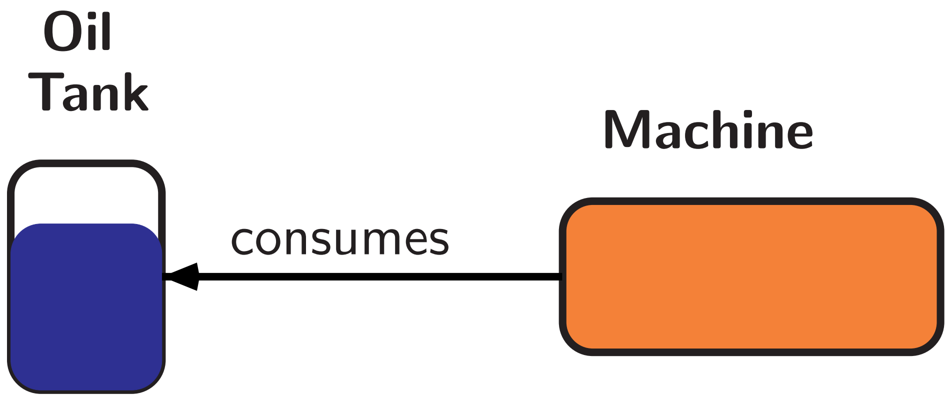
On the Interval-Bound Problem for Weighted Timed Automata

Karin Quaas

27th of May, 2011

Example from the real world



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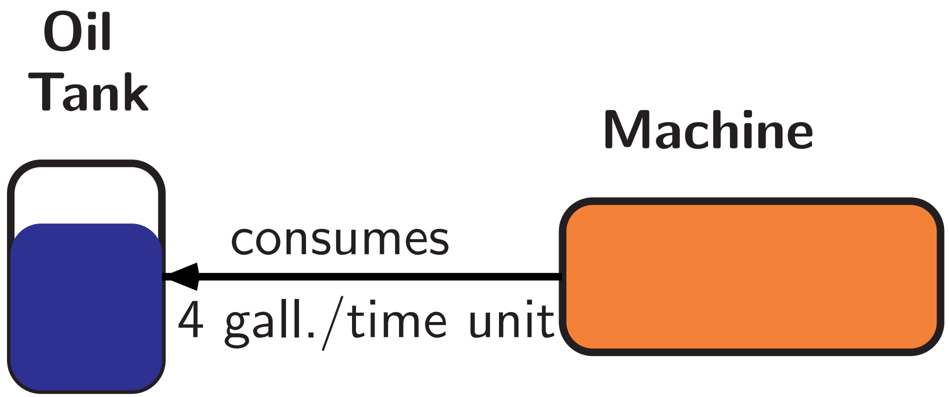

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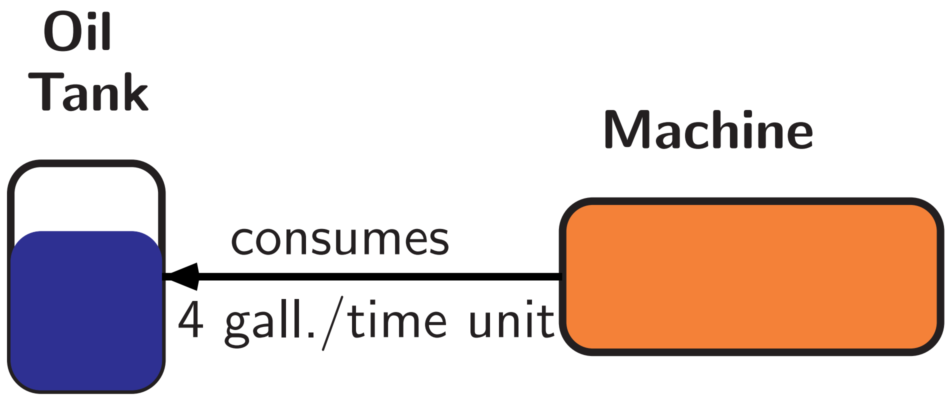

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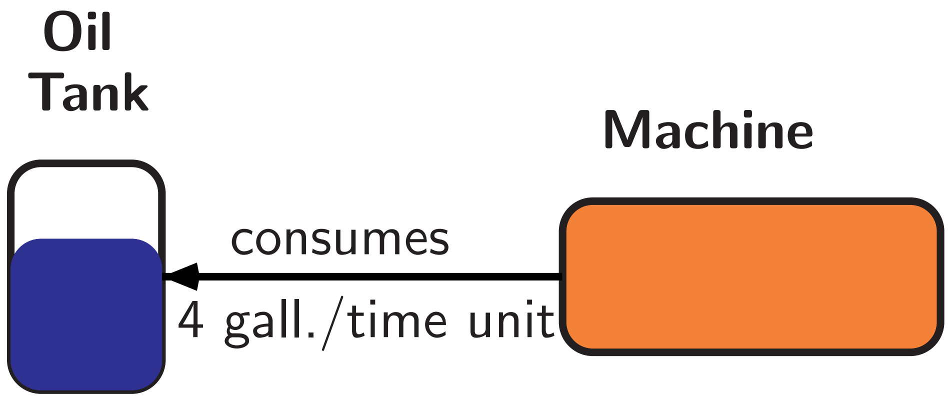

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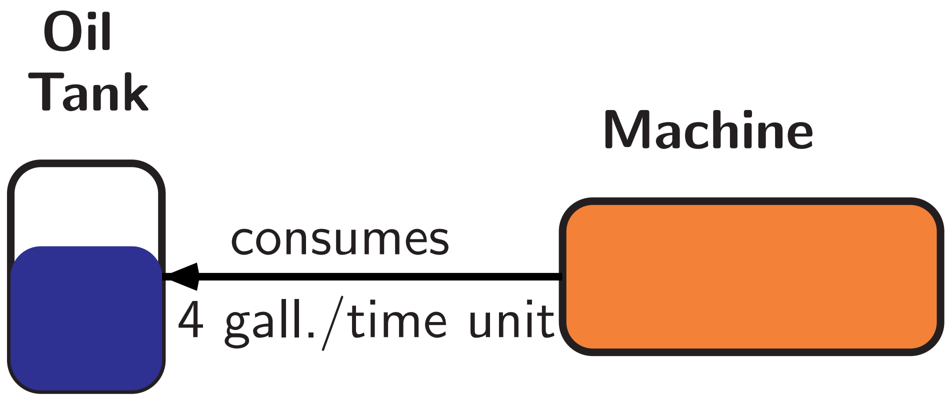

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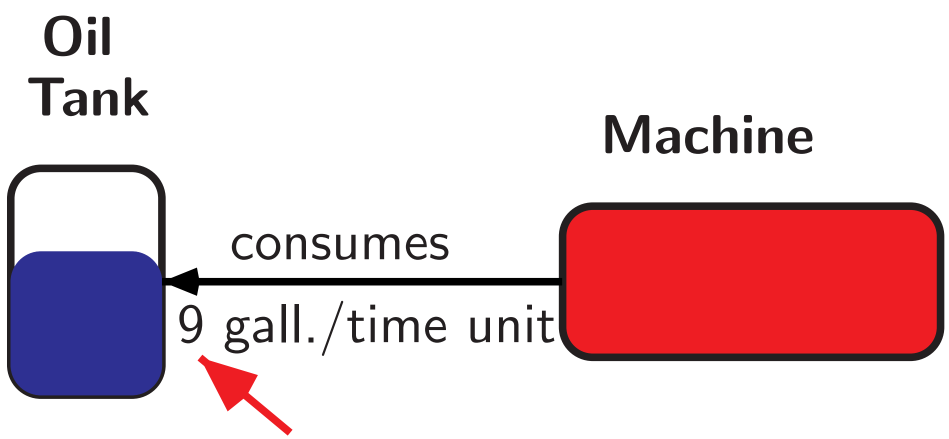

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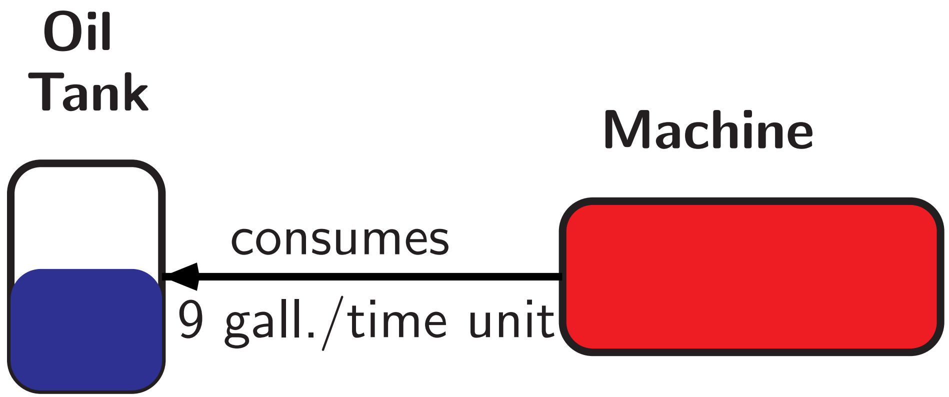

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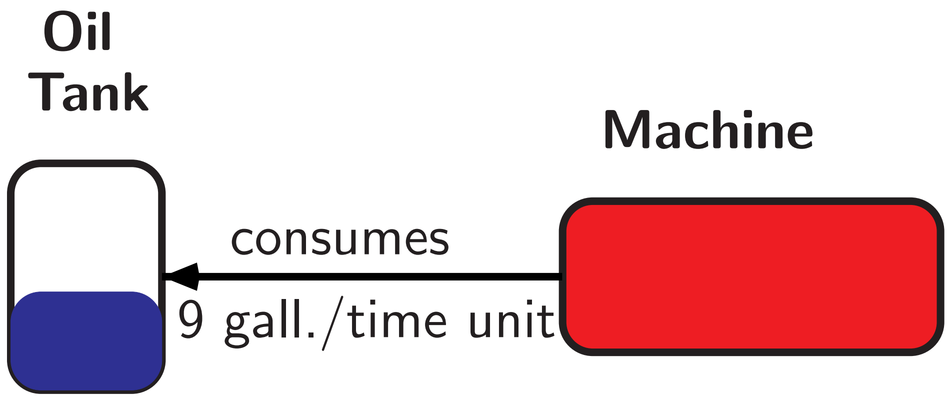

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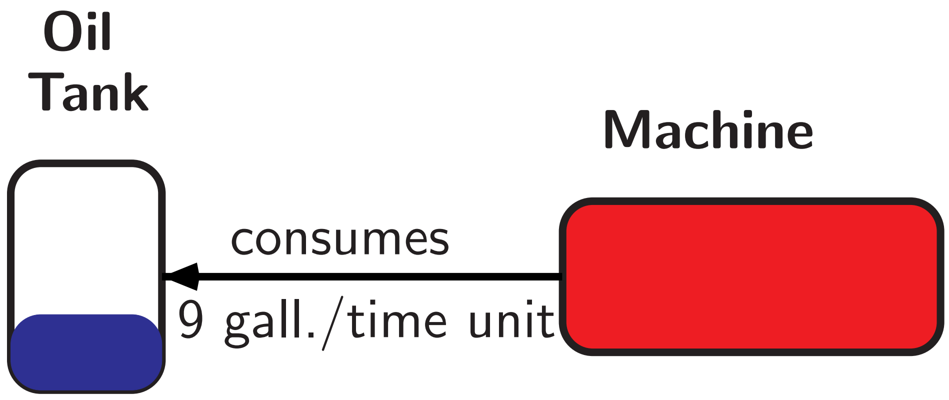

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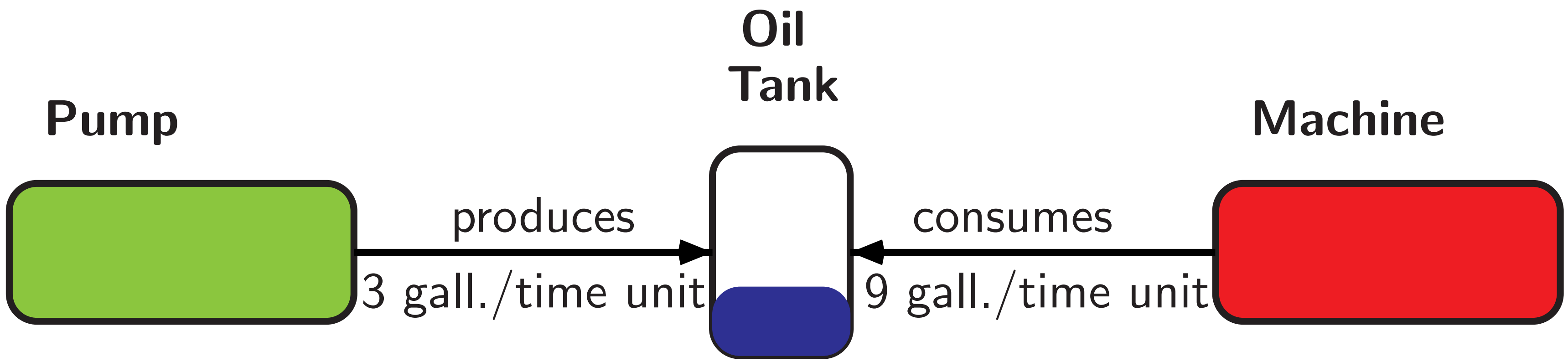

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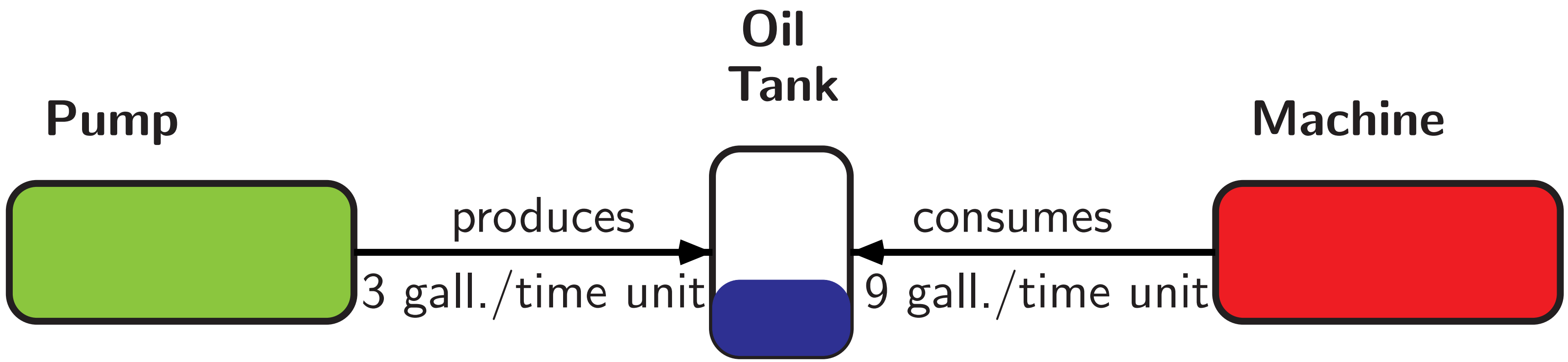

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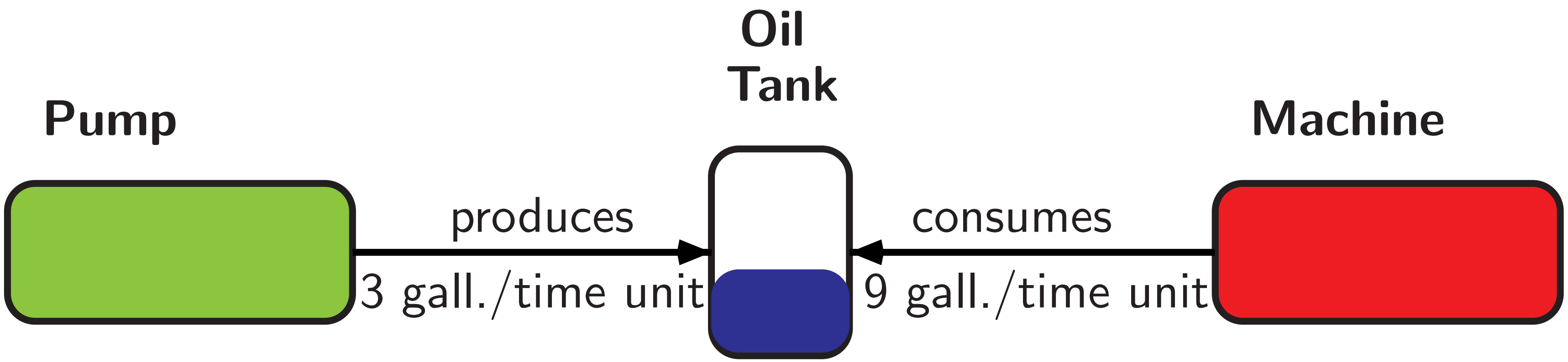

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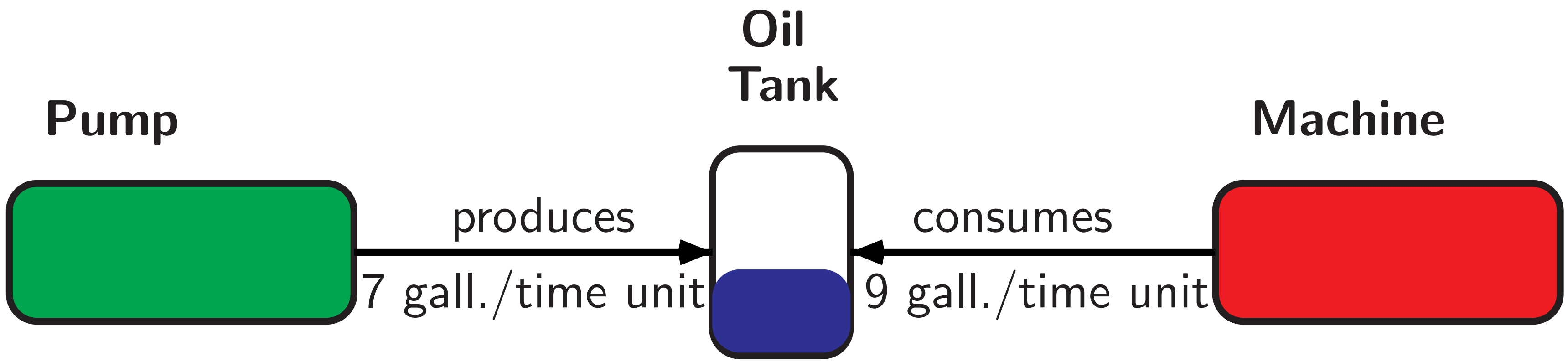

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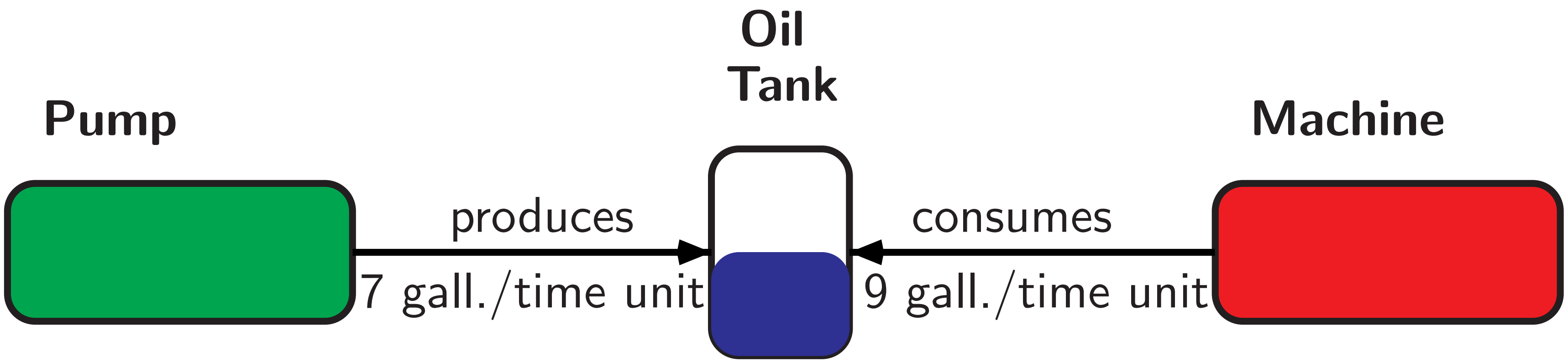

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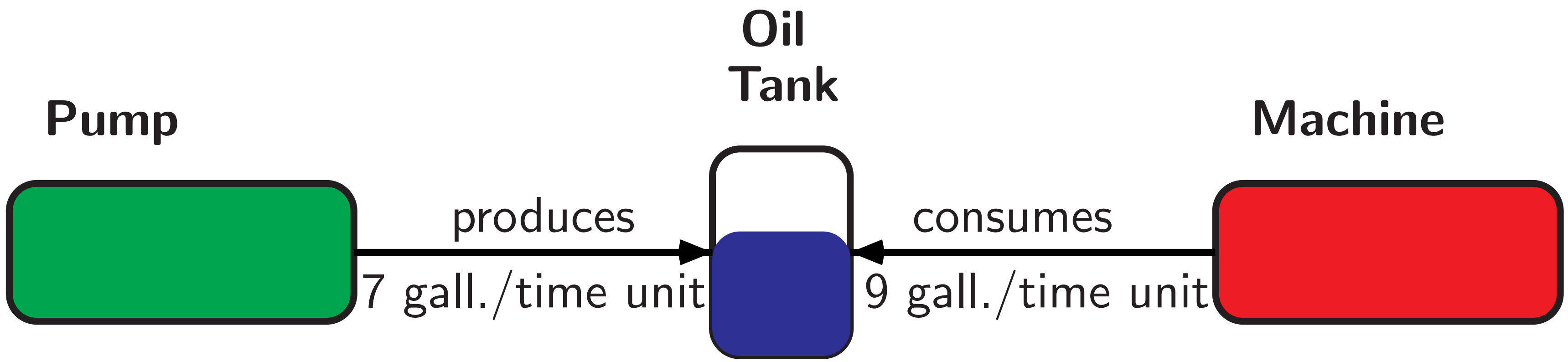

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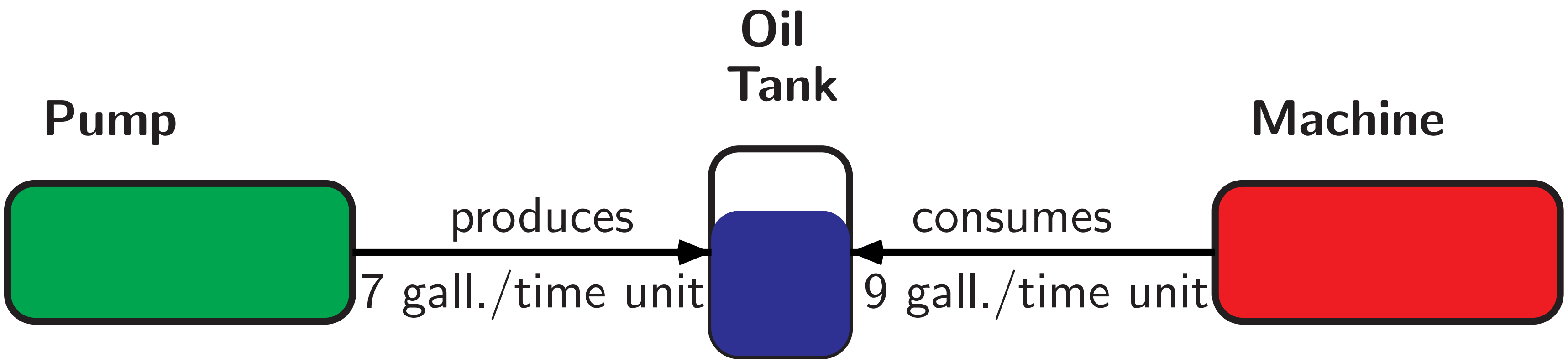
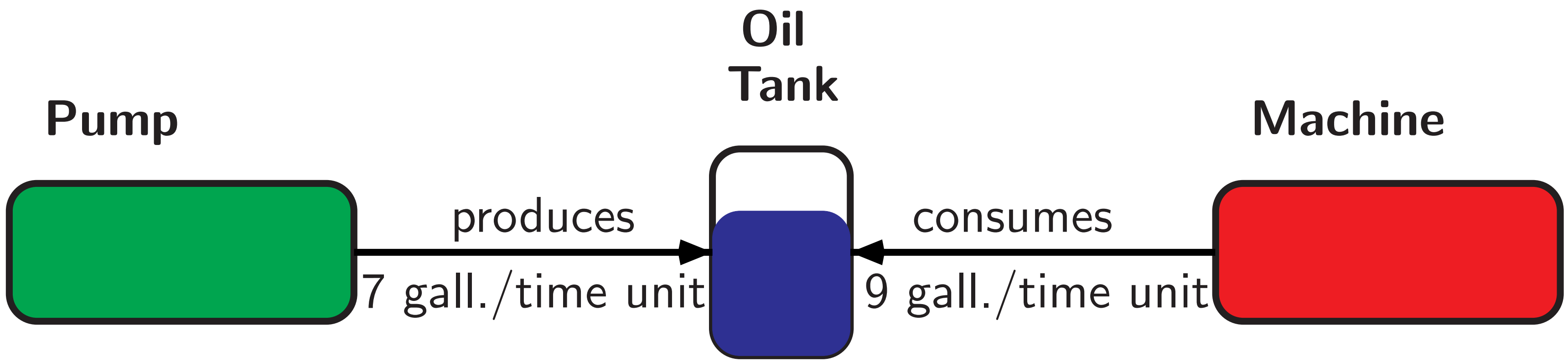


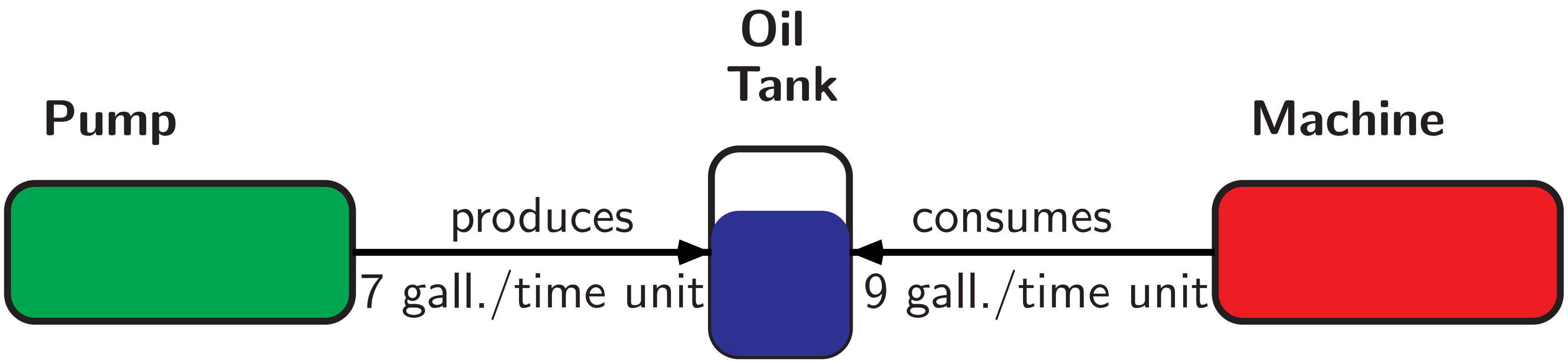
- Initial filling of the tank
- Machine should never break down

Example from the real world



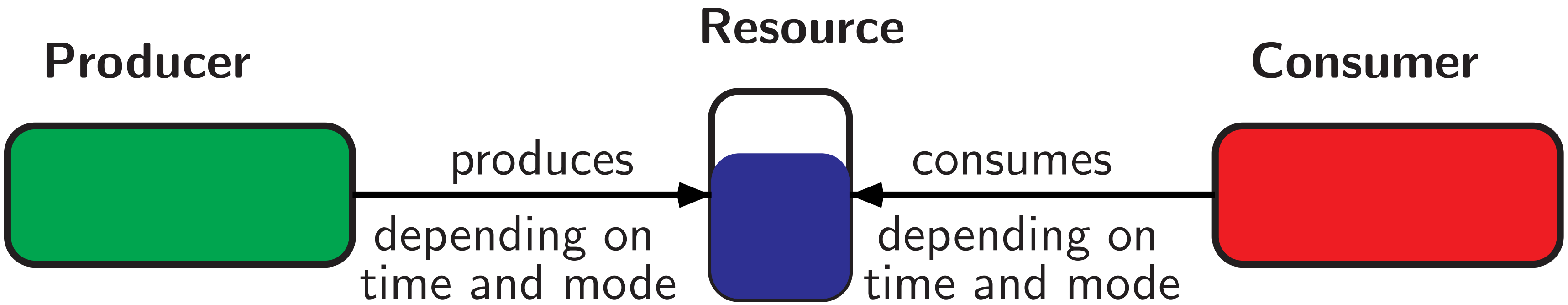
- Initial filling of the tank
- Machine should never break down
- Oil tank should never be empty

Example from the real world



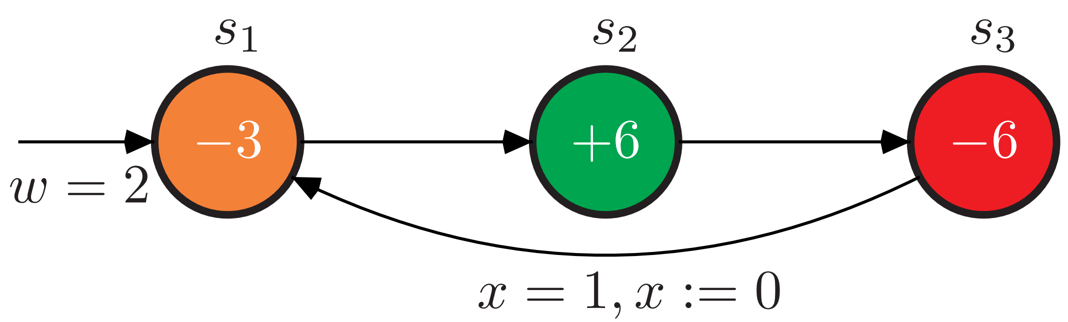
- Initial filling of the tank
- Machine should never break down
- Oil tank should never be empty
- Oil tank should never overflow

Weighted Timed Systems with Constraints on Weights



- Initial value ι of resource
- System should never terminate
- Value of resource should never be less than 0
- Value of resource should never exceed upper bound b

Weighted Timed Automaton(1,1)



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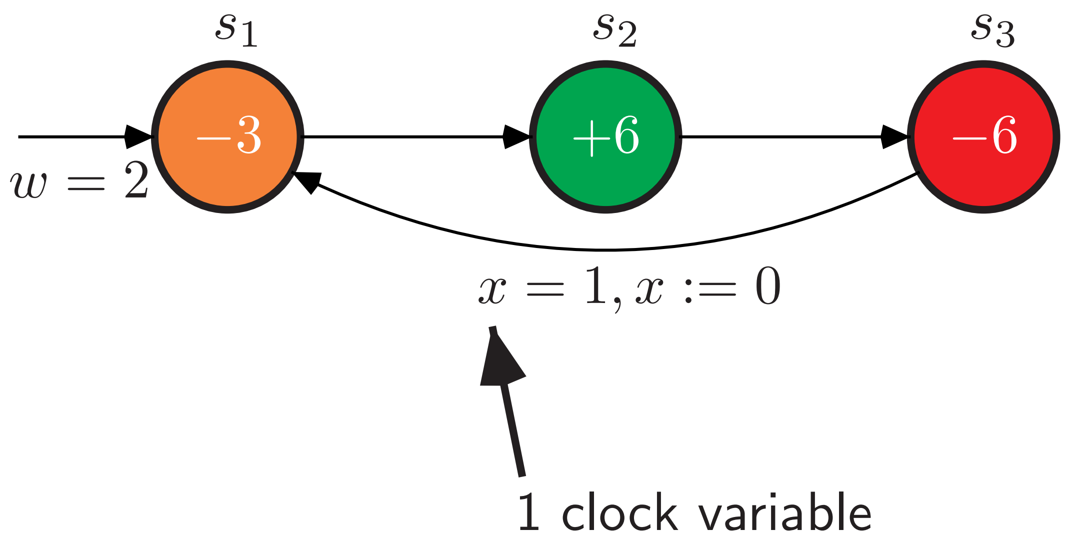

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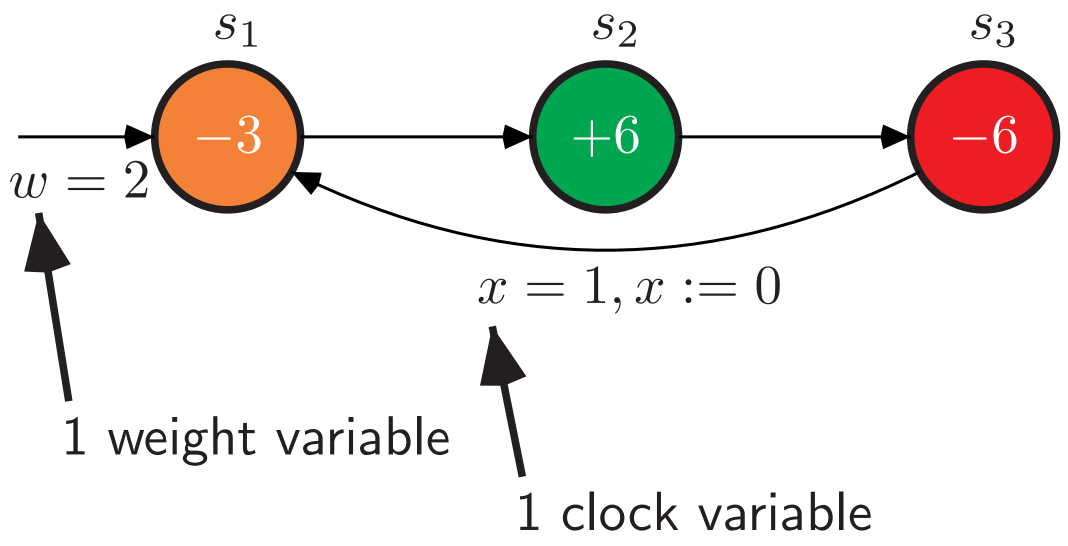

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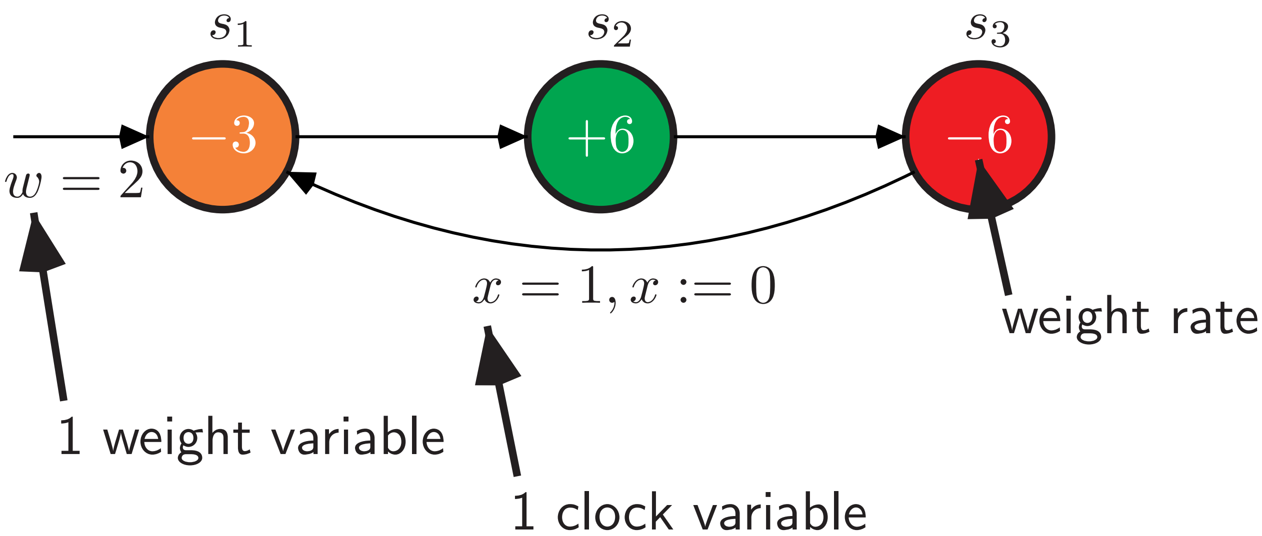

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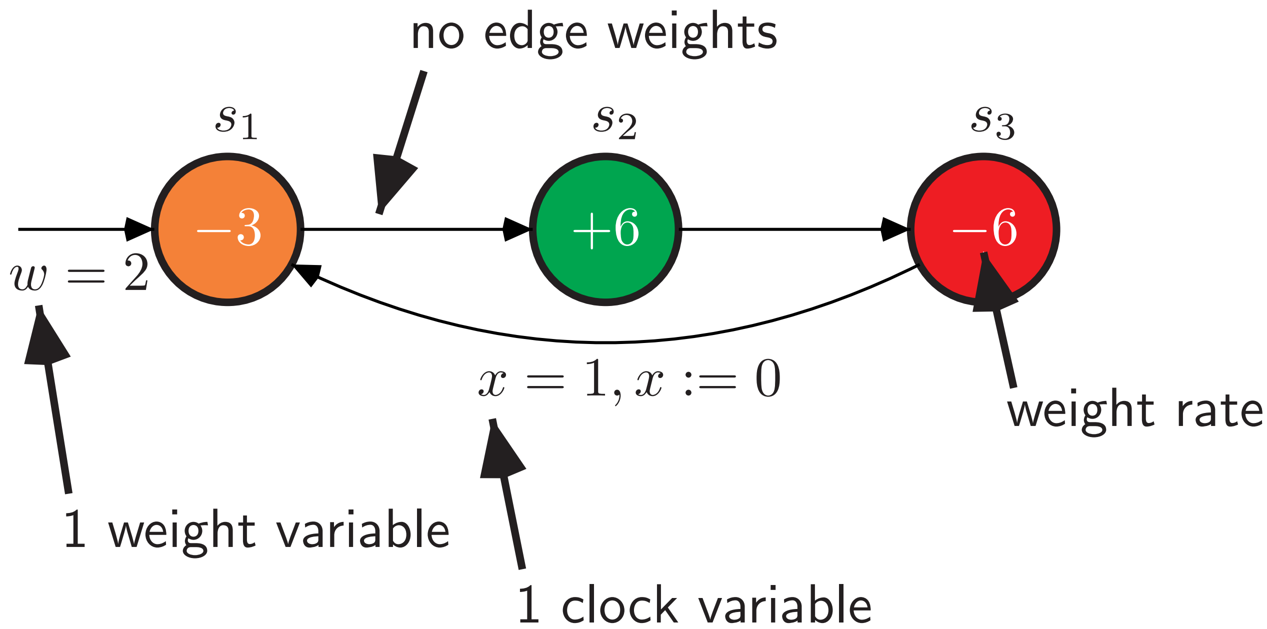

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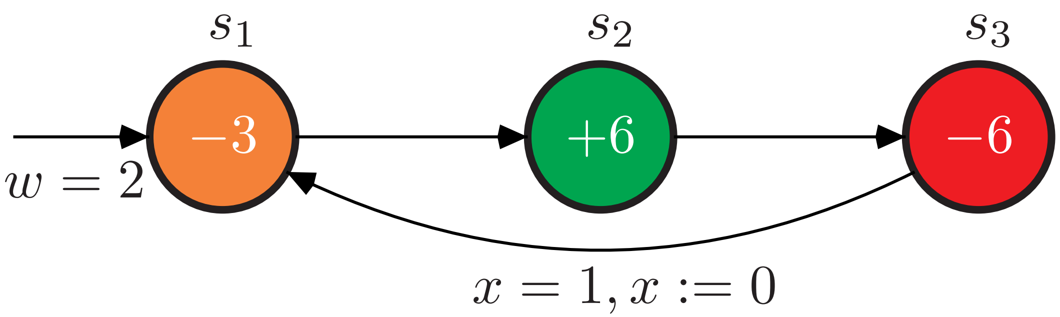
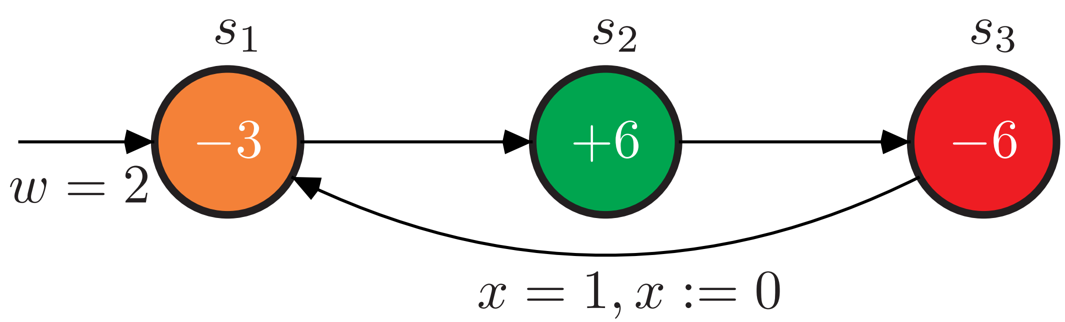


$$(s_1, 0, 2) \xrightarrow{\frac{2}{3}} (s_2, \tfrac{2}{3}, 0) \xrightarrow{\frac{1}{3}} (s_3, 1, 2) \xrightarrow{0} (s_1, 0, 2) \xrightarrow{\frac{2}{3}} (s_2, \tfrac{2}{3}, 0) \xrightarrow{\frac{1}{3}} \dots$$

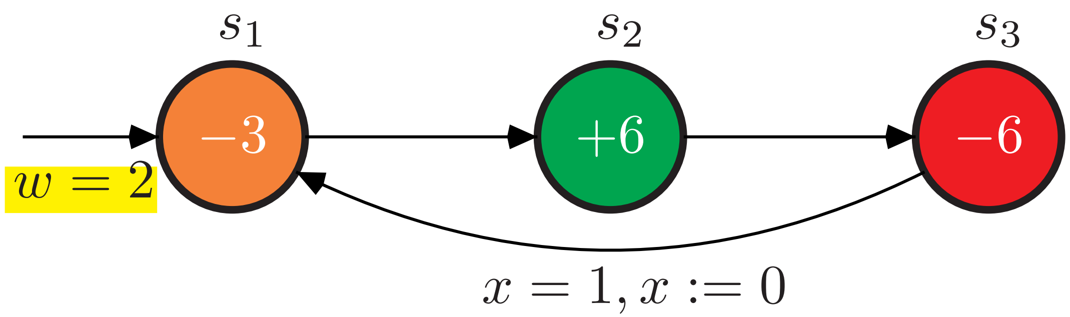
Weighted Timed Automaton(1,1)



$$(s_1, 0, 2) \xrightarrow{\frac{2}{3}} (s_2, \tfrac{2}{3}, 0) \xrightarrow{\frac{1}{3}} (s_3, 1, 2) \xrightarrow{0} (s_1, 0, 2) \xrightarrow{\frac{2}{3}} (s_2, \tfrac{2}{3}, 0) \xrightarrow{\frac{1}{3}} \ldots$$

There is an infinite 2-run such that the value of w is always within $[0, 2]$.

Weighted Timed Automaton(1,1)



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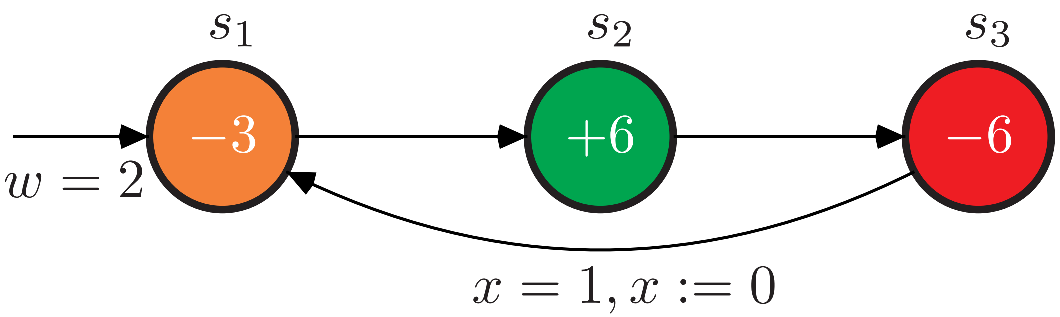

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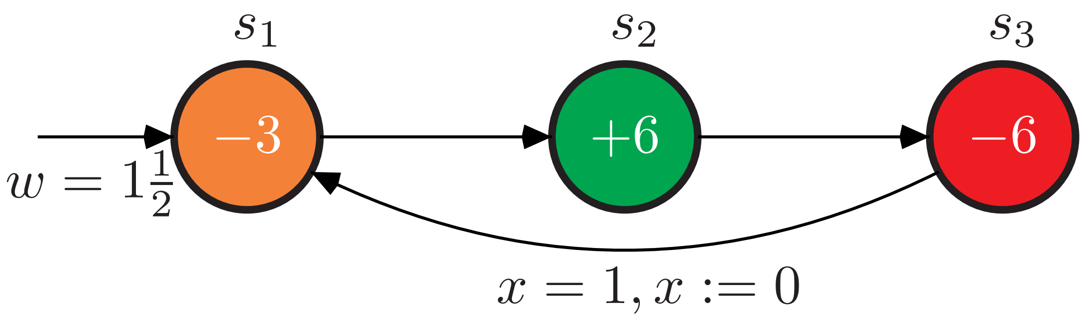
There is an infinite 2-run such that the value of w is always within $[0, 2]$.

Weighted Timed Automaton(1,1)



$$(s_1, 0, 1\tfrac{1}{2}) \xrightarrow{\frac{1}{2}} (s_2, \tfrac{1}{2}, 0) \xrightarrow{\frac{1}{3}} (s_3, \tfrac{5}{6}, 2) \xrightarrow{\frac{1}{6}} (s_1, 0, 1) \xrightarrow{\frac{1}{3}} (s_2, \tfrac{1}{3}, 0) \xrightarrow{\frac{1}{3}} \ldots$$

Is there is an infinite $1\frac{1}{2}$-run such that the value of w is always within $[0, 2]$?

The Interval-Bound Problem

Input: A weighted timed automaton $\mathcal{A}(m, n)$ with
- m clock variables,
- n weight variables,
- initial weight value $\iota \in \mathbb{Q}^n$,
- upper bound $b \in \mathbb{N}^n$.

Question: $\exists$ infinite ι-run in $\mathcal{A}$ such that $w_i \in [0, b_i]$ for each $i \in \{1, ..., n\}$?

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Question: $\exists$ infinite ι-run in $\mathcal{A}$ such that $w_i \in [0, b_i]$ for each $i \in \{1, ..., n\}$?

$\exists$ infinite run.	$\mathcal{A}(1,1)$ without edge weight	$\mathcal{A}(1,1)$ with edge weight
$w_i \in [0, \infty)$	P	EXPTIME
$w_i \in [0, b]$	?	?

Bouyer et al.: Infinite Runs in Weighted Timed Automata with Energy Constraints. FORMATS 2008.
Bouyer et al.: Timed Automata with Observers under Energy Constraints. HSCC 2010.

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Question: $\exists$ infinite ι-run in $\mathcal{A}$ such that $w_i \in [0, b_i]$ for each $i \in \{1, ..., n\}$?

$\exists$ infinite run.	$\mathcal{A}(1,1)$		$\mathcal{A}(2,2)$	
	without edge weight	with edge weight	without edge weight	with edge weight
$w_i \in [0, \infty)$	P	EXPTIME	?	?
$w_i \in [0, b]$	?	?	undecidable	undecidable

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Q: On the Interval-Bound Problem for Weighted Timed Automata. LATA 2011.

Two-Counter Machines

A two-counter machine is a finite sequence $\mathcal{M} = (I_1, ..., I_n)$ of instructions of the form (where $i \in \{1, 2\}$, $j, k, m \in \{1, ..., n\}$):

increment	$I_j : C_i := C_i + 1$; `go to` I_k
zero test/decrement	$I_j :$ `if` $C_i = 0$ `then go to` I_k `else` $C_i := C_i - 1$; `go to` I_m
stop	$I_j :$ `stop`

A configuration of $\mathcal{M}$ is a triple $\gamma \in \{I_1, ..., I_n\} \times \mathbb{N} \times \mathbb{N}$.

A computation of $\mathcal{M}$ is a sequence $(\gamma_i)_{i \geq 0}$, where $\gamma_0 = (I_1, 0, 0)$ and γ_{i+1} is the result of executing I_i on γ_i.

The Infinite Computation Problem

Input: A two-counter machine $\mathcal{M}$.
Question: $\exists$ infinite computation of $\mathcal{M}$?

How to Encode a Two-Counter Machine

Two-counter machine $\mathcal{M}$ $\xrightarrow{\text{encoded by}}$ Weighted timed automaton $\mathcal{A}_{\mathcal{M}}(2,2)$

- initial weight value $(4,0)$
- upper bound $(5,5)$

$\exists$ infinite computation of $\mathcal{M}$

$\Leftrightarrow$

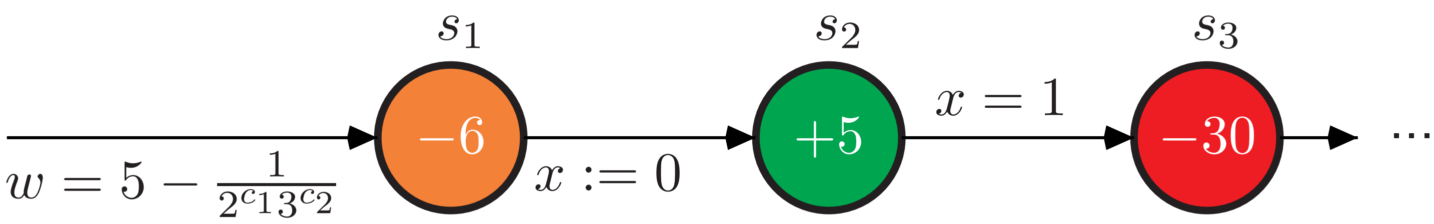
$\exists$ infinite $(4,0)$-run in $\mathcal{A}_{\mathcal{M}}$ such that $w \in [0,5]$ and $w' \in [0,5]$.

The counters C_1 and C_2 are encoded by the weight variable w:

$$w = 5 - \frac{1}{2^{c_1}3^{c_2}}$$

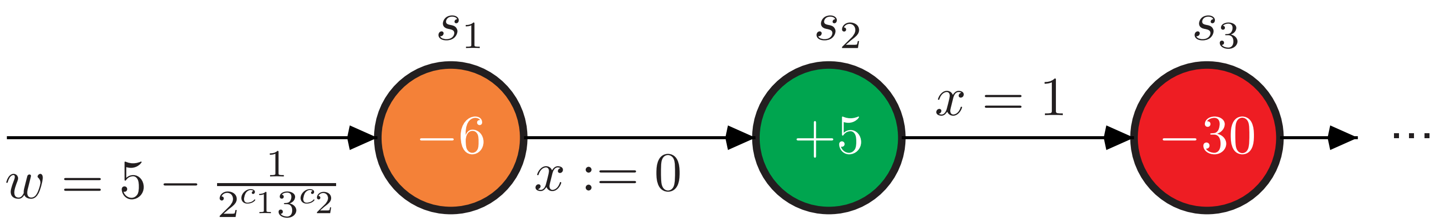
The interval-bounds and clock constraints are used to control the value of w.

How To Encode The Counters - Example



Is there is an infinite $5 - \frac{1}{2^{c_1}3^{c_2}}$-run such that the value of w is within $[0, 5]$?

How To Encode The Counters - Example



Is there is an infinite $5 - \frac{1}{2^{c_1}3^{c_2}}$-run such that the value of w is within $[0, 5]$?

$$(s_1, 0, w) \xrightarrow{\delta} (s_2, 0, w - 6\delta) \longrightarrow \ldots$$

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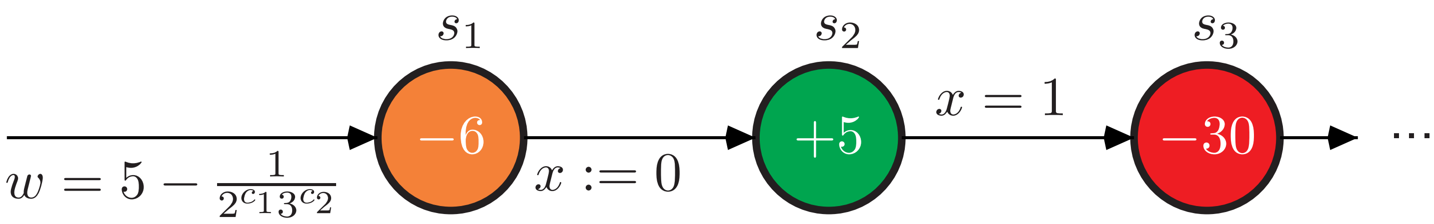

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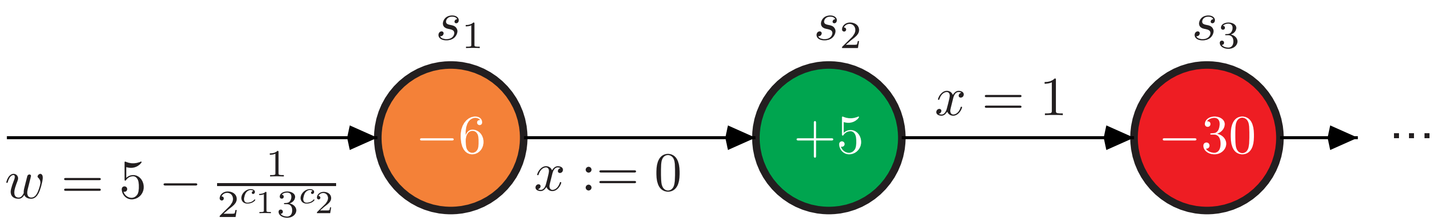
$(s_1, 0, w) \xrightarrow{\delta} (s_2, 0, w - 6\delta) \longrightarrow \ldots$

δ must satisfy $0 \leq w - 6\delta \leq 5 \quad \Rightarrow \quad \frac{w-5}{6} \leq \delta \leq \frac{w}{6}$

How To Encode The Counters - Example



Is there is an infinite $5 - \frac{1}{2^{c_1}3^{c_2}}$-run such that the value of w is within $[0, 5]$?

$$(s_1, 0, w) \xrightarrow{\delta} (s_2, 0, w - 6\delta) \xrightarrow{1} (s_3, 1, w - 6\delta + 5) \longrightarrow \ldots$$

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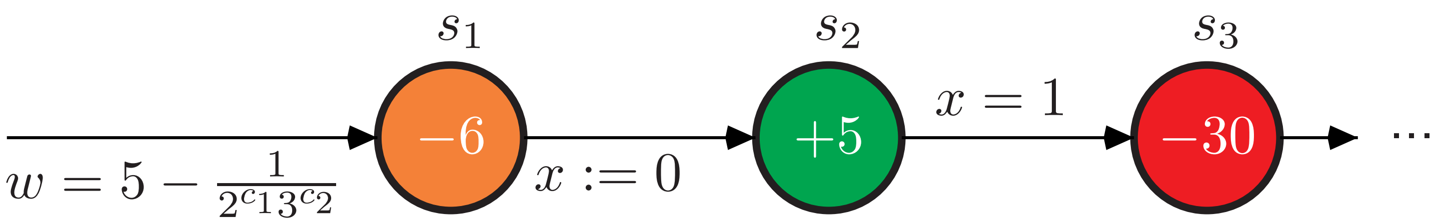

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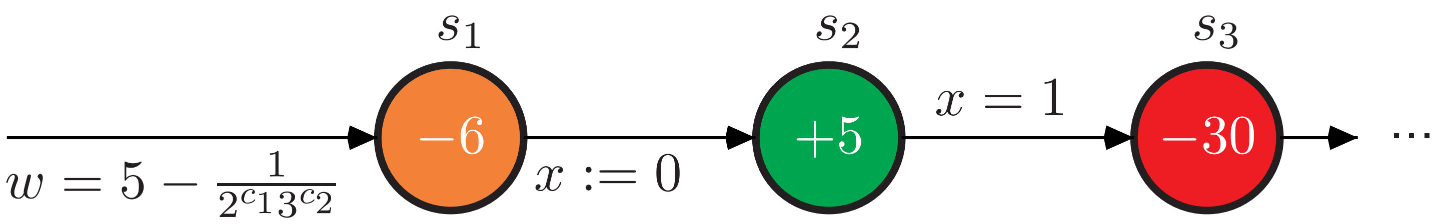

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$(s_1, 0, w) \xrightarrow{\delta} (s_2, 0, w - 6\delta) \xrightarrow{1} (s_3, 1, w - 6\delta + 5) \longrightarrow \ldots$

δ must satisfy $0 \leq w - 6\delta + 5 \leq 5 \quad \Rightarrow \quad \delta = \frac{w}{6}$

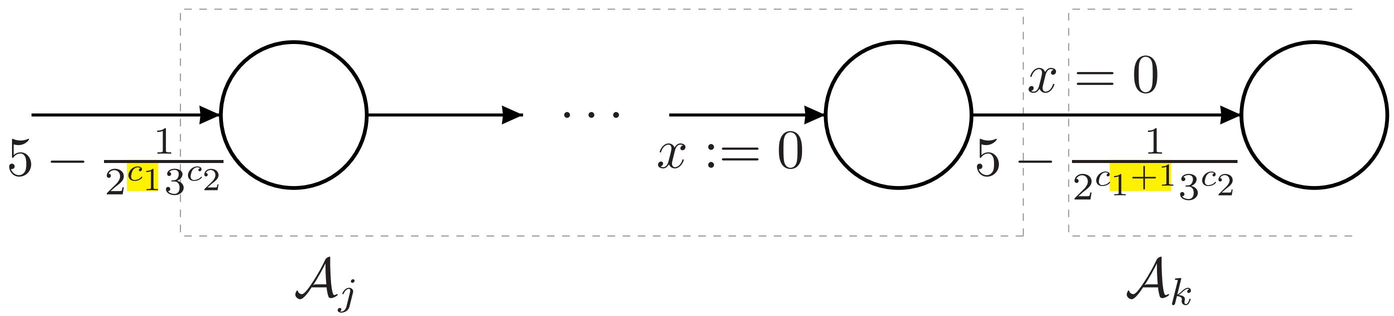
How to Encode a Two-Counter Machine

For each instruction I_j of $\mathcal{M}$, we construct a corresponding $\mathcal{A}_j$

e.g.,

increment	$I_j : C_1 := C_1 + 1$; go to I_k

:



The second clock and weight variable are needed for encoding the instruction

zero test/decrement	I_j : if $C_i = 0$ then go to I_k else $C_i := C_i - 1$; go to I_m

First Main Result

Two-counter machine $\mathcal{M}$ $\xrightarrow{\text{encoded by}}$ Weighted timed automaton $\mathcal{A}_{\mathcal{M}}(2,2)$
- initial weight value $(4,0)$
- upper bound $(5,5)$

$\exists$ infinite computation of $\mathcal{M}$

$\Leftrightarrow$

$\exists$ infinite $(4,0)$-run in $\mathcal{A}_{\mathcal{M}}$ such that $w \in [0,5]$ and $w' \in [0,5]$.

Theorem: The interval-bound problem for weighted timed automata with two clocks, two weight variables, and without edge weights is undecidable.

The Interval-Bound Problem

$\exists$ infinite run.	$\mathcal{A}(1,1)$ without edge weight	$\mathcal{A}(1,1)$ with edge weight	$\mathcal{A}(2,2)$ without edge weight	$\mathcal{A}(2,2)$ with edge weight
$w_i \in [0, \infty)$	P	EXPTIME	?	?
$w_i \in [0, b]$	?	?	undecidable	undecidable

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Q: On the Interval-Bound Problem for Weighted Timed Automata. LATA 2011.

The Interval-Bound Problem

∃ infinite run.	$\mathcal{A}(1,1)$ without edge weight	$\mathcal{A}(1,1)$ with edge weight	$\mathcal{A}(2,2)$ without edge weight	$\mathcal{A}(2,2)$ with edge weight
$w_i \in [0, \infty)$	P	EXPTIME	?	?
$w_i \in [0, b]$	?	?	undecidable	undecidable

$\mathcal{A}(1,2)$ without edge weight	$\mathcal{A}(1,2)$ with edge weight
?	undecidable

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The Interval-Bound Problem

$\exists$ infinite run.	$\mathcal{A}(1,1)$ without edge weight	$\mathcal{A}(1,1)$ with edge weight	$\mathcal{A}(2,2)$ without edge weight	$\mathcal{A}(2,2)$ with edge weight
$w_i \in [0,\infty)$	P	EXPTIME	?	?
$w_i \in [0,b]$	?	?	undecidable	undecidable

$\exists$ infinite run.	$\mathcal{A}(2,1)$ without edge weight	$\mathcal{A}(2,1)$ with edge weight	$\mathcal{A}(1,2)$ without edge weight	$\mathcal{A}(1,2)$ with edge weight
$w_i \in [0,b]$	?	undecidable	?	undecidable

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Q: On the Interval-Bound Problem for Weighted Timed Automata. LATA 2011.

Bouyer-Decitre, Markey. April 2011. Unpublished.

The Interval-Bound Problem for Discrete-Time

Weighted Discrete-Timed Automata: All time delays are in $\mathbb{N}$.

Input: A weighted discrete-timed automaton $\mathcal{A}(m, n)$ with
- m clock variables,
- n weight variables,
- initial weight value $\iota \in \mathbb{Q}^n$,
- upper bound $b \in \mathbb{N}^n$.

Question: $\exists$ infinite ι-run in $\mathcal{A}$ such that $w_i \in [0, b_i]$ for each $i \in \{1, ..., n\}$?

Theorem: The interval-bound problem for weighted discrete-timed automata is PSPACE-complete.

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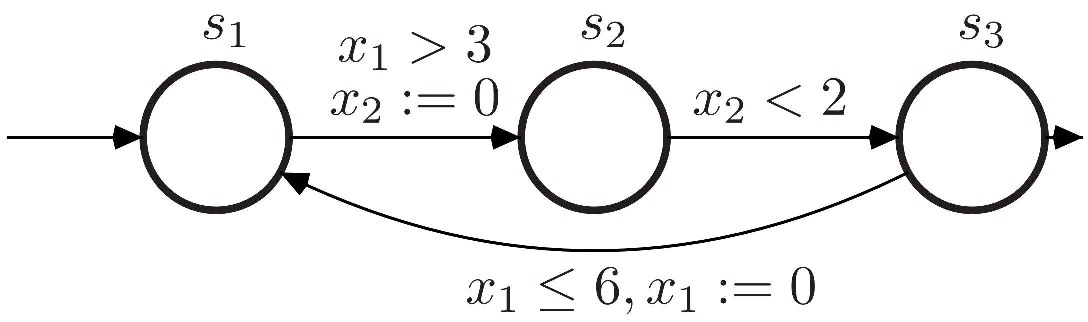
Question: $\exists$ infinite ι-run in $\mathcal{A}$ such that $w_i \in [0, b_i]$ for each $i \in \{1, ..., n\}$?

Theorem: The interval-bound problem for weighted discrete-timed automata is PSPACE-complete.

Q: On the Interval-Bound Problem for Weighted Timed Automata. LATA 2011.

Discrete-Timed Automaton(2)



$$(s_1, 0, 0) \xrightarrow{4} (s_2, 4, 0) \xrightarrow{1} (s_3, 5, 1) \xrightarrow{0} (s_1, 0, 1) \xrightarrow{4} (s_2, 4, 0) \xrightarrow{0} \ldots$$

The Recurrent Reachability Problem

Input: A discrete-timed automaton $\mathcal{A}(n)$ with n clocks.
Question: $\exists$ infinite Büchi-accepting run of $\mathcal{A}$?

The recurrent reachability problem is PSPACE-complete if $n \geq 3$.

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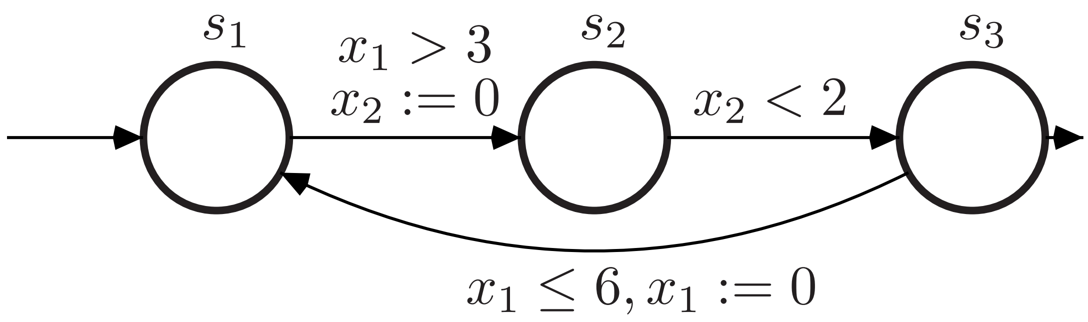

All time delays are in $\mathbb{N}$

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How to Encode a Weighted Discrete -Timed Automaton

Weighted D -T Automaton $\mathcal{A}(m, n)$
- initial weight value $\iota \in \mathbb{N}^n$
- upper bound $b \in \mathbb{N}^n$

encoded by $\longrightarrow$

Discrete -Timed Automaton
$\mathcal{A}'(m+n+c)$

$\exists$ infinite ι-run in $\mathcal{A}$ such that $w_i \in [0, b(i)]$ for each $i \in \{1, ..., n\}$

$\Leftrightarrow$

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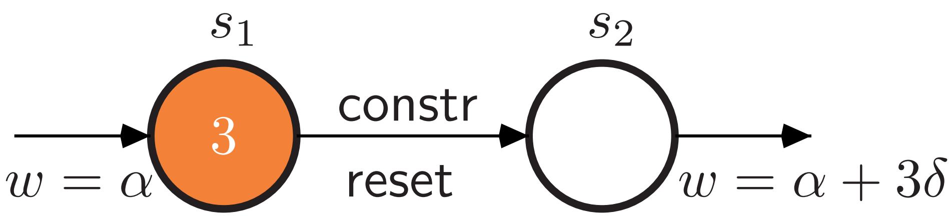
A weight variable w of $\mathcal{A}$ is encoded by clock variable w' in $\mathcal{A}'$.

How to Encode a Weight Variable



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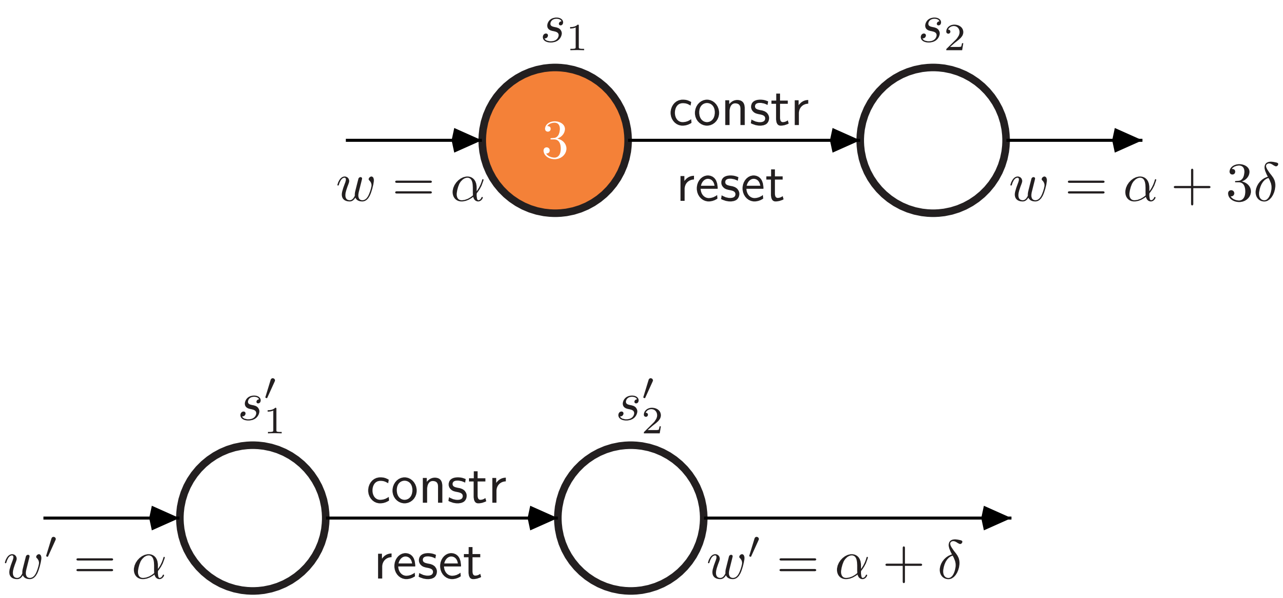

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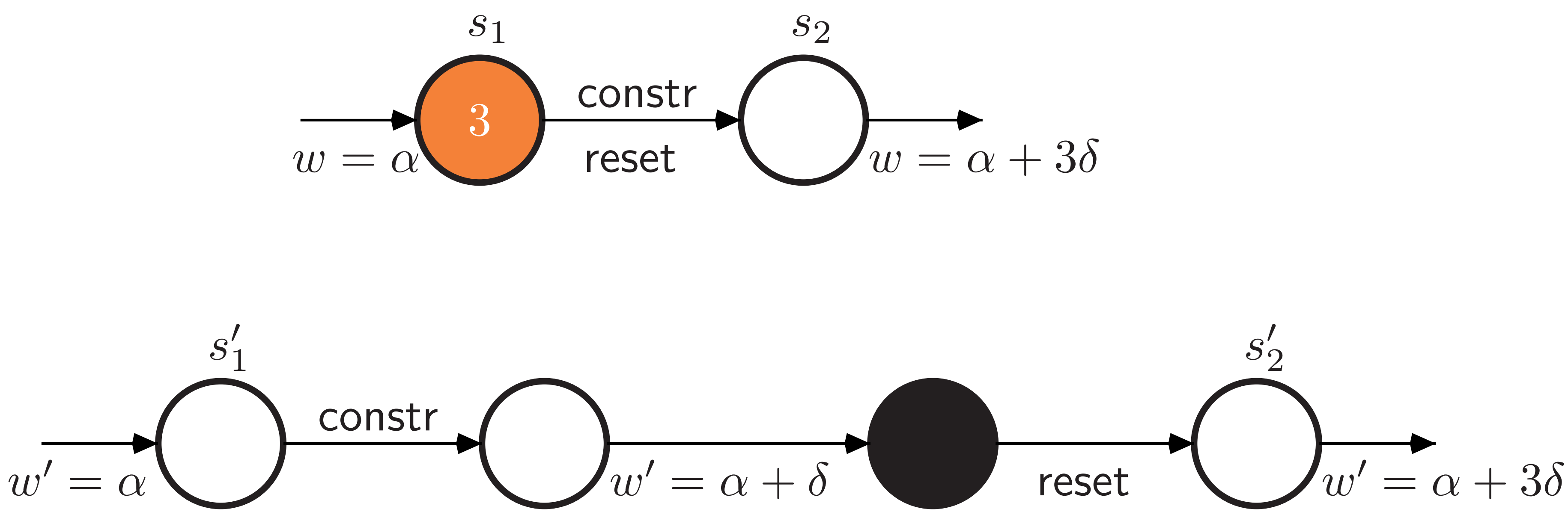

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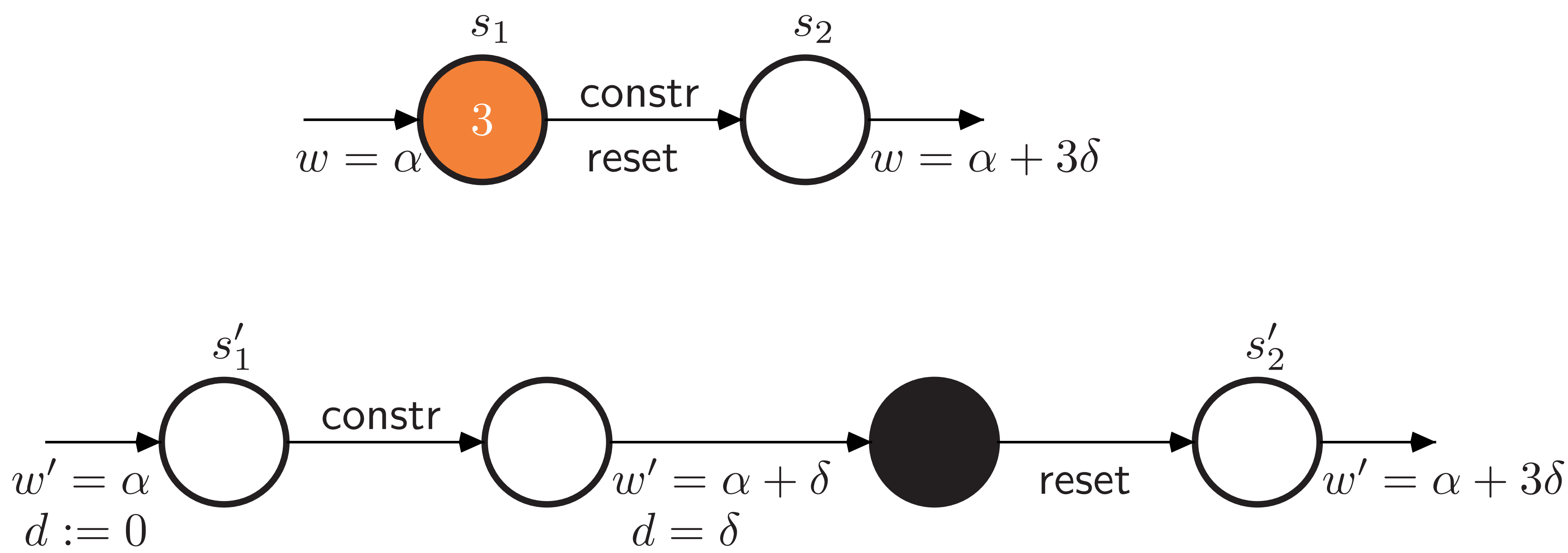

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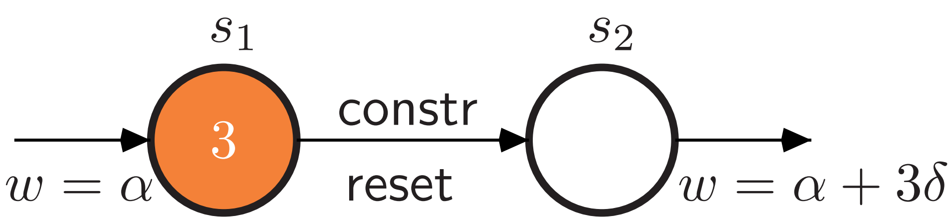
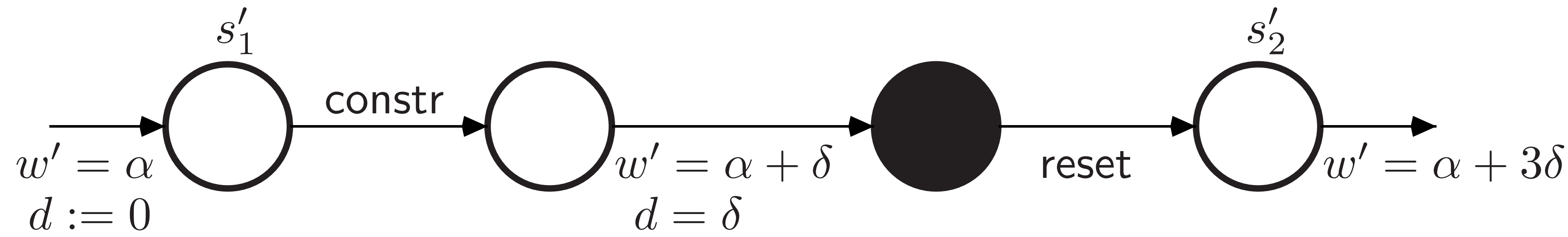
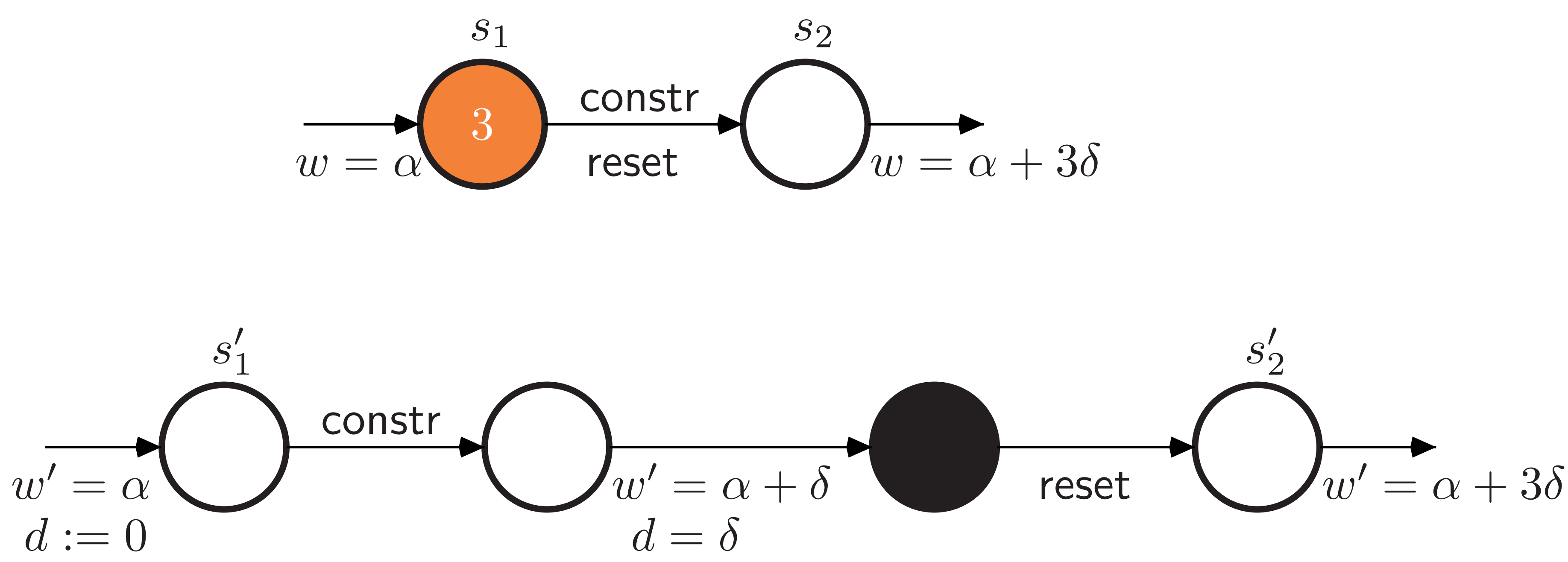




- read out value of d

How to Encode a Weight Variable



- read out value of d
- add value of d to value of w'

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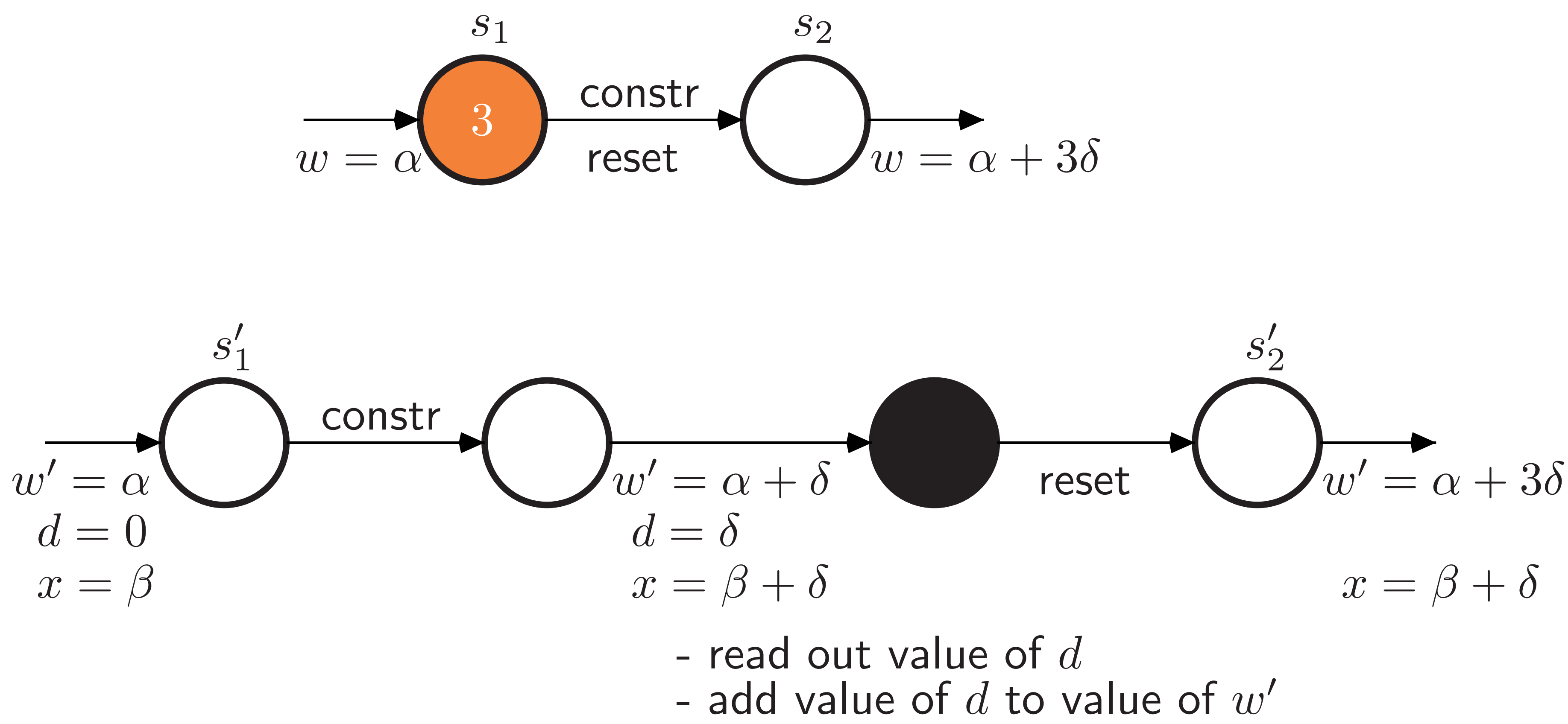

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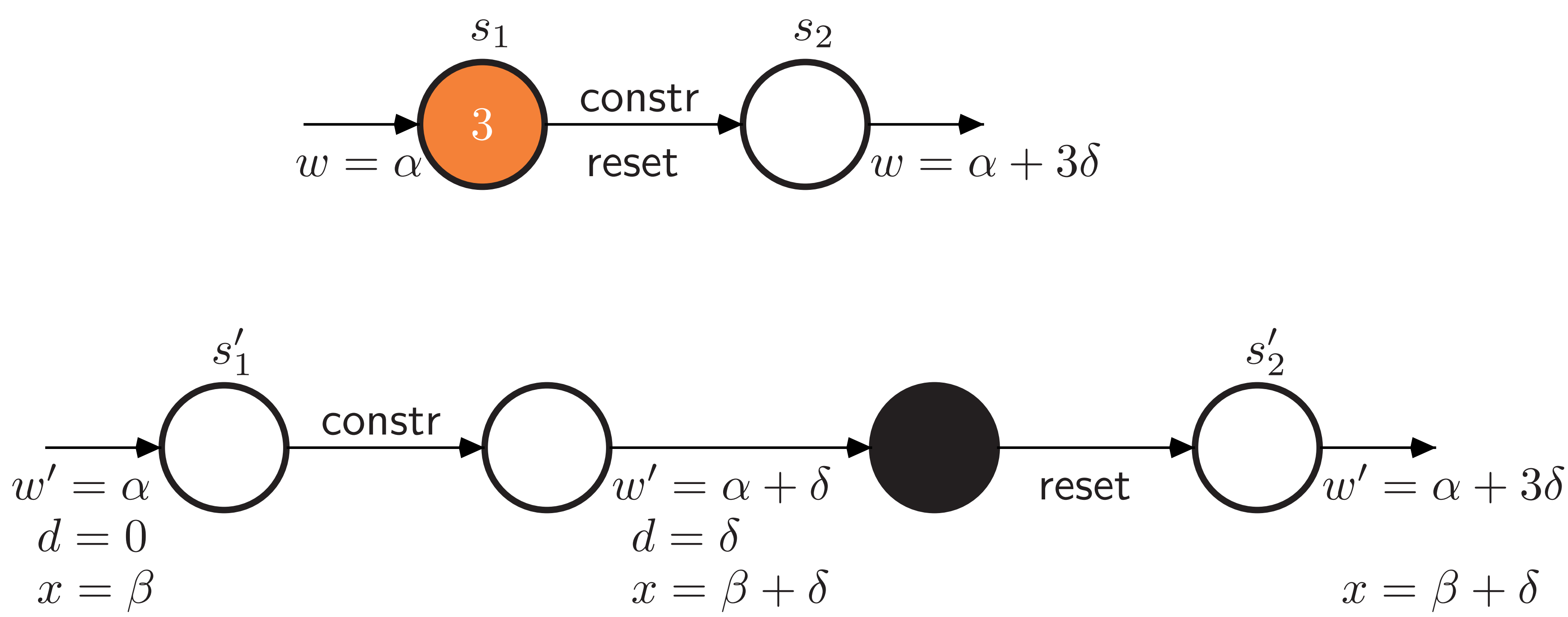
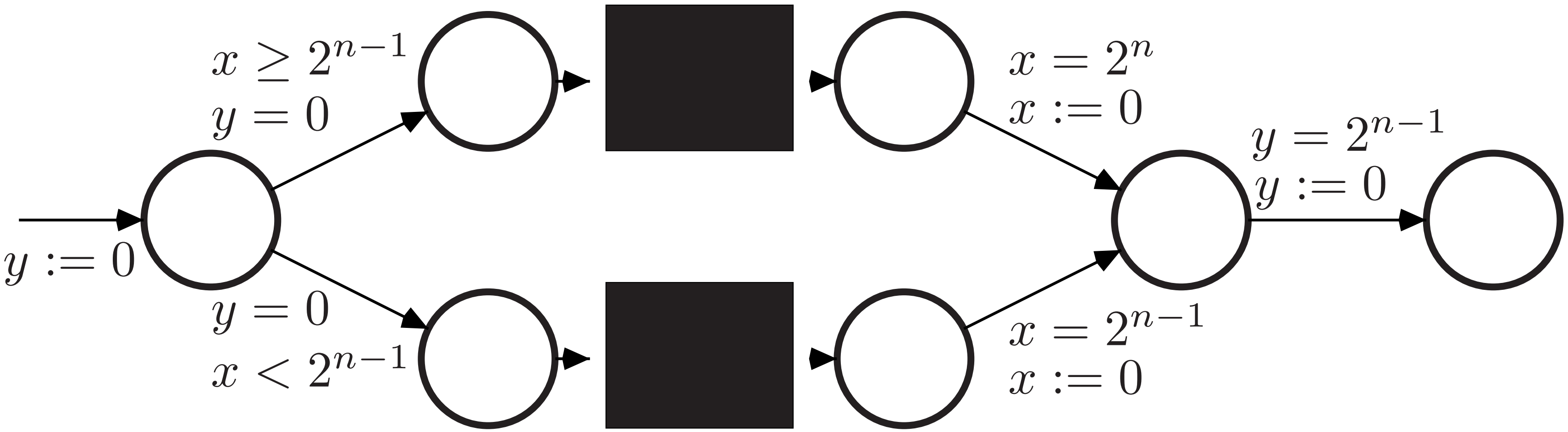


- read out value of d
- add value of d to value of w'
- maintain value of x

How to read out the value of a clock variable*



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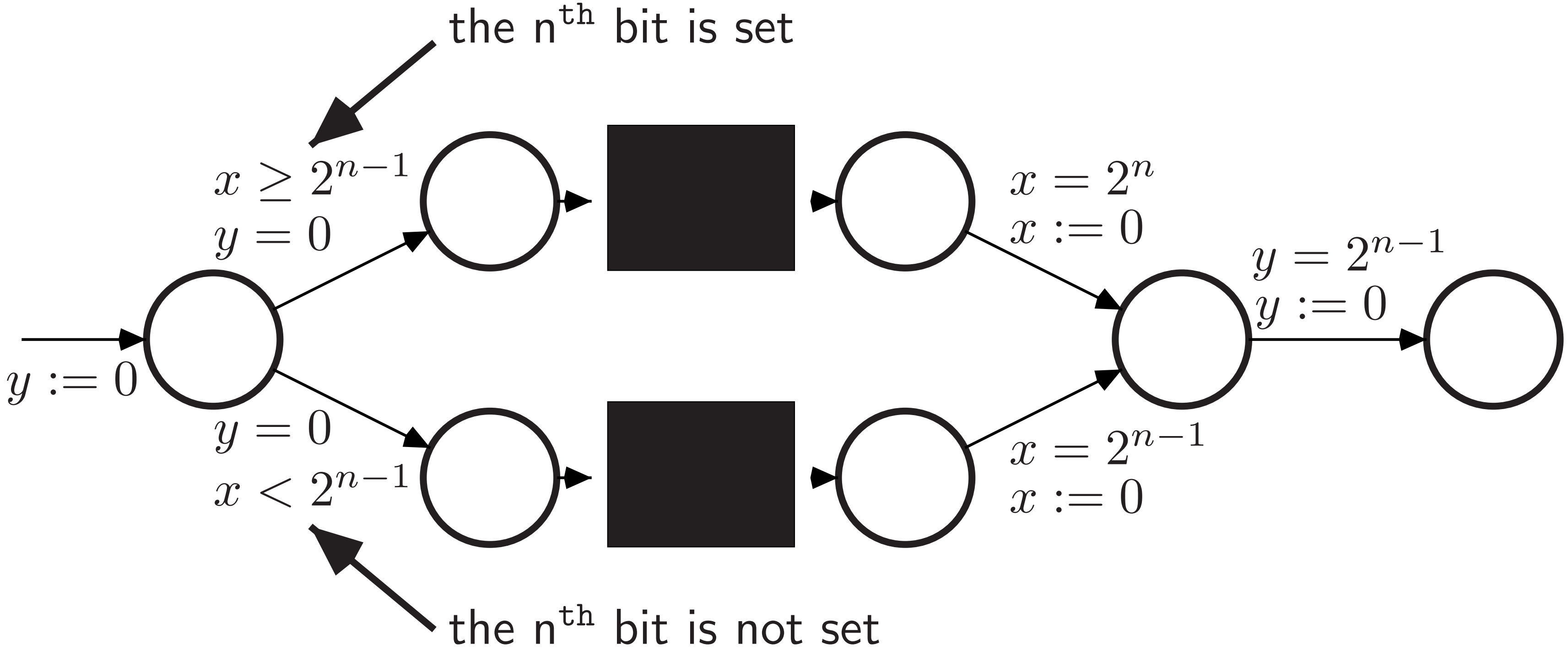

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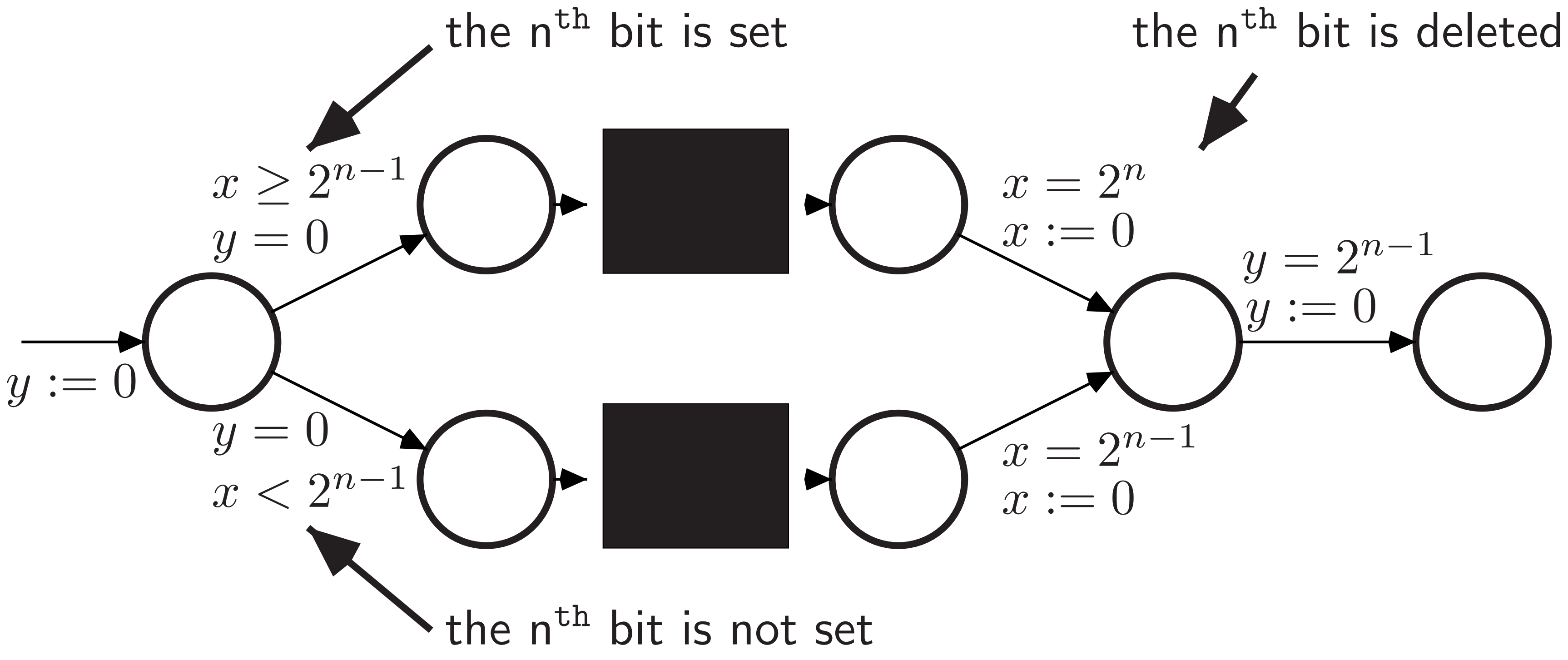

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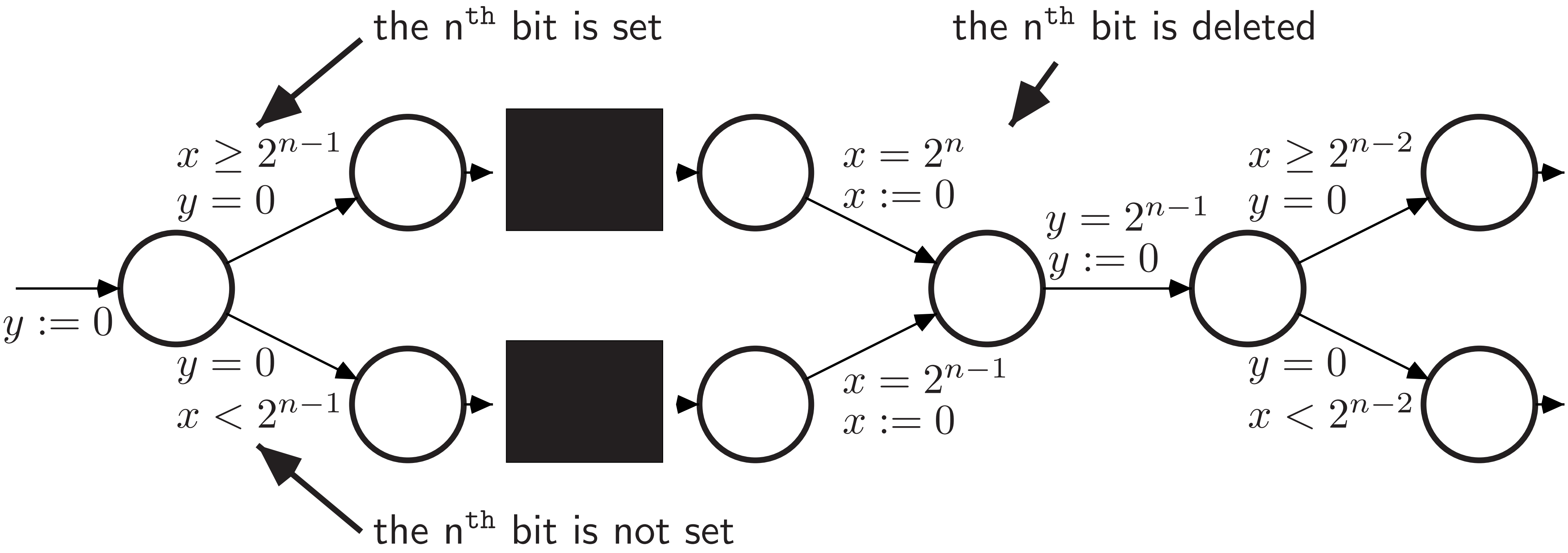

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Second Main Result

Weighted D -T Automaton $\mathcal{A}(m,n)$
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encoded by →

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PSPACE-hardness follows from PSPACE-completeness of recurrent reachability problem for discrete-timed automata.

Summary and Open Problems

∃ infinite run.	$\mathcal{A}(1,1)$		$\mathcal{A}(2,2)$	
	without edge weight	with edge weight	without edge weight	with edge weight
$w_i \in [0, \infty)$	P	EXPTIME	?	?
$w_i \in [0, b]$	?	?	undecidable	undecidable

∃ infinite run.	$\mathcal{A}(2,1)$		$\mathcal{A}(1,2)$	
	without edge weight	with edge weight	without edge weight	with edge weight
$w_i \in [0, b]$	?	undecidable	?	undecidable

Bouyer et al.: Infinite Runs in Weighted Timed Automata with Energy Constraints. FORMATS 2008.
Bouyer et al.: Timed Automata with Observers under Energy Constraints. HSCC 2010.
Q: On the Interval-Bound Problem for Weighted Timed Automata. LATA 2011.
Bouyer-Decitre, Markey. April 2011. Unpublished.