Universität Leipzig
Institut für Informatik
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Exercises

Verification for Infinite-State Systems

Exercise Sheet 1 (for 22.10.2026)

Exercise 1.1

(a) Can you write a program that, for every input pushdown automaton $\mathcal{A}_S$ and finite automaton $\mathcal{A}_\varphi$, decides whether every word accepted by $\mathcal{A}_S$ is also accepted by $\mathcal{A}_\varphi$?

(b) Can you write a program that, for every input finite automaton $\mathcal{A}_S$ and pushdown automaton $\mathcal{A}_\varphi$, decides whether every word accepted by $\mathcal{A}_S$ is also accepted by $\mathcal{A}_\varphi$?

Exercise 1.2

Let $\mathcal{R} = \{r_1, \ldots, r_4\}$. Define $\nu_1 = \nu_\perp$, $\nu_2 = \nu_1[\{r_1, r_2\} \leftarrow 4]$, and $\nu_3 = \nu_2[\{r_1, r_4\} \leftarrow 8]$.

(a) Give a register test ϕ such that $\nu_1, 5 \models \phi$.

(b) Give a datum $d \in \mathbb{N}$ such that $\nu_2, d \models r_2 = \mathbf{d} \wedge r_3 \neq \mathbf{d}$.

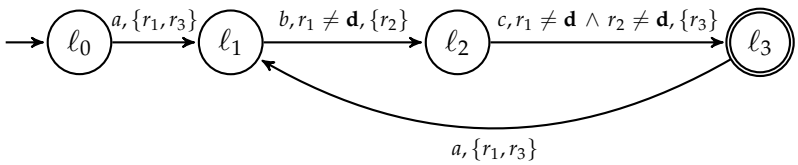
(c) Give a register test ϕ such that $\nu_3, 8 \models \phi$ and ϕ both uses an equality and an inequality test.

(d) Give a datum $d \in \mathbb{N}$ such that $\nu_3, d \models r_1 = \mathbf{d}$.

(e) Give a register test ϕ such that $\nu_3, d \not\models \phi$ for all $d \in \mathbb{N}$.

Exercise 1.3

Consider the register automaton depicted below:



(a) Give a data word w such that $|w| = 6$ and $w \in L(\mathcal{A})$. (Prove that $w \in L(\mathcal{A})$!)

(b) Give data values $d_1, d_2, d_3 \in \mathbb{N}$ such that the data word $(a, d_1)(b, d_2)(c, d_3)$ is not in $L(\mathcal{A})$.

(c) Define $L(\mathcal{A})$.

Exercise 1.4

Prove that the satisfiability problem for register tests is in **P**.

Exercise 1.5

Prove the following statement.

Let $\mathcal{A} = (Q, \Sigma, Q_{\text{in}}, \delta, Q_{\text{acc}})$ be a finite automaton. The following two statements are equivalent.

(a) There exists some initialized accepting run of $\mathcal{A}$.
(b) There exists some initialized accepting run of $\mathcal{A}$ of length[1] strictly smaller than the number of states in Q.

[1] By the length of a run we mean the number of participating transitions.