Universität Leipzig
Institut für Informatik
Winter 2026/2027

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Exercises

Games in Automata and Logic

Exercise Sheet 4 (for 9.11.2026)

Exercise 4.1

In the proof of Theorem 7, we (only) show $W_E(G') \subseteq W_A(G)$ and $W_A(G') \subseteq W_E(G)$ (inclusion). Why can we set $W_A(G) = W_E(G')$ and $W_E(G) = W_A(G')$ (equalities)?

Exercise 4.2 (Closure of Attractor under Attractor)

Prove Lemma 10:

$$\text{For all } T \subseteq V,\ \text{Attr}_E(T) = \text{Attr}_E(\text{Attr}_E(T)).$$

Exercise 4.3

Prove or disprove the following statement: for every arena (V, V_A, V_E, E) and $T \subseteq V$, we have

$$\text{Force}_E(\text{Attr}_E(T)) = \text{Force}_A(V \setminus \text{Attr}_A(T)).$$

Exercise 4.4

Let $A = (V, V_A, V_E, E)$ and let $T \subseteq V$. Let $G = (A, \text{Reach}^2(A, T))$, where $\text{Reach}^2(A, T)$ is defined by

$$\{v_0 v_1 v_2 \cdots \in \text{Plays}(A) \mid \text{there exist } i < j \text{ such that } v_i \in T \text{ and } v_j \in T\}.$$

We define $R_2 = T \setminus (\text{Force}_A(V \setminus \text{Attr}_E(T))$. We aim to prove that G is memoryless determined and $W_E(G) = \text{Attr}_E(R_2)$:

(a) Prove that there exists a memorless winning strategy g for Eve such that g is winning for every $v \in \text{Attr}_E(R_2)$.

(b) Prove that there exists a memoryless winning strategy f for Alice such that f is winning for every $v \notin \text{Attr}_E(R_2)$.

Exercise 4.5

Let $A = (V, V_A, V_E, E)$ and let $T \subseteq V$. Given a strategy $g : V_E \to V$ for Eve, define the arena A_g to be the arena obtained from A by removing all edges $(v, u) \in E$ such that $v \in V_E$ and $u \neq g(v)$.

We define *a path* to be a sequence $v_0, v_1, \ldots, v_k$ satisfying $(v_{i-1}, v_i) \in E$ for all $1 \leq i \leq k$. A path $v_0, v_1, \ldots, v_k$ is a *cycle* if $v_0 = v_k$. Given two vertices $v, u \in V$, we say that *u is reachable from v* if there exists a path $v_0, v_1, \ldots, v_k$ with $v_0 = v$ and $v_k = u$.

Given a cycle $\pi = v_0, v_1, \ldots, v_k$, we say that π *is reachable from* v if there exists some $0 \leq i \leq k$ such that v_i is reachable from v.

Prove or disprove the following two claims, for the Büchi game $G = (A, \text{Büchi}(A, T))$. For every $v \in V$ and every $g : V_{\text{E}} \to V$:

(a) If g is winning for Eve from v, then every cycle in A_g that is reachable from v intersects T.

(b) If every cycle in A_g that is reachable from v intersects T, then g is winning for Eve from v.

Exercise 4.6

Consider the following decision problem: given an arena $A = (V, V_{\text{A}}, V_{\text{E}}, E)$, target vertices $T \subseteq V$, $v \in V$, and a memoryless strategy $g : V_{\text{E}} \to V$ for Eve, decide whether g is winning for Eve from v in the co-Büchi game $G = (A, \text{coBüchi}(A, T))$. What is the computational complexity of deciding this problem?