Universität Leipzig
Institut für Informatik
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Prof. Dr. Karin Quaas

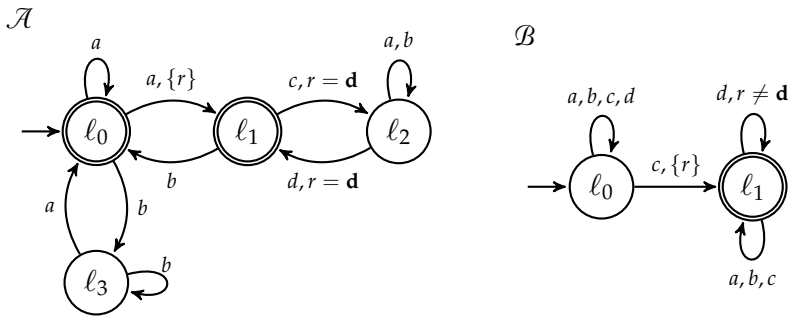
Exercises

Verification for Infinite-State Systems

Exercise Sheet 3 (for 5.11.2026)

Exercise 3.1

Construct the product automaton $\mathcal{A} \times \mathcal{B}$ for $\mathcal{A}$ and $\mathcal{B}$ depicted below.



Exercise 3.2

Can you give a construction that, given two deterministic register automata $\mathcal{A}_1$ and $\mathcal{A}_2$ over Σ, produces a *deterministic* register automaton $\mathcal{A}$ such that $L(\mathcal{A}) = L(\mathcal{A}_1) \cup L(\mathcal{A}_2)$?

Exercise 3.3

Use Lemma 10 to prove that the data language

$$\{(a, d_1) \dots (a, d_n) \mid n \geq 1, d_n = 6\}$$

is not data regular.

Exercise 3.4

Let $\mathcal{R} = \{r_1, r_2, r_3, r_4\}$.

(a) List all possible equivalence relations over $\mathcal{R}$.

(b) Count the number of equivalence relations for each number of equivalence classes, that is, how many equivalence relations over $\mathcal{R}$ have precisely 1 (2, 3, and 4, respectively) induced equivalence class(es)?

(c) Look-up the Bell number $\mathbf{B}_4$ and verify that your total matches it.

Exercise 3.5

Let $\nu = (2, 7, \bot, 7, 8)$ be a register valuation.

(a) Give $\nu' \in \mathbb{N}^{\mathcal{R}}$ such that $\nu \neq \nu'$ and $\nu \equiv \nu'$.

(b) Give $\nu'' \in \mathbb{N}^{\mathcal{R}}$ such that $\nu \equiv \nu''$ does not hold.

(c) Give the 5-type representing $[\nu]_{\equiv}$.

Exercise 3.6

Let $\mathcal{R}$ be a finite set of n registers and let $C = \{c_1, \ldots, c_k\} \subseteq \mathbb{N}$ be a finite set of distinguished constants. We extend the register automaton model by allowing register tests of the form $r = c$, where $r \in \mathcal{R}$ and $c \in C$.

(a) Define an equivalence relation on register valuations that takes the constants into account. In particular, characterize when two valuations are equivalent in terms of a bijection on $\mathbb{N}_{\bot}$.

(b) Prove that this equivalence relation has only finitely many equivalence classes. Give an explicit formula for the number of classes in terms of the number n of registers and the number $k = |C|$ of constants.