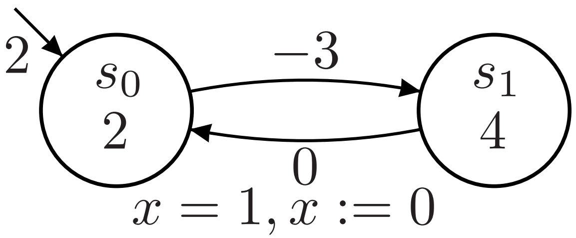

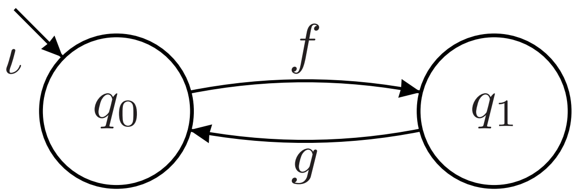

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$\Rightarrow$ non-obvious reduction from WTA over $\mathbb{Z}$ and LIN to weighted automata over semiring of energy functions

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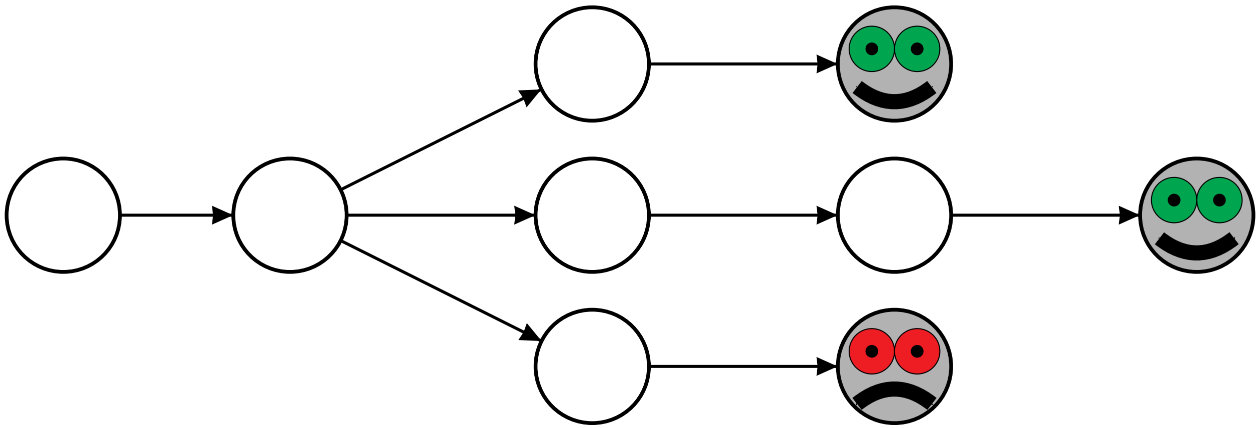

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(Koymans, 1990)

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$(w, i) \models \varphi_1 U_I \varphi_2$ if $\exists j \geq i (w, j) \models \varphi_2$ and $t_j - t_i \in I$ and $\forall i \leq k < j.(w, k) \models \varphi_1$

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wMTL Model Checking of WTA over $\mathbb{N}$ and LIN

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- Finitary model checking of WTA with one clock and one stopwatch is decidable
- Finitary model checking is undecidable for WTA with
    - two clocks, one stopwatch
    - one clock, two stopwatches
    - one clock, one weight variable with arbitrary rates

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- e.g. $(\mathbb{Q}^+, +, 0, \leq)$, $(\mathbb{N}\backslash\{0\}, \cdot, 1, \leq)$, $(\mathbb{Q} \cup \{\infty\}, \min, \infty, \geq)$

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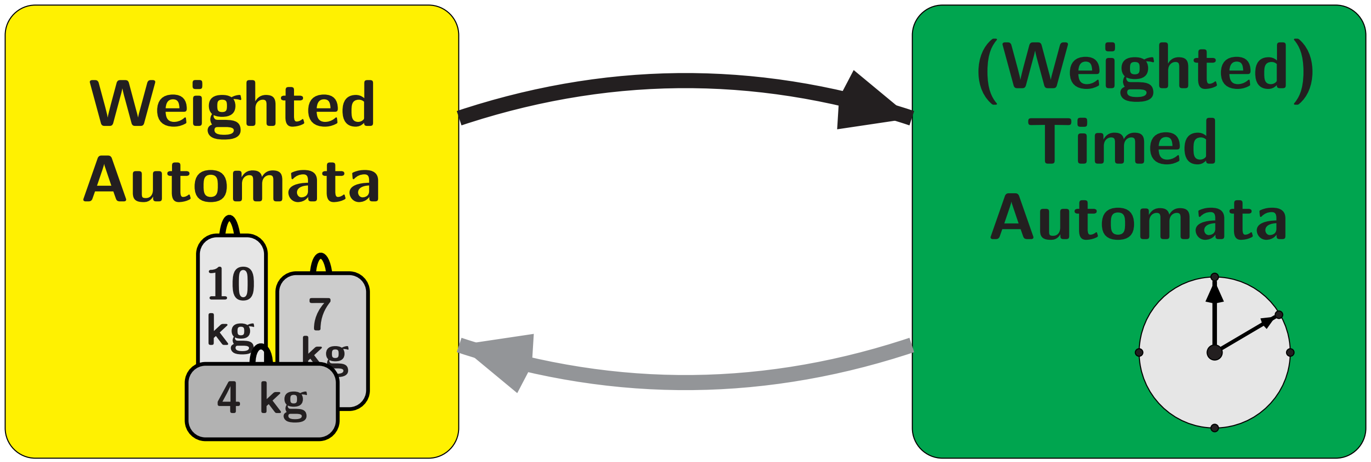
Open problem:
Can we restrict the logic in such a way that it is still reasonably expressive and model checking is decidable?

MTL Model Checking Timed Automata + VASS

-wMTL model checking for WTA over $\mathbb{Z}$ and LIN is undecidable

Open problem:
Is MTL model checking decidable for timed automata extended with weight variable ranging over $\mathbb{N}$, but whose value can be increased and decreased in a discrete manner?
(Transitions which decrease the value below zero are blocked.)

A Fruitful Interplay!



Introduction of WTA and Optimal Reachability

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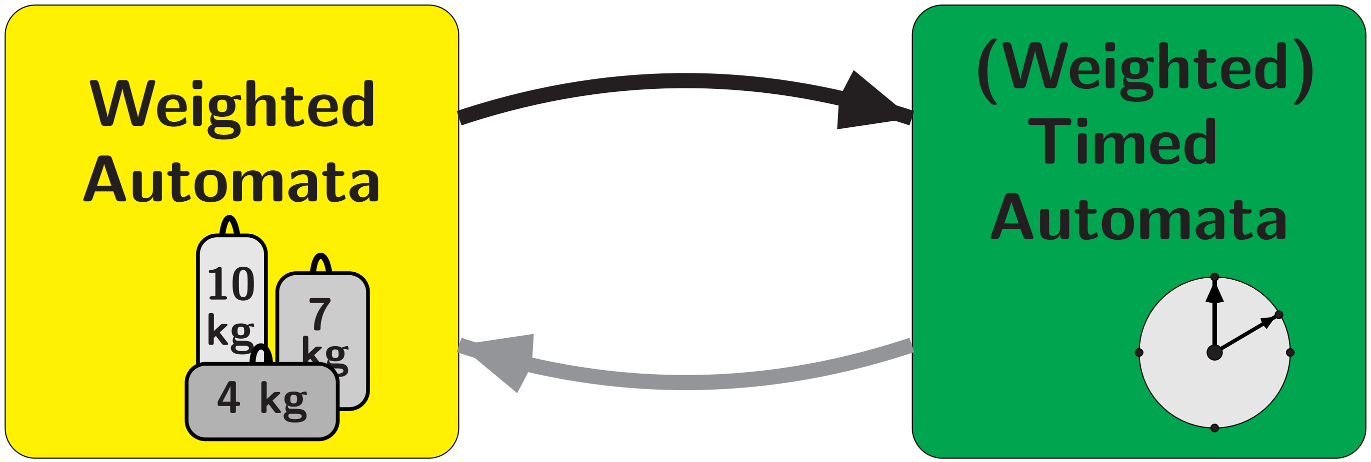

Unifying Definition of WTA, Timed series, Büchi, Kleene-Schützenberger, Supports...

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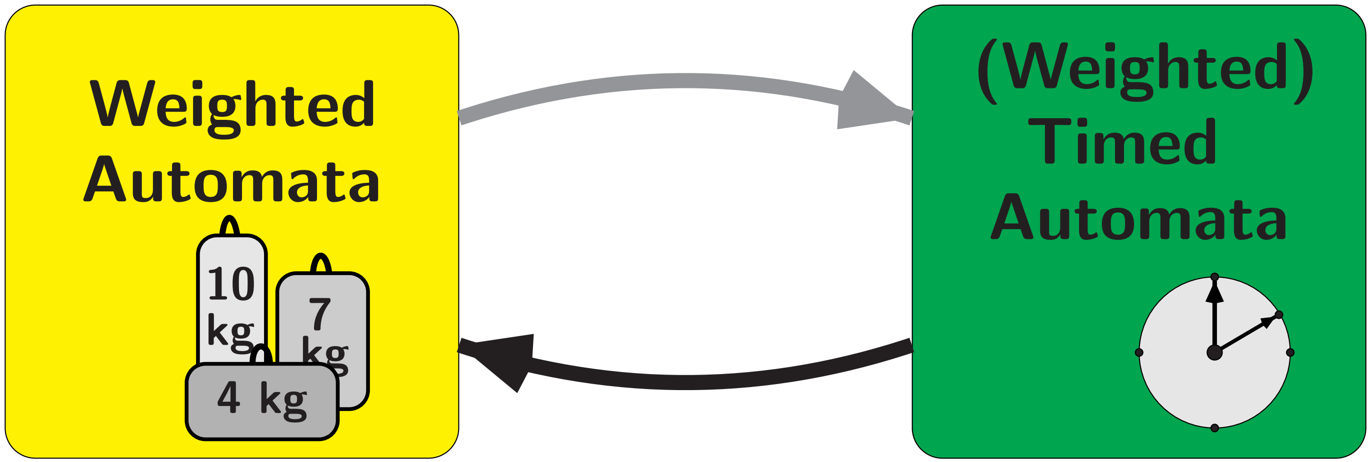

Energy Problems, Energy Automata

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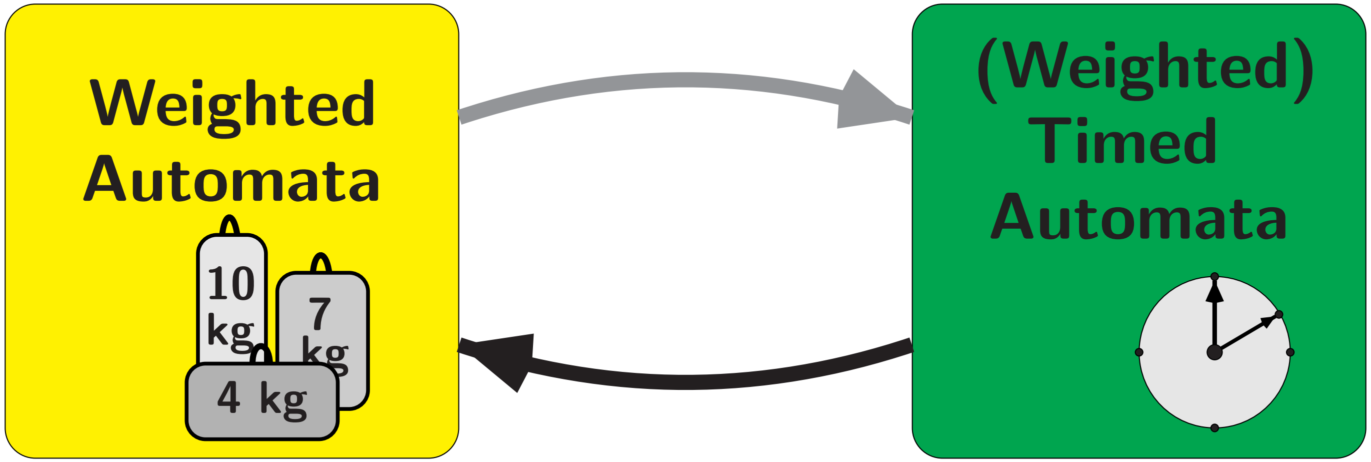

Model Checking MTL-like temporal logics

Future Research

- Unifying frameworks for non-semiring WTA
- Generalizing known results for WTA
- Energy Problems:
  - $[0, b]$-problem for 1-clock WTA
  - for other weight structures
  - for other kinds of weighted automata
- Model checking restriction of weighted LTL over $\mathbb{Z}$
- Model checking MTL of timed automata $+$ VASS