Universität Leipzig
Institut für Informatik
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Exercises

Games in Automata and Logic

Exercise Sheet 3 (for 2.11.2026)

Exercise 3.1

Prove or disprove the truth of the following FO formulas.

- $\exists g \forall v\, \varphi(g, v) \;\rightarrow\; \forall v \exists g\, \varphi(g, v)$
- $\forall v \exists g\, \varphi(g, v) \;\rightarrow\; \exists g \forall v\, \varphi(g, v)$

Exercise 3.2

Let $A = (V, V_{\mathrm{A}}, V_{\mathrm{E}}, E)$ and let $T \subseteq V$. Consider the algorithm for computing the attractor $\mathrm{Attr}_{\mathrm{E}}(T)$, presented in Lecture 3:

$$\mathrm{Attr}_{\mathrm{E}}(T) := \bigcup_{i \geq 0} \mathrm{Attr}^i_{\mathrm{E}}(T),$$

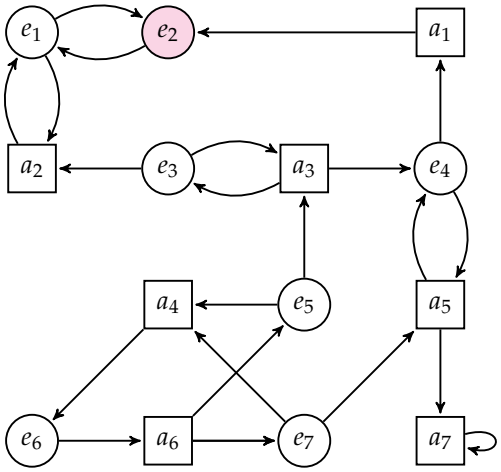
where

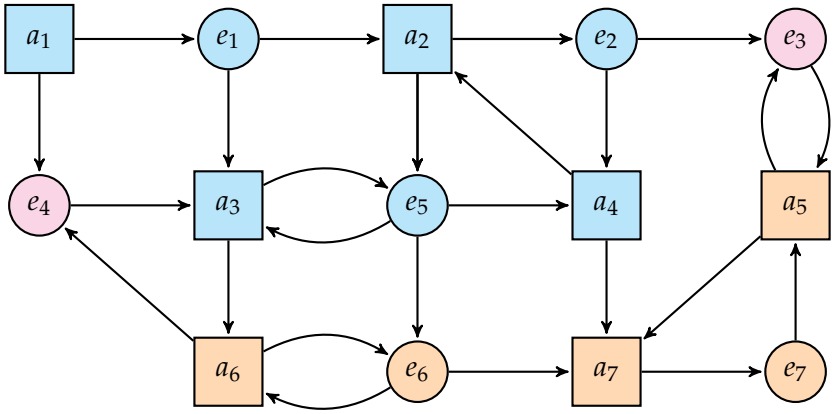
$$\begin{aligned} \mathrm{Attr}^0_{\mathrm{E}}(T) &:= T \\ \mathrm{Attr}^{i+1}_{\mathrm{E}}(T) &:= \mathrm{Attr}^i_{\mathrm{E}}(T) \cup \mathrm{Force}_{\mathrm{E}}(\mathrm{Attr}^i_{\mathrm{E}}(T)) \end{aligned}$$

and

$$\begin{aligned} \mathrm{Force}_{\mathrm{E}}(U) \;:=\; & \{v \in V_{\mathrm{E}} \mid (v,u) \in E \text{ for some } u \in U\} \cup \\ & \{v \in V_{\mathrm{A}} \mid (v,u) \in E \text{ implies } u \in U\} \end{aligned}$$

for every $U \subseteq V$. A naïve implementation of the attractor algorithm repeatedly scans all vertices until a fixed point is reached, yielding a quadratic running time. Show how to implement the algorithm in linear time. For explanation purposes, you may apply your linear time algorithm to the arena depicted on page 2, with $T = \{e_2\}$.



Exercise 3.3

Consider the following arena. Eve wins if she can force a play to staying in the set of blue vertices, that is, $\{a_1, e_1, a_2, e_2, a_3, e_5, a_4\}$. From which vertices does Eve have a winning strategy?