Universität Leipzig
Institut für Informatik
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Exercises

Games in Automata and Logic

Exercise Sheet 1 (for 19.10.2026)

Exercise 1.1

Explain the meaning of the following statements over natural numbers. Prove or disprove whether the statements are true.

(a) $\forall x \exists y \; x < y$

(b) $\exists y \forall x \; x < y$

(c) $\forall x \forall z \exists y \; x + y > z$

(d) $\exists x \forall y \exists z \; x + y = z$

Exercise 1.2

We play a game with two players, Alice and Eve. The game consists of one round and the rules are:

- Alice picks a natural number x.
- Eve picks a natural number y.

Eve wins the game if, and only if, $x < y$.

(a) Does Eve have a winning strategy?

(b) Does Alice have a winning strategy?

(c) Write two quantified formulas in first-order logic over the natural numbers that express:

- Eve has a winning strategy.
- Alice has a winning strategy.

Exercise 1.3

We play the two-player-game from Lecture 1. The rules of the game are:

- Alice picks a bit in $\{0, 1\}$,
- Eve responds with a bit in $\{0, 1\}$,

- this goes on forever.

A play is an infinite sequence $(a_1, e_1)(a_2, e_2)\ldots$ over $\{0,1\}^2$. For each of the following winning conditions for Eve, decide whether Eve has a winning strategy. If yes, give a strategy and decide whether it is memoryless and/or finite-memory. If no, explain why not.

(a) There exists $n \geq 1$ such that $e_i = 1$ for all $i \geq n$.

(b) For all $n \geq 1$ there exists $i \geq n$ such that $e_i = a_i$.

(c) For all $n \geq 1$ there exists $i \geq n$ such that $e_i = a_i$, but infinitely often $e_i \neq a_i$.

(d) For all $i \geq 1$, if i is odd, then $e_i = 0$, and if i is even, then

$$e_i = \begin{cases} 1 & \text{if the number of 1's in } a_1, \ldots, a_i \text{ is even} \\ 0 & \text{otherwise} \end{cases}.$$

(e) For all $i \geq 1$, $e_i = 1$ if the number of 1's in $a_1, \ldots, a_i$ is equal to the number of 0's in $a_1, \ldots, a_i$; otherwise $e_i = 0$.

(f) If Alice plays infinitely many 1's, then Eve must play only finitely many 1's. If Alice plays only finitely many 1's, then Eve plays infinitely many 1's.