Universität Leipzig
Institut für Informatik
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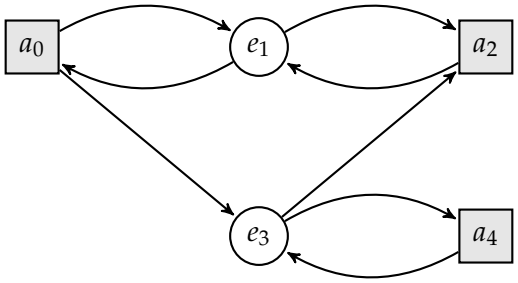
Exercises

Games in Automata and Logic

Exercise Sheet 2 (for 26.10.2026)

Exercise 2.1 (Reachability and Safety)

Consider the arena $A = (V, V_A, V_E, E)$ depicted below, where $V_A = \{a_0, a_2, a_4\}$ and $V_E = \{e_1, e_3\}$. Let $T = \{a_0, e_3, a_4\}$.



(a) Give a play from a_0 that is in Safety(A, T).

(b) Give a play from a_0 that is in Reach(A, T) but not in Safety(A, T).

(c) Can you give a play from a_0 that is in Safety(A, T) but not in Reach(A, T)? Prove your answer.

(d) Give a play from a_0 that is in Reach$(A, V \setminus T)$. Is this play in Safety(A, T)? Prove your answer.

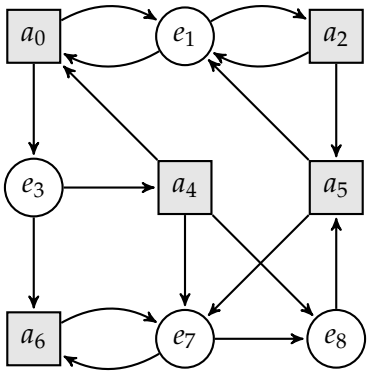
Exercise 2.2

Prove that for every arena $A = (V, V_A, V_E, E)$ and every target set $T \subseteq V$, we have

$$\text{Safety}(A, T) = \text{Plays}(A) \setminus \text{Reach}(A, V \setminus T).$$

Exercise 2.3

Consider the following arena from Lecture 2.



Let $T = \{a_4\}$. We play the reachability game $(A, \text{Reach}(A, T))$. We define the following strategy $g : \{e_1, e_3, e_7, e_8\} \to V$ for Eve by

$$g(e_1) = a_2, \quad g(e_3) = a_4, \quad g(e_7) = a_6, \quad g(e_8) = a_5.$$

(a) Is g a memoryless strategy?

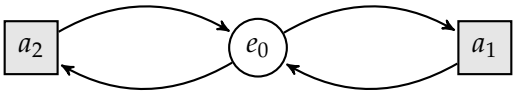
(b) Is g a winning strategy for Eve from e_3?

(c) Is g a winning strategy for Eve from e_7?

(d) Is g a winning strategy for Eve from a_0?

Exercise 2.4

Consider the following arena.



Consider the following strategy g for Eve:

- the first time Eve visits e_0, she moves to a_1;
- the second time Eve visits e_0, she moves to a_2;
- the third time Eve visits e_0, she moves to a_1;
- and so on, alternating between a_1 and a_2.

(a) Is g a strategy for Eve?

(b) Is g memoryless? Explain your answer.

(c) Give two histories π and π' such that $\text{last}(\pi) = \text{last}(\pi') = e_0$, but $g(\pi) \neq g(\pi')$.

Exercise 2.5

Recall the infinite game with two players Alice and Eve from Lecture 1, where

- Alice picks a bit in $\{0, 1\}$,
- Eve responds with a bit in $\{0, 1\}$, and
- this goes on forever.

A play is therefore an infinite sequence $(a_1, e_1)(a_2, e_2)(a_3, e_3) \cdots$. Eve wins if, and only if, $e_i = 1 - a_i$ for all $i \geq 1$.

Transform this game into an equivalent game played on a finite arena whose winning condition is a reachability or safety condition. Your construction should:

- Specify the vertices of the new arena and indicate which vertices belong to Alice and which belong to Eve.
- Specify all possible edges and explain how they correspond to the choices made in the original game.
- Give the reachability or safety set.
- Prove that the new game is equivalent to the original game, i.e. that a play is winning for Eve in the original game if and only if the corresponding play in the constructed arena satisfies the new winning condition.
- Briefly explain why your construction preserves the existence of winning strategies for both players.