Ehrenfeucht-Fraïssé Games for MTL and TPTL over Non-Monotonic Data Words

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MTL and TPTL

- Metric Temporal Logic (MTL) [Koymans, 1990]
- Timed Propositional Temporal Logic (TPTL) [Alur&Henzinger, 1993]
- extensions of Linear Temporal Logic (LTL)
- express properties about concrete data values in data words

- e.g. $(P_0, 0.5)(P_1, 1.7)(P_2, 3.5)(P_3, 4.7)(P_4, 7.9)\dots$

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- What is the relative expressiveness of these two logics?

Relative Expressiveness of TPTL and MTL

- When evaluated over monotonic real-timed words $(P_0, d_0)(P_1, d_1)(P_2, d_2)\ldots$, where $d_0 < d_1 < d_2 < \cdots \in \mathbb{R}_{\geq 0}$:

  TPTL is strictly more expressive than MTL [Bouyer et al, 2010]

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- When evaluated over **non-monotonic** discrete data words $(P_0, d_0)(P_1, d_1)(P_2, d_2)\dots$, where $d_0, d_1, d_2, \dots \in \mathbb{N}$:

  What is the relative expressiveness of TPTL and MTL?

Metric Temporal Logic

Syntax:

P... a finite set of propositional variables.

$\varphi ::= p \mid \neg\varphi \mid \varphi \vee \varphi \mid \varphi \mathsf{U}_I \varphi \quad (p \in P,\ I \subseteq \mathbb{Z})$

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Semantics:

Let $w = (P_0, d_0)(P_1, d_1)(P_2, d_2)\ldots,\ i \geq 0.$

$(w, i) \models \varphi_1 \mathsf{U}_I \varphi_2 \Leftrightarrow \exists j > i.(w, j) \models \varphi_2, d_j - d_i \in I, \forall i < k < j.(w, k) \models \varphi_1$

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Syntactical abbreviations:

$\texttt{true} = p \vee \neg p$
$\mathsf{F}_{=0}\varphi = \texttt{true}\mathsf{U}_{[0,0]}\varphi$

Timed Propositional Temporal Logic

Syntax:

P... a finite set of propositional variables, X...a set of register variables

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$(p \in P$, $x \in X$, $c \in \mathbb{Z}$, $\sim \in \{<, \leq, =, \geq, >\})$

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$$(w, i, \nu) \models x.\varphi \quad \Leftrightarrow (w, i, \nu[x \mapsto d_i]) \models \varphi$$
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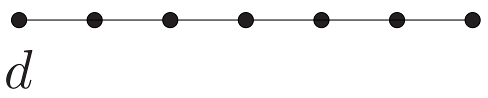

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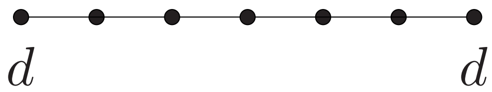

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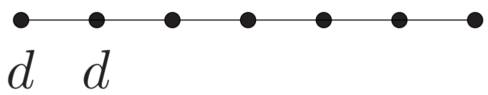

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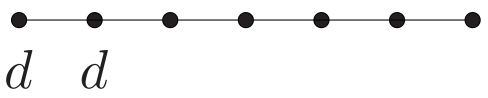

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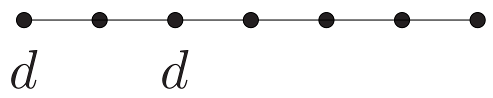

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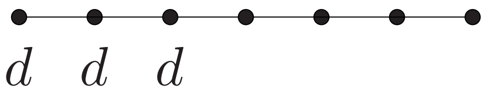

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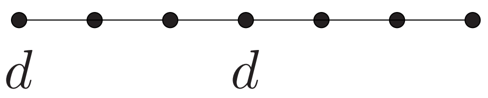

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- Played in k rounds on two data words w_1 and w_2

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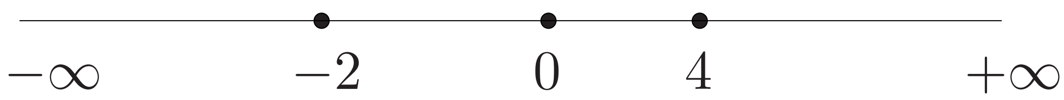

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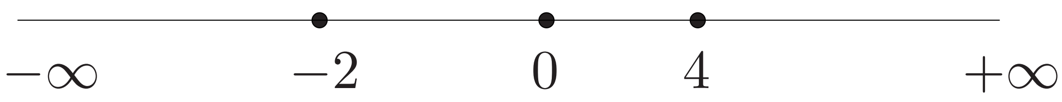

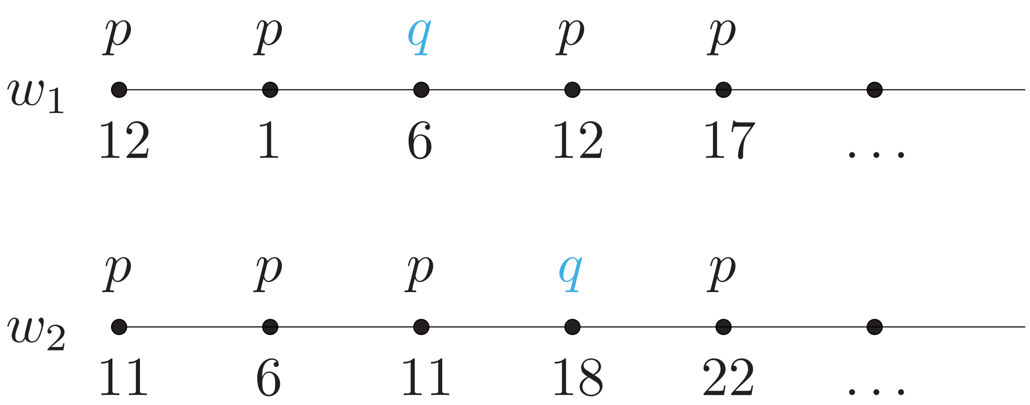

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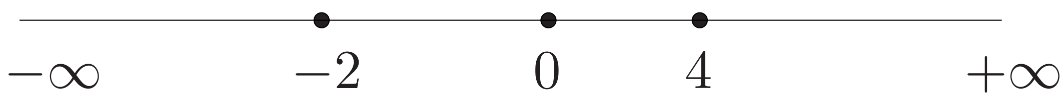

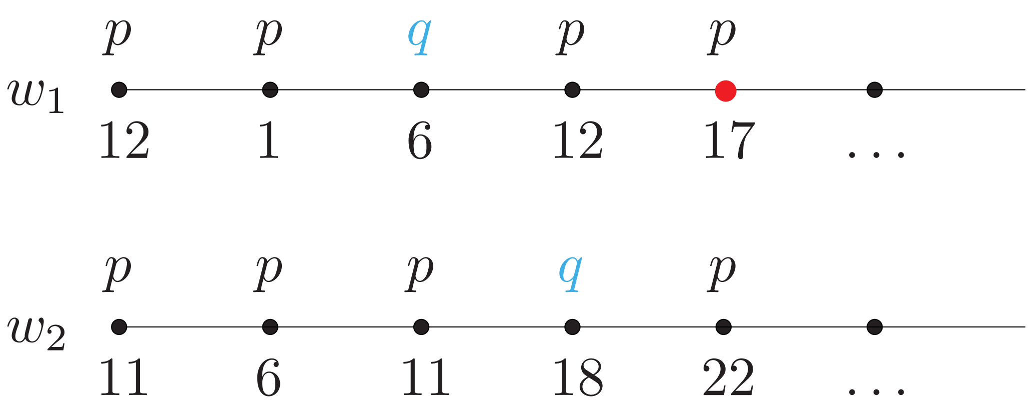

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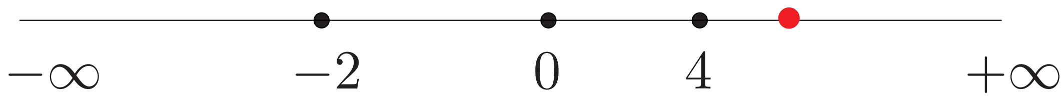

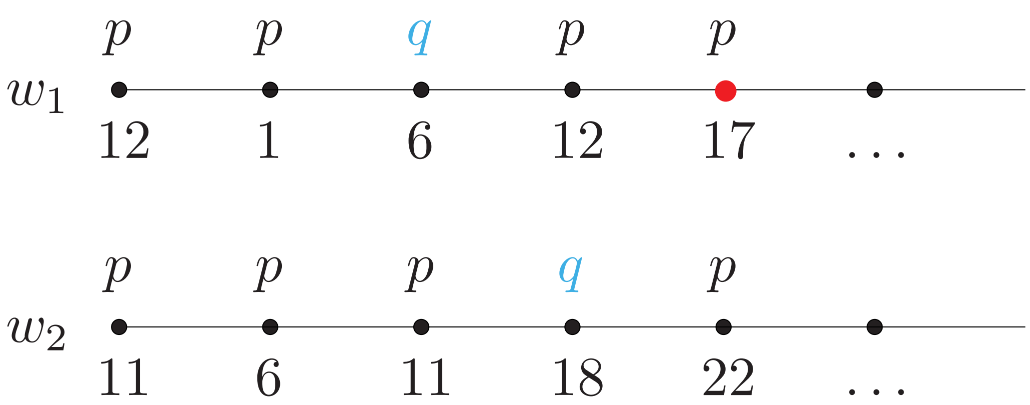

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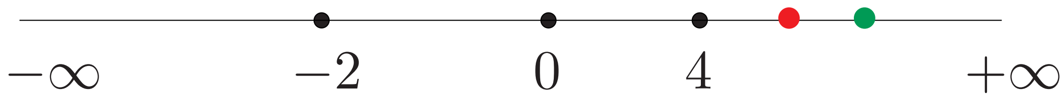

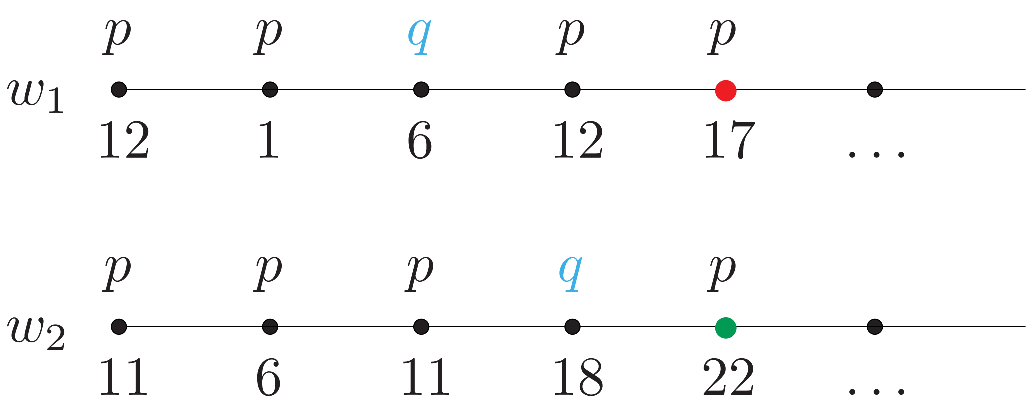

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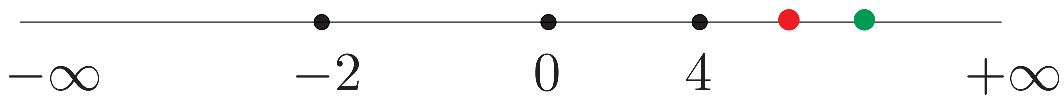

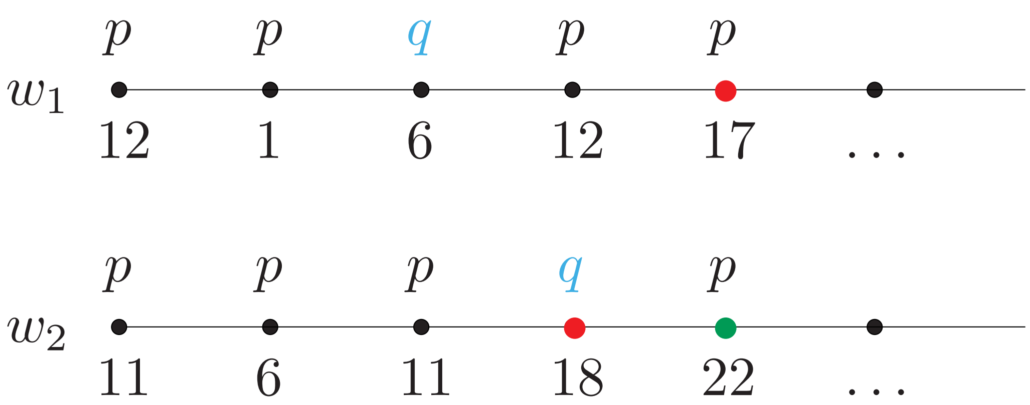

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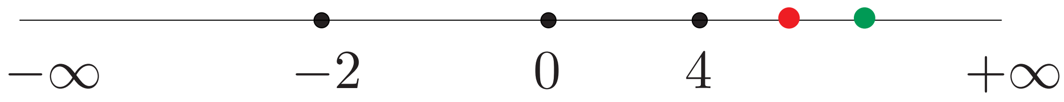

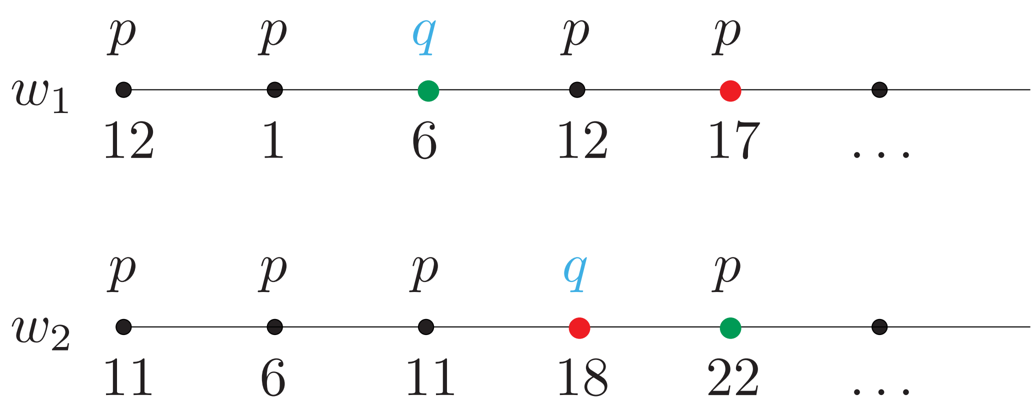

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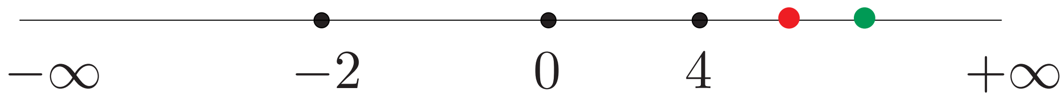

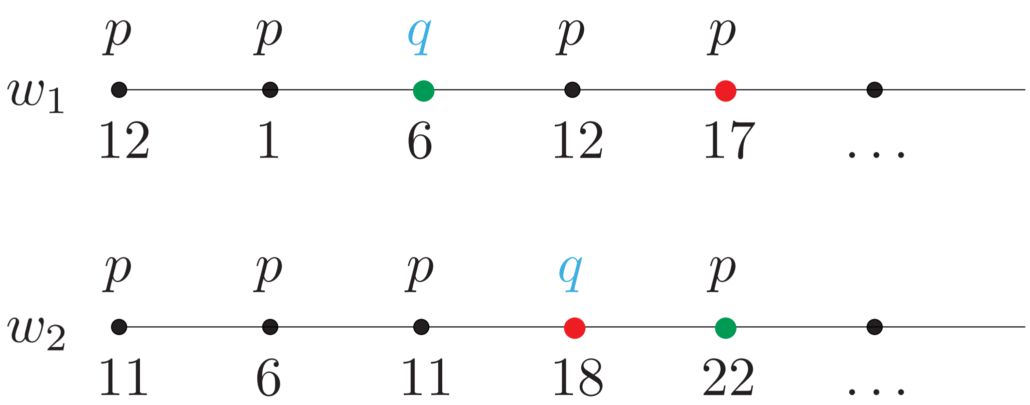

$\mathsf{GAME}_k^I(w_1, w_2)$

Main Theorems

Theorem 1. The following statements are equivalent:

1. Duplicator has a winning strategy in $\mathsf{GAME}_k^I(w_1, w_2)$.
2. w_1 and w_2 satisfy the same formulas in MTL_k^I.

MTL_k^I:
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Theorem 2. For each $\varphi \in \mathsf{TPTL}$, the following statements are equivalent:

1. φ is not definable in MTL.
2. For all I, k there exist data words w_1, w_2 such that
   - $w_1 \models \varphi$,
   - $w_2 \not\models \varphi$, and
   - Duplicator has a winning strategy in $\mathsf{GAME}_k^I(w_1, w_2)$.

TPTL is strictly more expressive than MTL

Proposition. The TPTL formula $x.\mathsf{FFF}(x = 0)$ is not definable in MTL.

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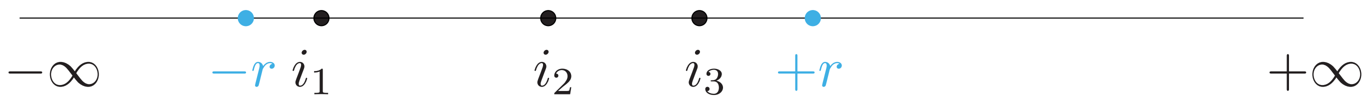

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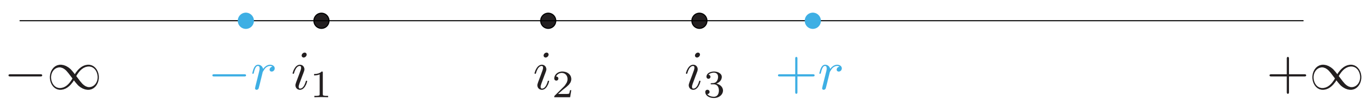

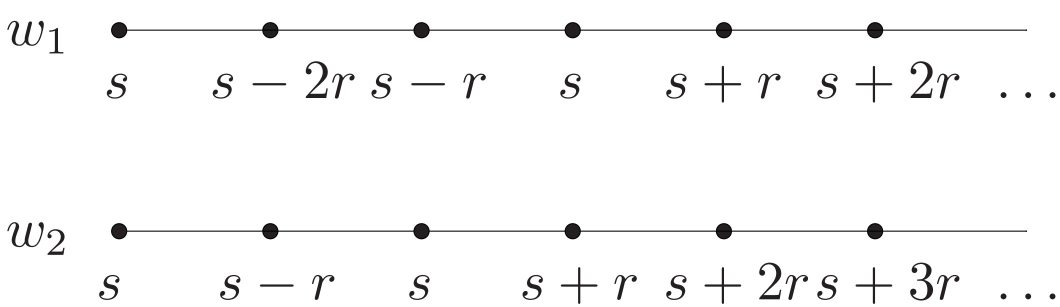

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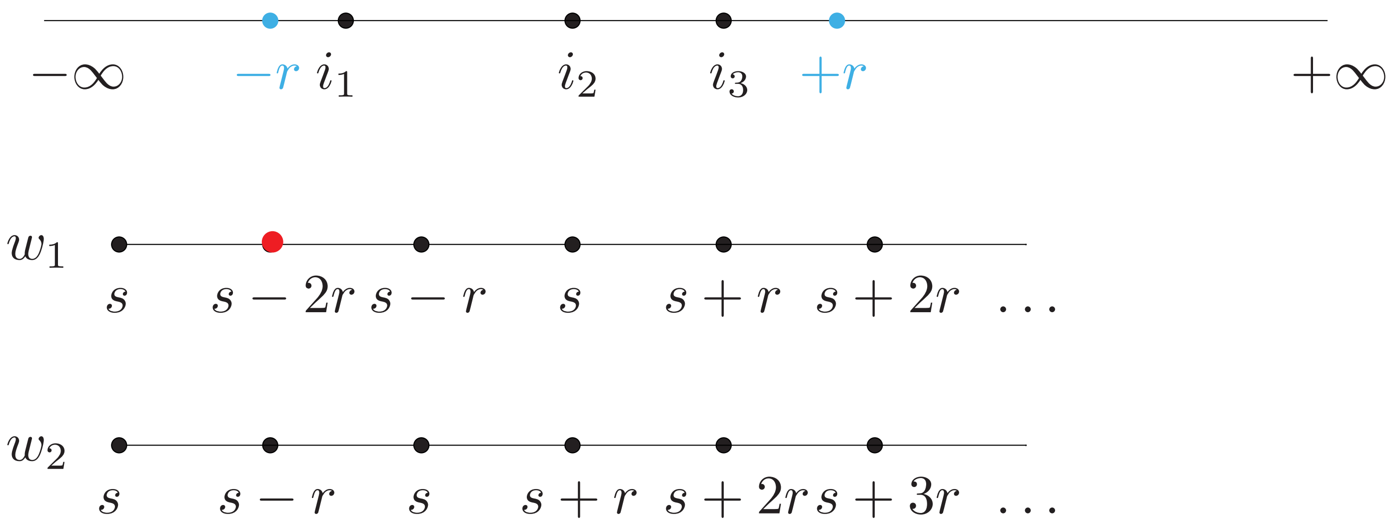

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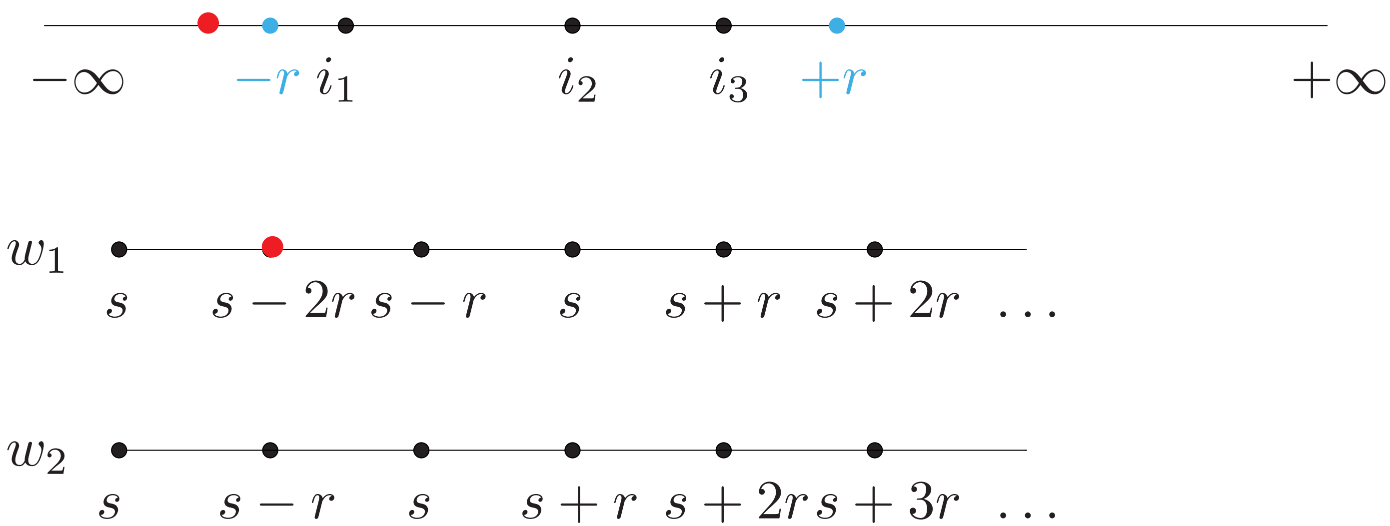

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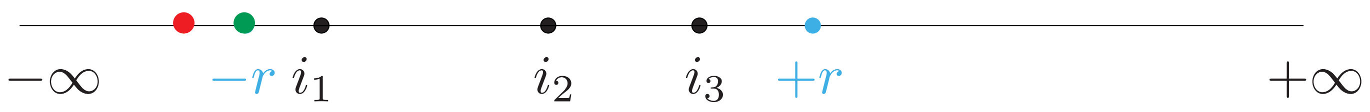
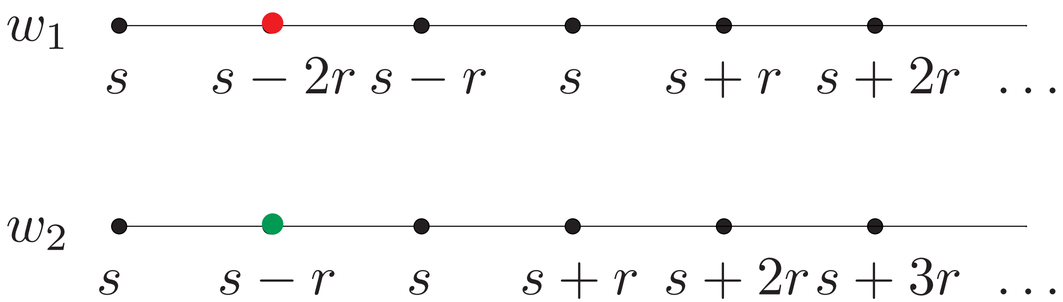
Proposition. The TPTL formula $x.\mathsf{FFF}(x = 0)$ is not definable in MTL.

Proof. For all I, k there exist data words w_1, w_2 such that

- $w_1 \models \varphi$, $w_2 \not\models \varphi$, and
- Duplicator has a winning strategy in $\mathsf{GAME}_k^I(w_1, w_2)$.

Define $r, s \in \mathbb{N}$ such that $i \in (-r, +r)$ for all $i \in I \setminus \{-\infty, +\infty\}$ and $s \geq 2r$.





Main Theorems

Theorem 3. TPTL is strictly more expressive than MTL.

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MTL Membership Problem

Instance: A TPTL formula φ
Question: Can φ be expressed in MTL?

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Theorem 4. The MTL membership problem is undecidable.

Proof. Reduction from the recurrent state problem for two-counter machines, using Ehrenfeucht-Fraïssé Games for MTL.

Conclusion

- New tool to show expressiveness results for MTL.
- We defined Ehrenfeucht-Fraïssé games also for TPTL.
- Used to show results on, e.g. register hierarchy, constraint hierarchy, until hierarchy.
- Can also be used for proving results for monotonic real-timed words.

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Thank you for your attention!