The Complexity of Flat Freeze LTL

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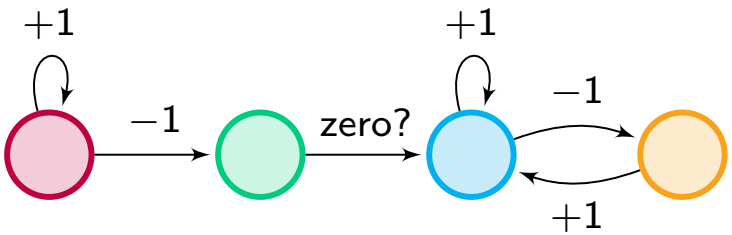
CONCUR 2017

Model Checking One-Counter Machines & Freeze LTL

One-Counter Machines



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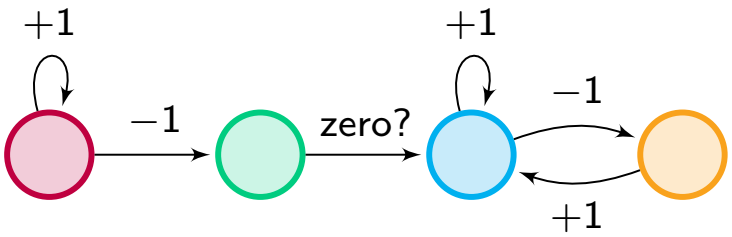

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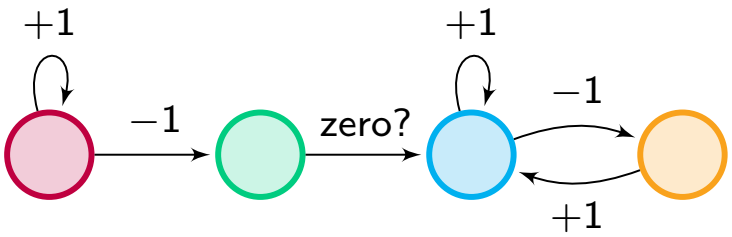

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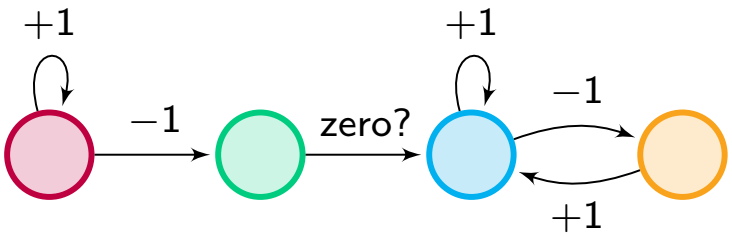

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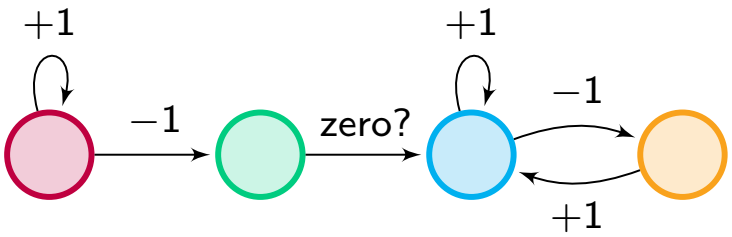

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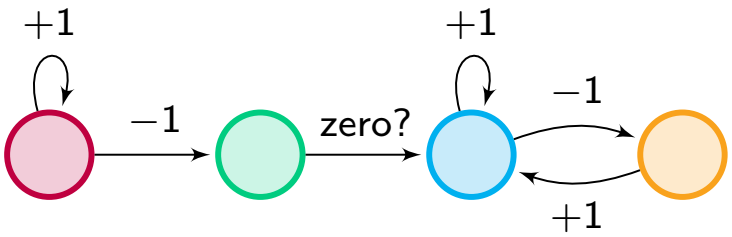

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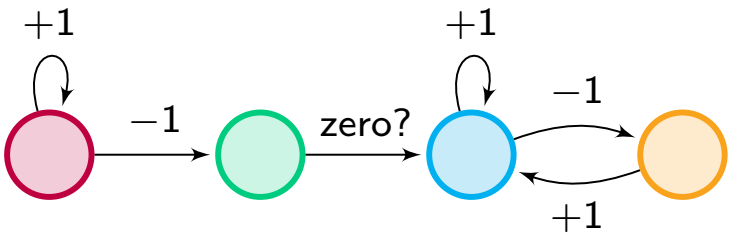

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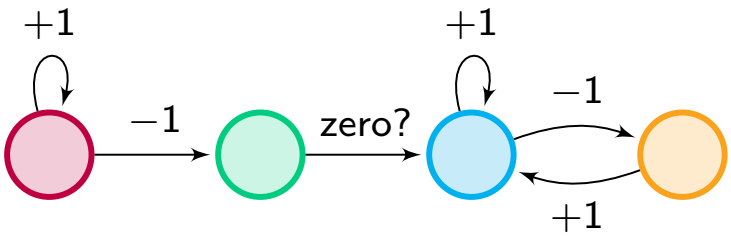

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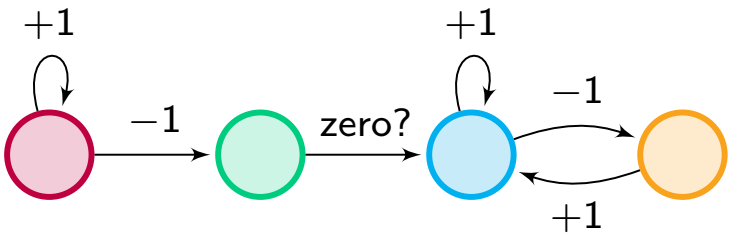

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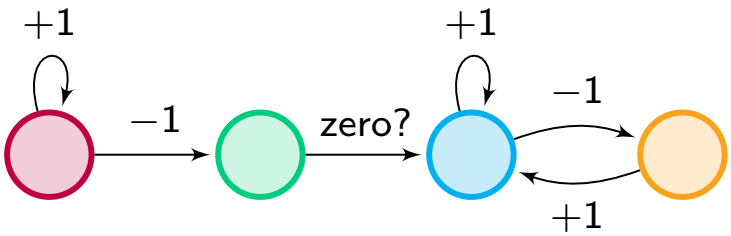

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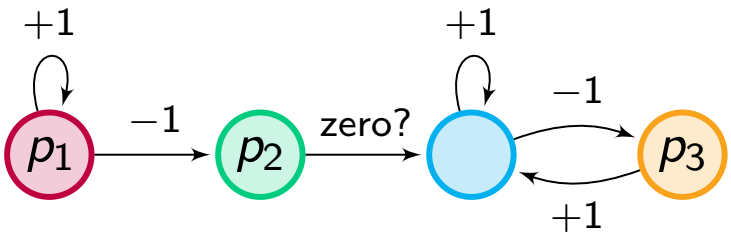

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The set of Freeze LTL formulas is defined by the grammar

$$\varphi ::= p \mid \mathsf{X}\varphi \mid \varphi \mathsf{U} \varphi \mid \neg\varphi \mid \varphi \vee \varphi \mid \downarrow r.\varphi \mid {=}r$$

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$\mathsf{F}\downarrow r.\mathsf{GF}{=}r$ Some counter value occurs infinitely often.

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Theorem (Demri, Lazic, Sangnier 2010)

Model checking one-counter machines against formulas of Freeze LTL is undecidable, even if the formulas only use a single register.

Flat Freeze LTL

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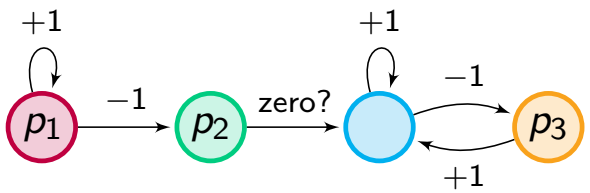

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red. / exp.

Reachability of
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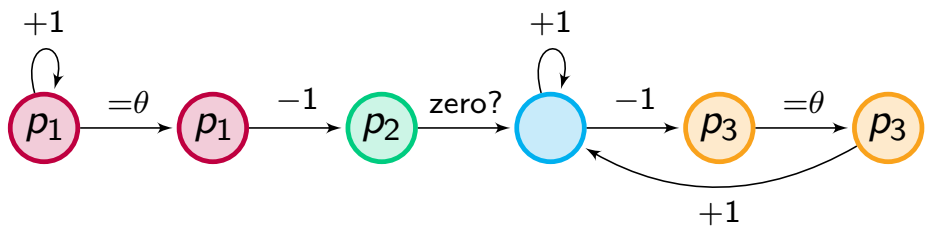

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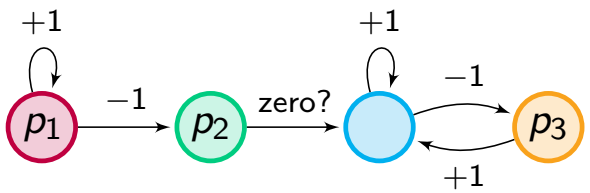

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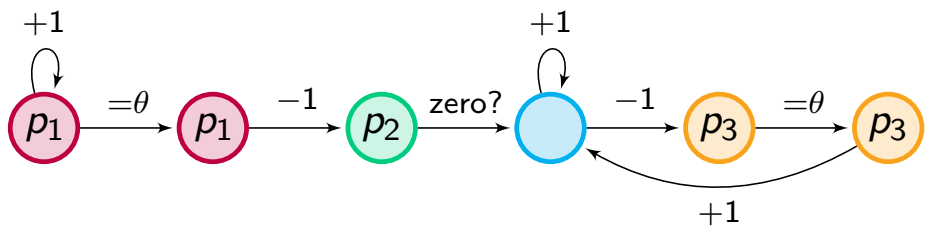

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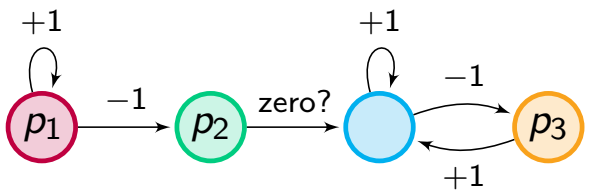

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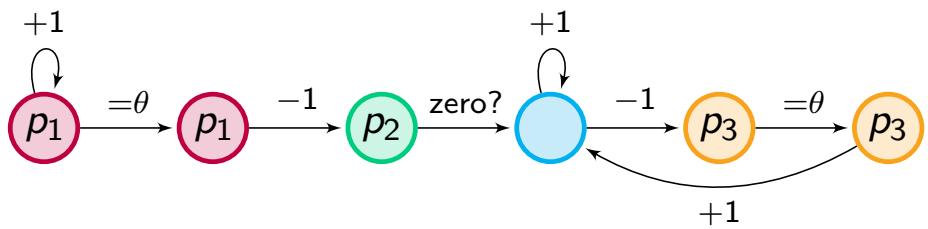

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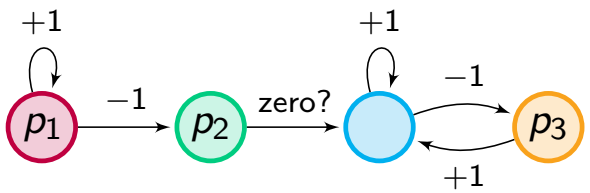

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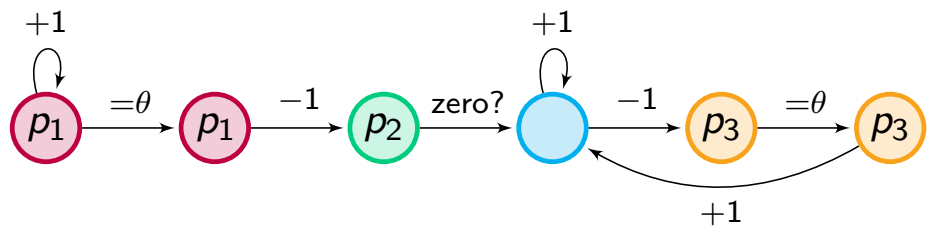

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- trivial 2NEXPTIME upper bound for model checking

Our Contribution

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Model checking one-counter machines against formulas of Flat Freeze LTL is NEXPTIME-complete.

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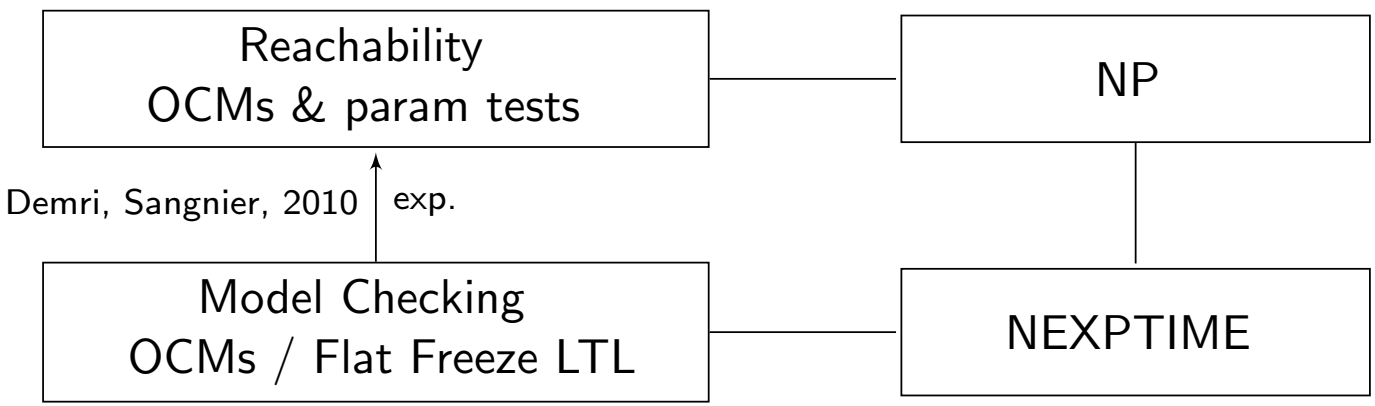

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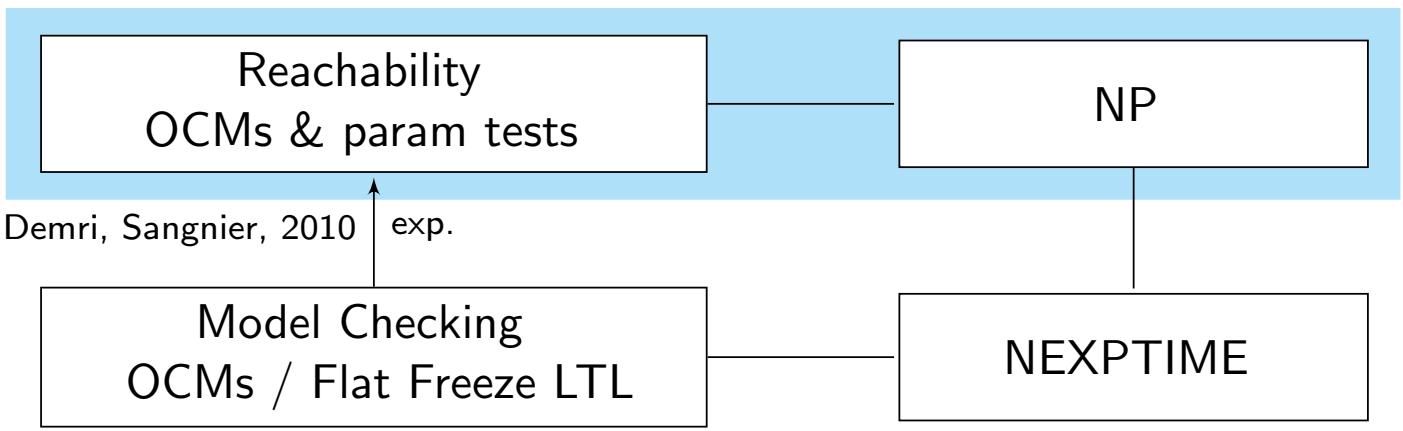

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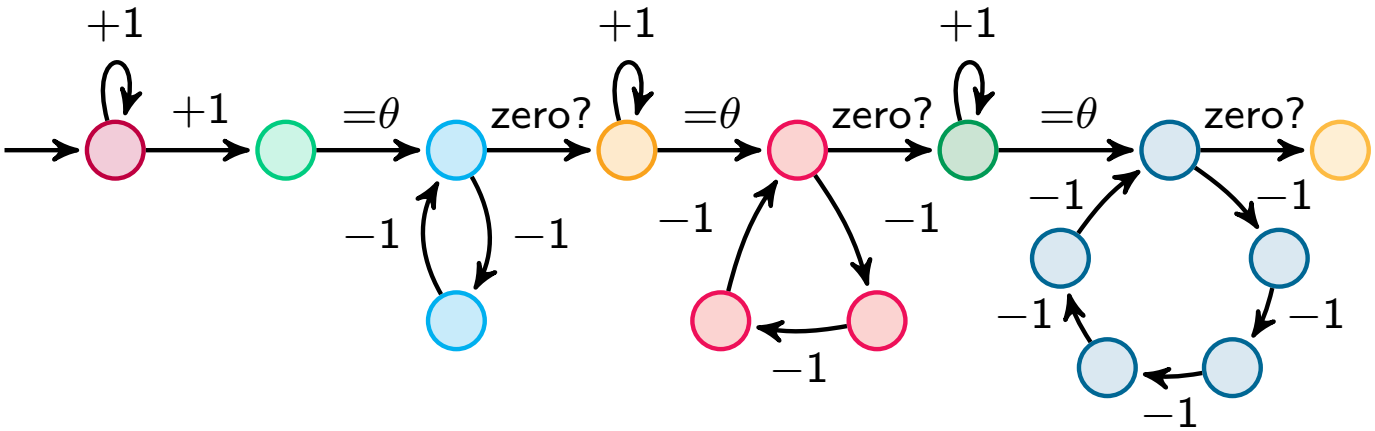

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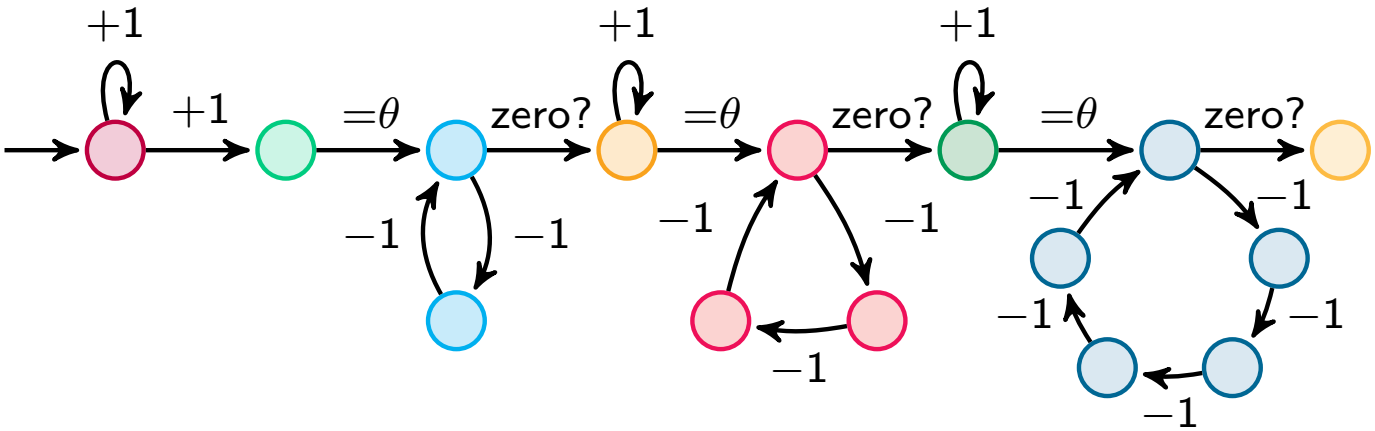

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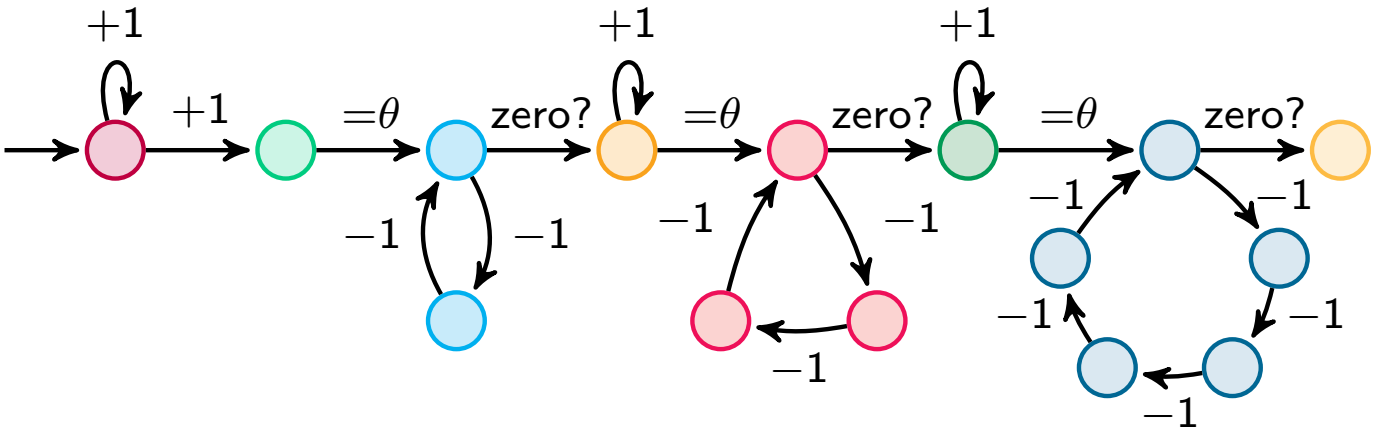

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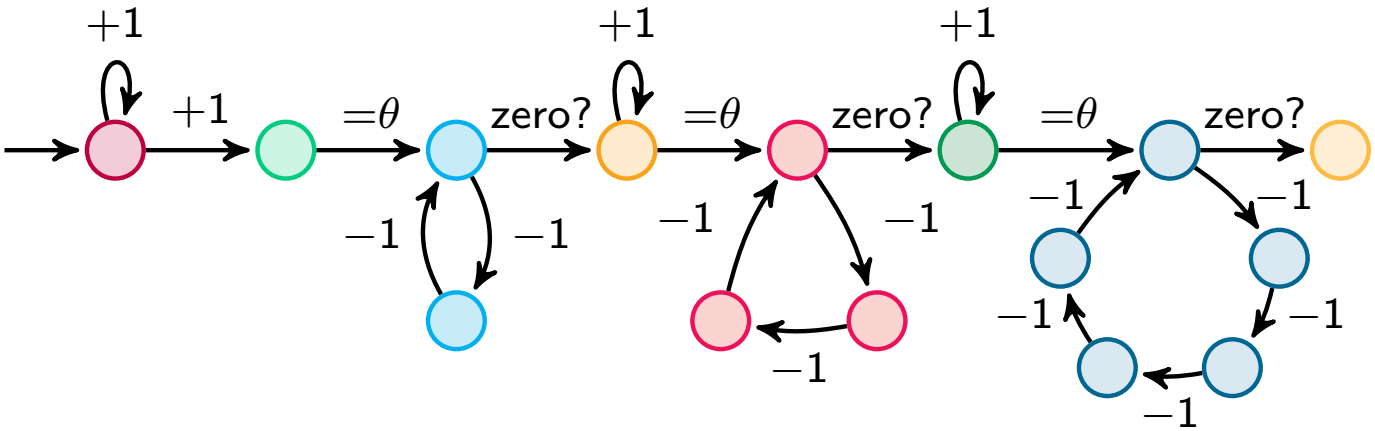

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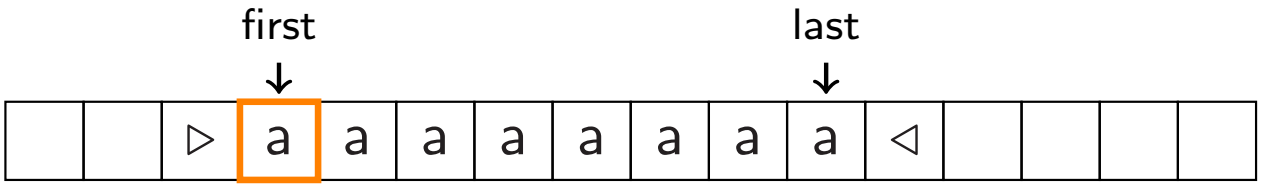
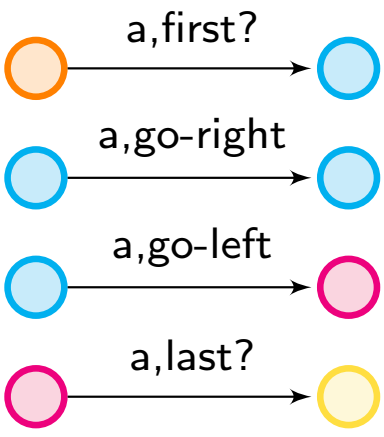
Reachability Problem

Given an OCM with parameterized tests and a target control state q, does there exist a parameter valuation γ and a γ-computation ending in q?

NP lower bound



Nonemptiness
Two-Way Automata



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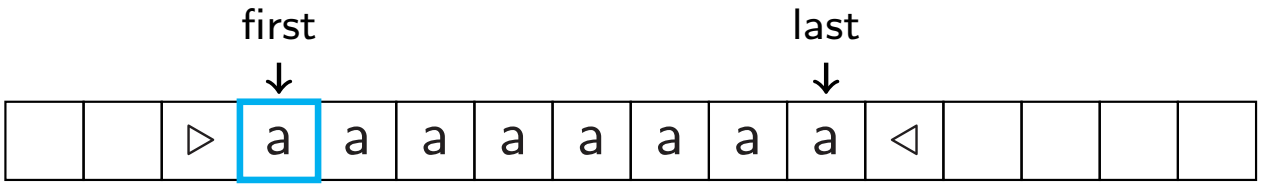

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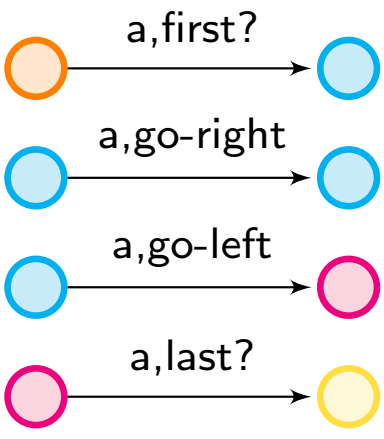

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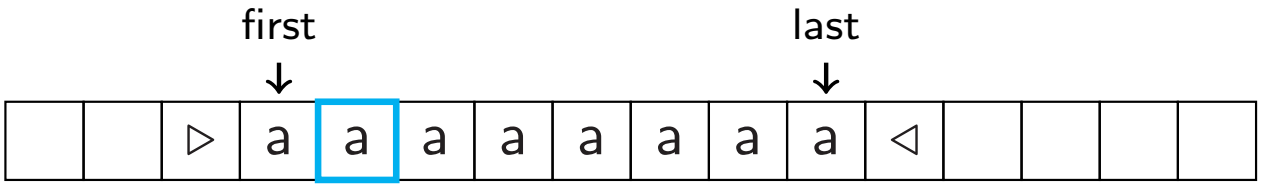

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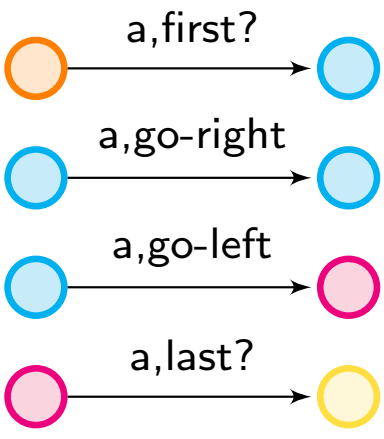
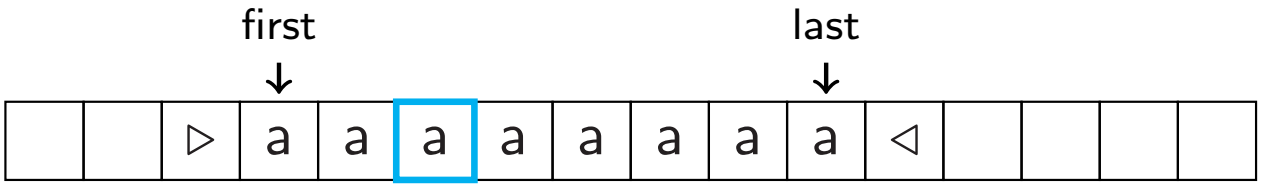
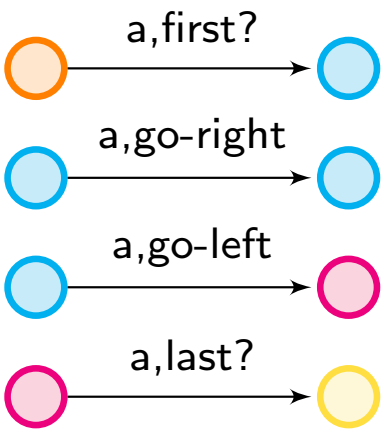


NP lower bound



Nonemptiness
Two-Way Automata



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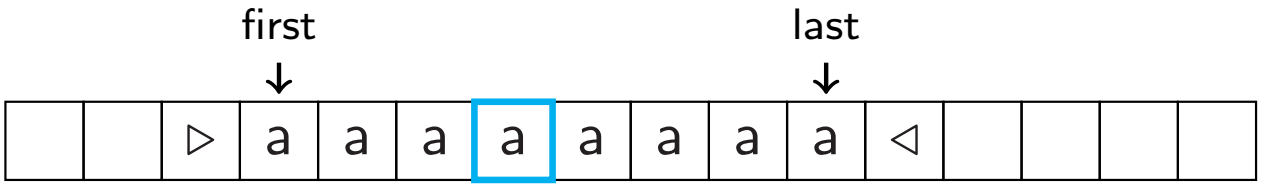

Nonemptiness
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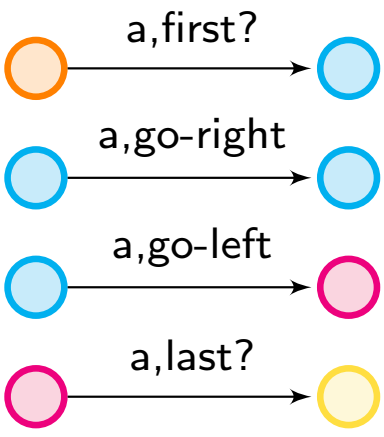

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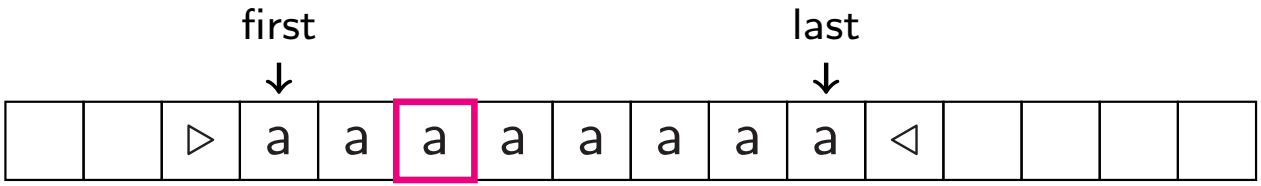

Nonemptiness
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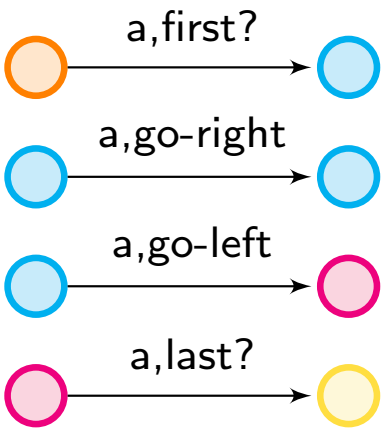

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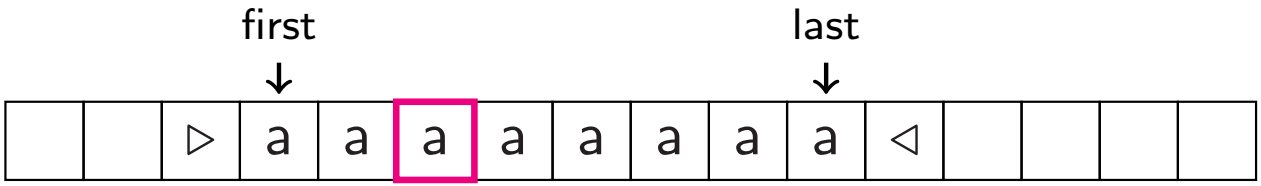

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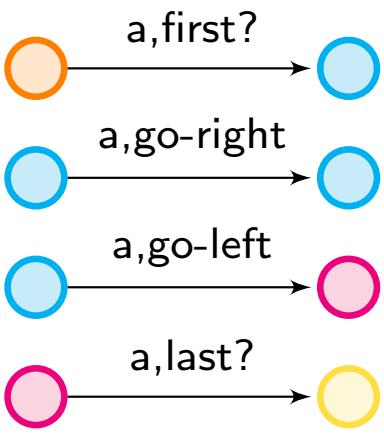

Theorem (Galil 1976)

The nonemptiness problem for two-way automata over a singleton alphabet and with two endmarkers is NP-complete.

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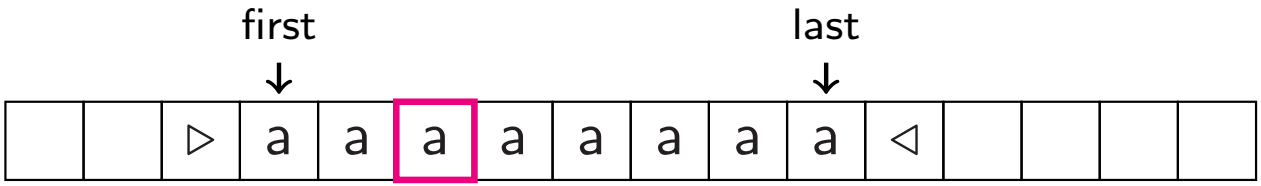
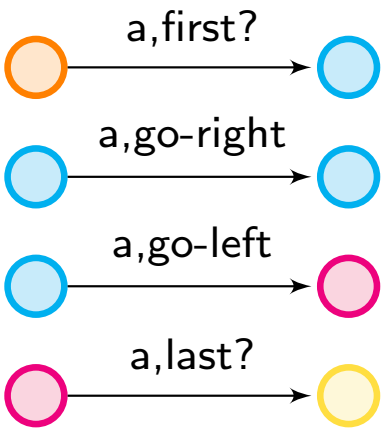


Nonemptiness
Two-Way Automata



red.
⟹
poly.

Reachability
OCMs with param tests

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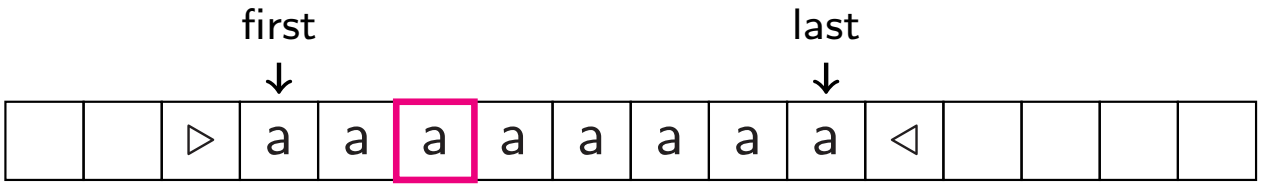

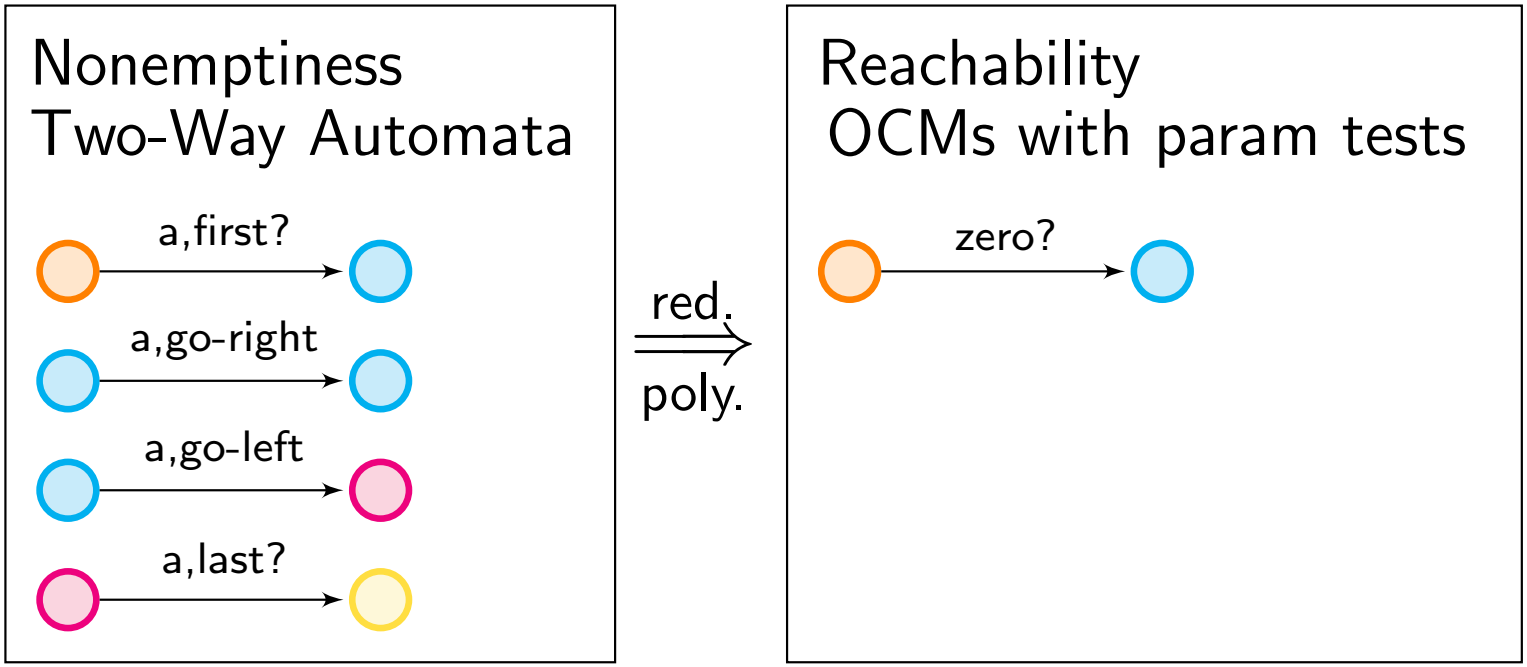

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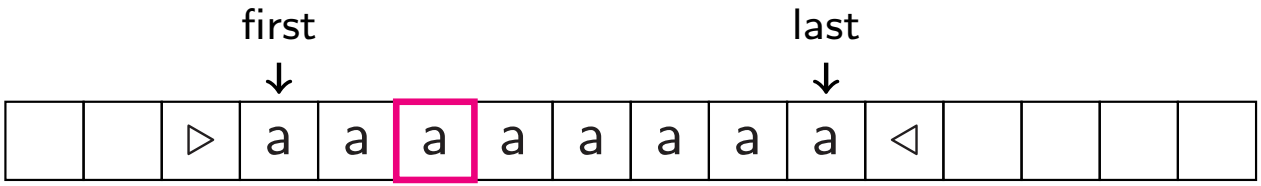

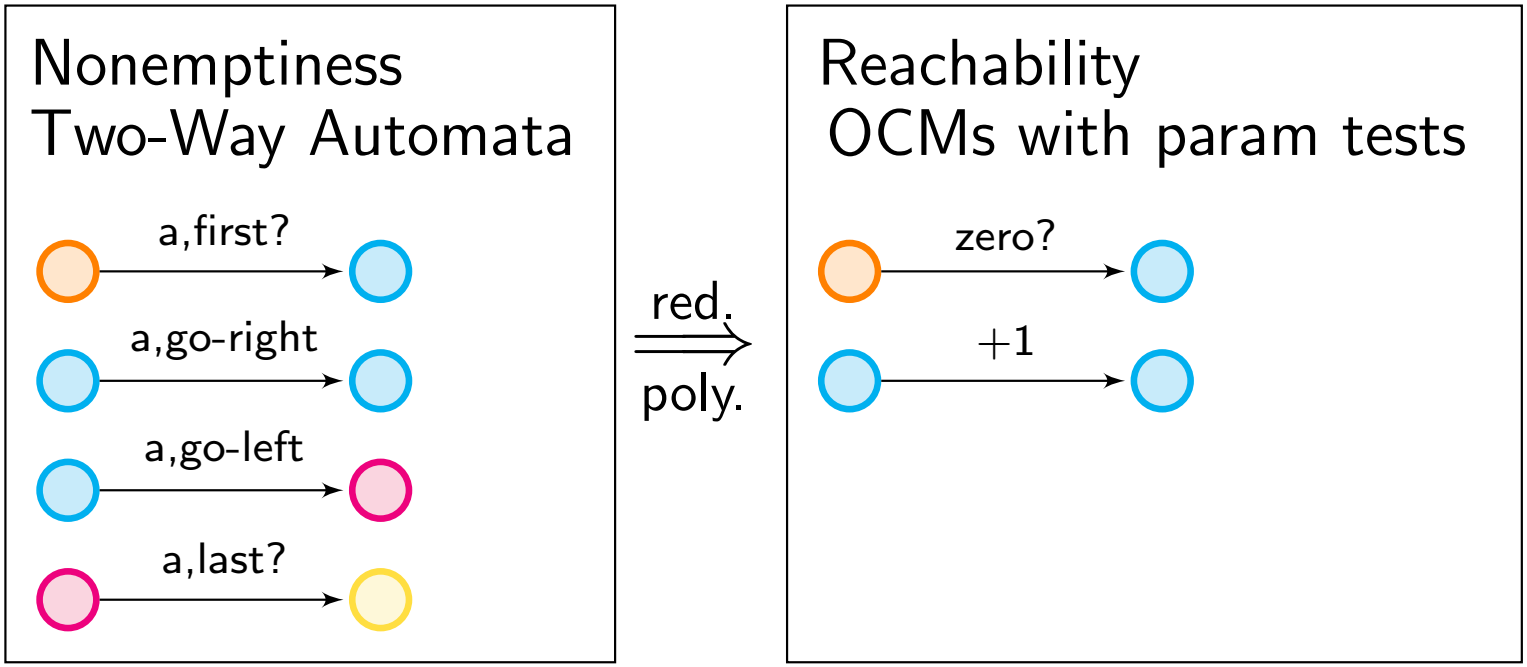

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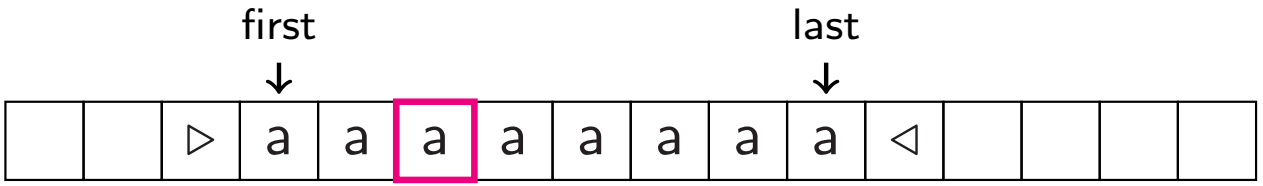

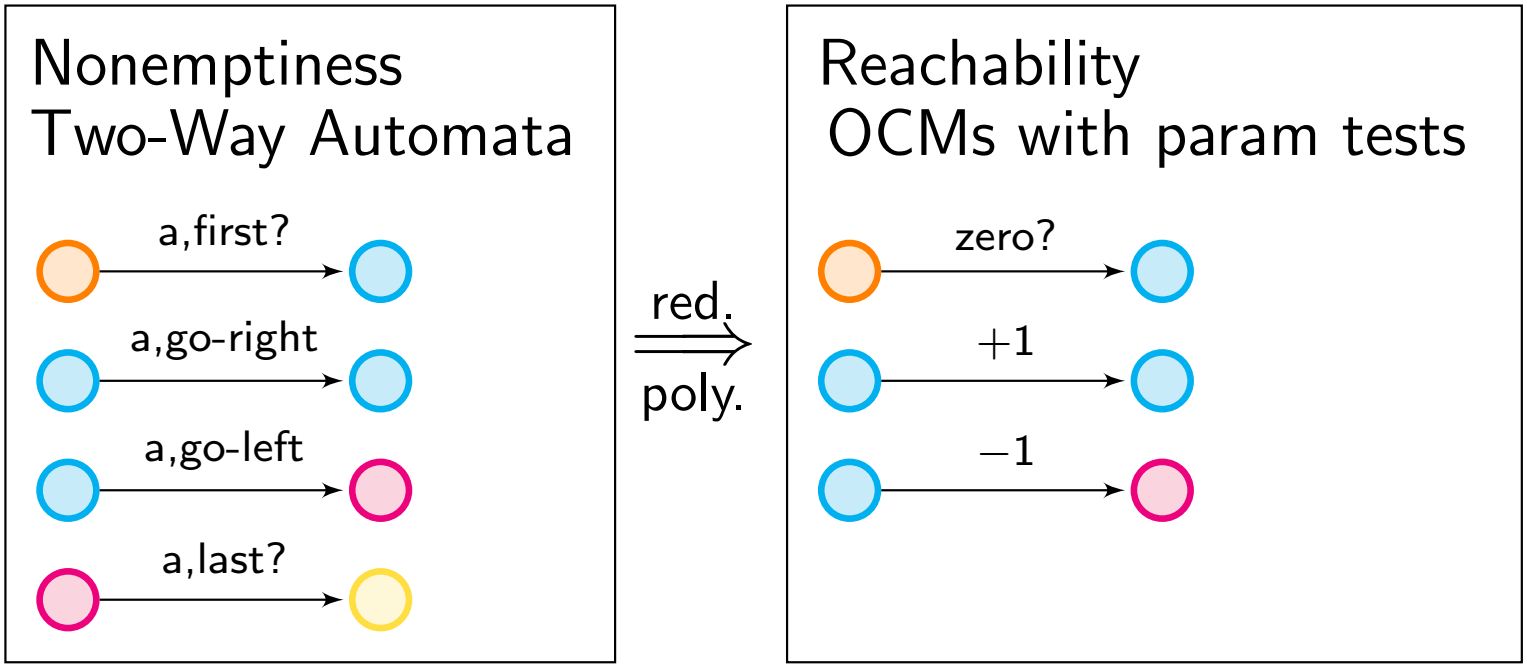

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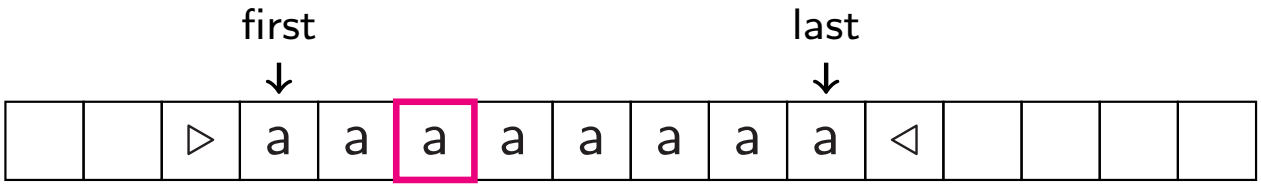

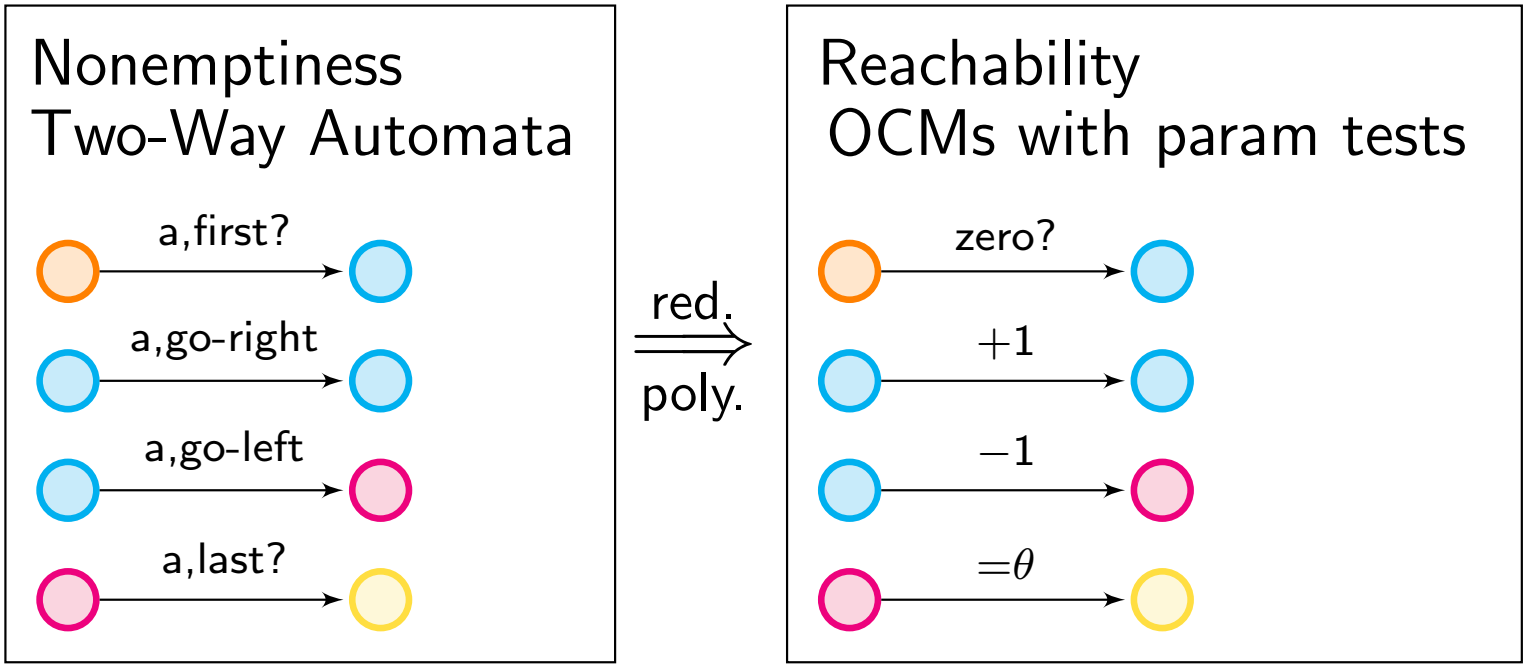

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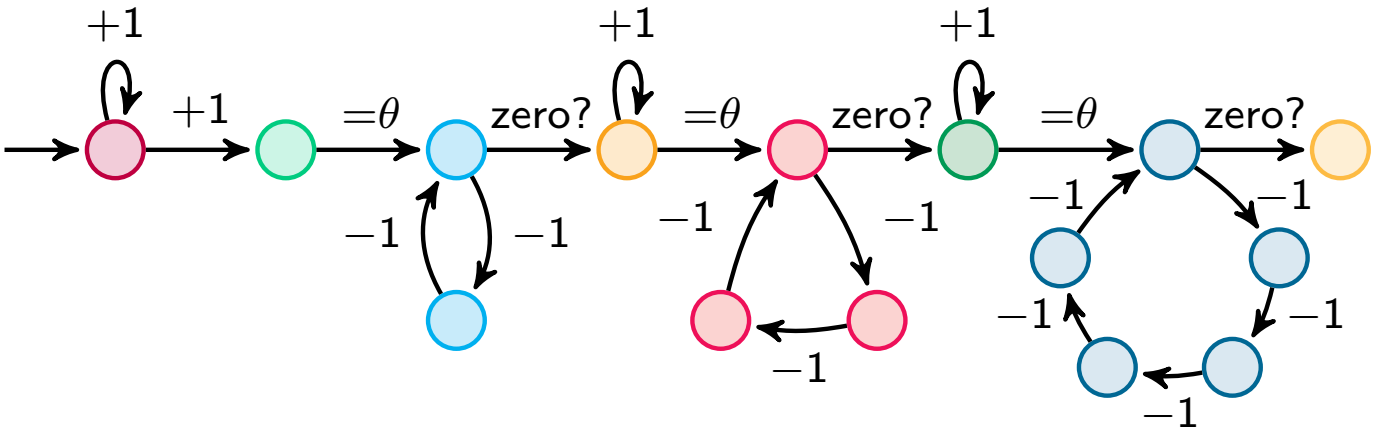
OCMs with Parameterized Tests



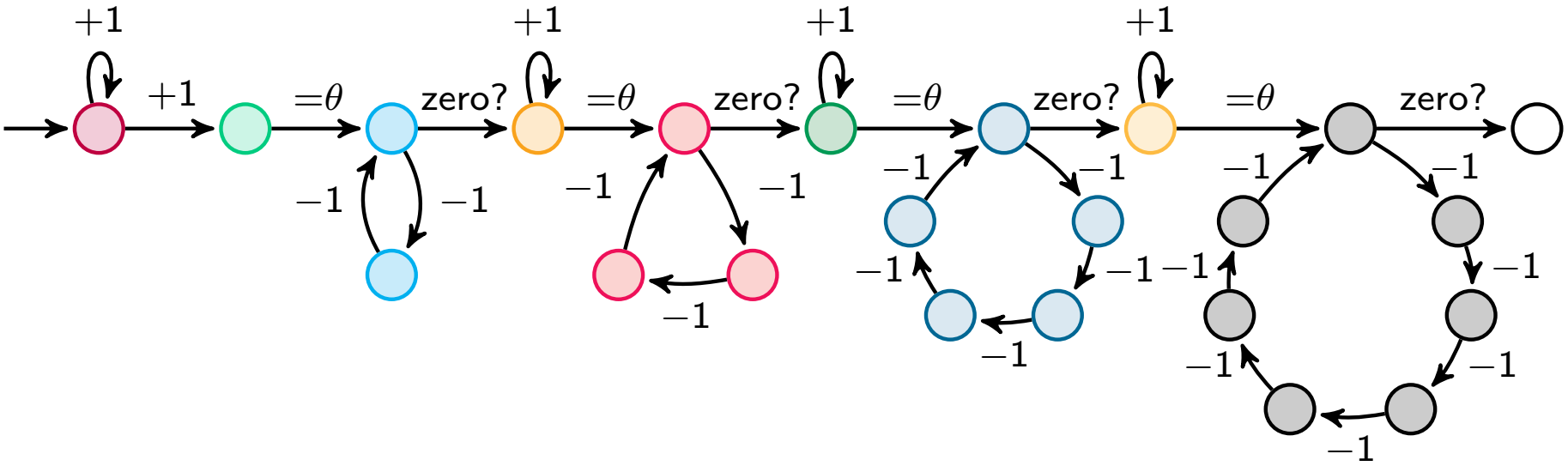
Parameter valuation $\gamma : \Theta \to \mathbf{N}$, e.g. $\gamma(\theta) = 30$

$0 \to_\gamma^{29} 29 \to_\gamma 30 \to_\gamma 30(\to_\gamma\to_\gamma)^{15}0 \to_\gamma 0 \to_\gamma^{30} 30 \to_\gamma 30(\to_\gamma\to_\gamma\to_\gamma)^{10}0\ldots$

Reachability Problem

Given an OCM with parameterized tests and a target control state q, does there exist a parameter valuation γ and a γ-computation ending in q (visiting q infinitely often)?

OCMs with Parameterized Tests



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NP upper bound (1)

Let $\gamma = \{\theta_1 \mapsto 5, \theta_2 \mapsto 2, \theta_3 \mapsto 5, \theta_4 \mapsto 8\}$ be a parameter valuation.

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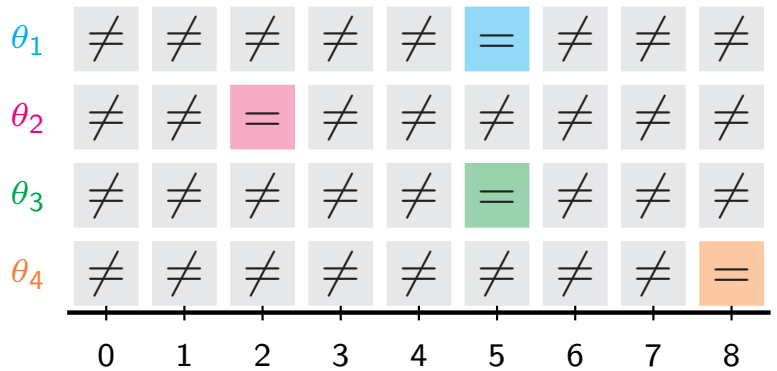
The following "parameter word" w_γ encodes γ.

$w_\gamma =$



$\in \Sigma^*$, where $\Sigma = \{=, \neq\}^4$

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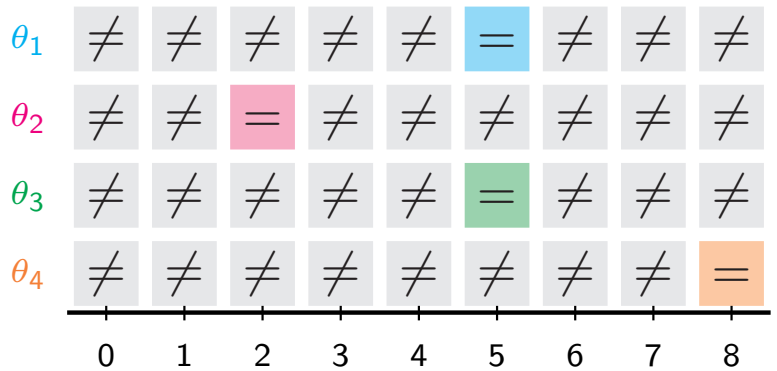
$w_\gamma =$ 

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Reachability of
OCMs with param tests

$\xRightarrow[\text{poly.}]{\text{red.}}$

Nonemptiness
Two-Way Automata

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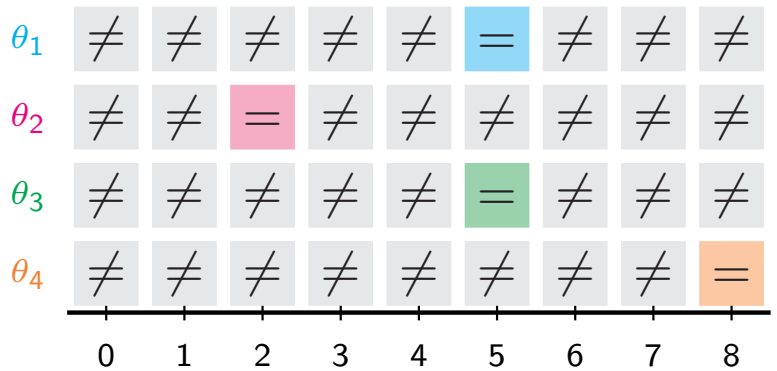
$w_\gamma =$ 

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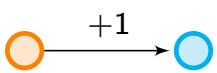
Reachability of
OCMs with param tests



red.
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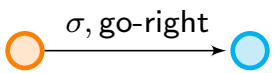
Nonemptiness
Two-Way Automata



$\sigma \in \Sigma$

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Reachability of
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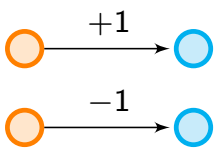

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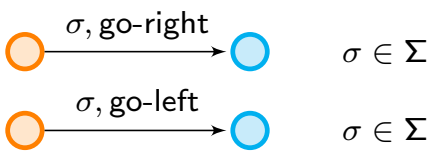
Nonemptiness
Two-Way Automata



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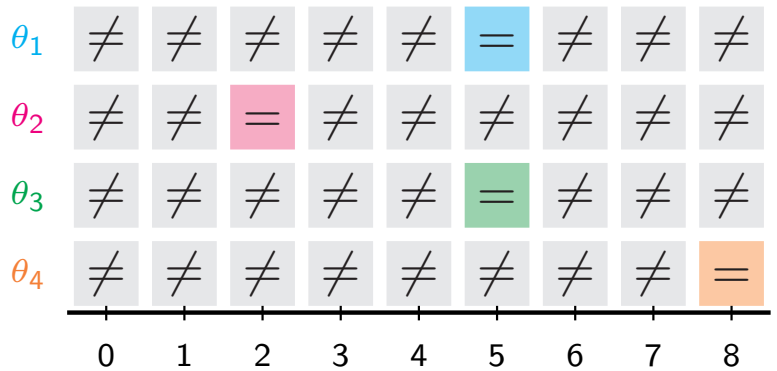
$w_\gamma =$ 

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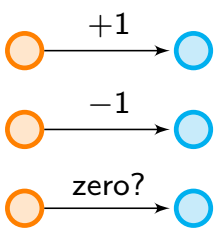
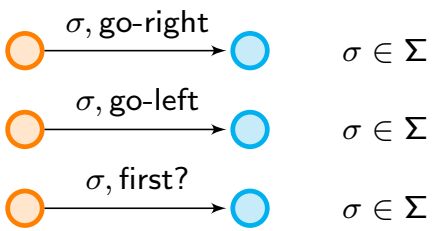
Reachability of OCMs with param tests



red. / poly.

Nonemptiness Two-Way Automata



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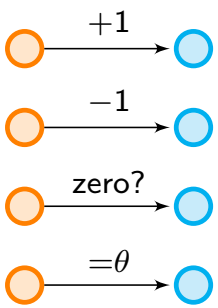
Reachability of OCMs with param tests



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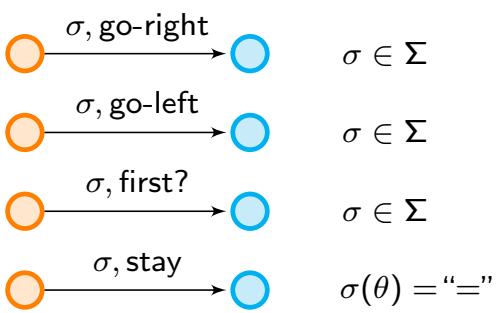
Nonemptiness Two-Way Automata



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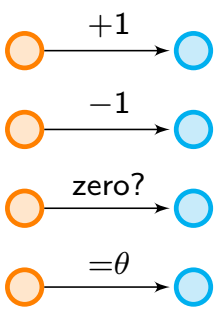
Reachability of OCMs with param tests



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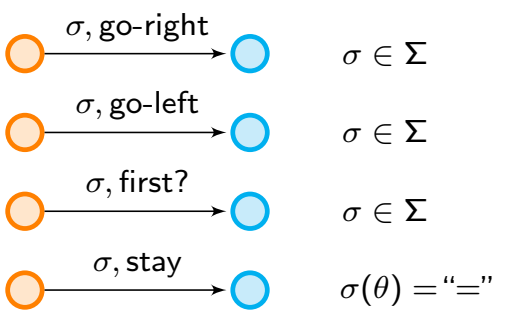
Nonemptiness Two-Way Automata



red. exp.

Nonemptiness Finite Automata

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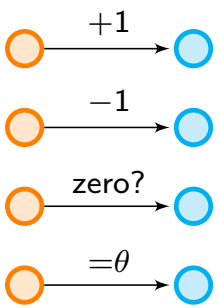

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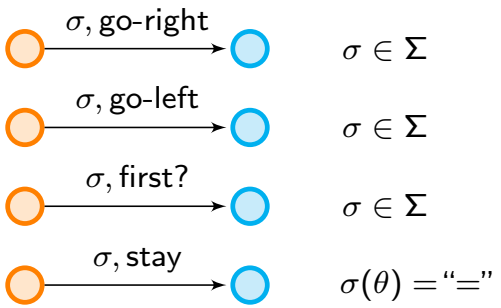
Nonemptiness Two-Way Automata



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Nonemptiness Finite Automata

length of smallest witness for nonemptiness is exponential in the size of the OCM

NP upper bound (1)

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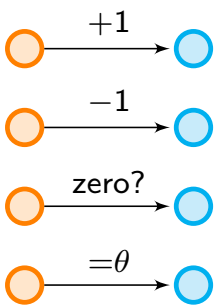
Reachability of
OCMs with param tests



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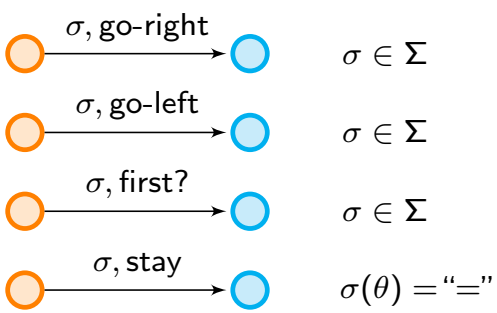
Nonemptiness
Two-Way Automata



red.
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Nonemptiness
Finite Automata

length of smallest
witness for nonemptiness
is exponential in
the size of the OCM

↷ parameter valuation
is bounded exponentially

NP upper bound (2)

Given an OCM with parameterized tests $\mathcal{A}$ and a target control state q,

- if q is reachable, then it is reachable under some parameter valuation γ such that $\gamma(\theta)$ is bounded exponentially in the size of $\mathcal{A}$, for every θ

NP upper bound (2)

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- if q is reachable, then it is reachable under some parameter valuation γ such that $\gamma(\theta)$ is bounded exponentially in the size of $\mathcal{A}$, for every θ
- $\curvearrowright$ if q is reachable, then it is reachable even if the counter values along the computation are bounded exponentially in the size of $\mathcal{A}$

NP upper bound (2)

Given an OCM with parameterized tests $\mathcal{A}$ and a target control state q,

- if q is reachable, then it is reachable under some parameter valuation γ such that $\gamma(\theta)$ is bounded exponentially in the size of $\mathcal{A}$, for every θ
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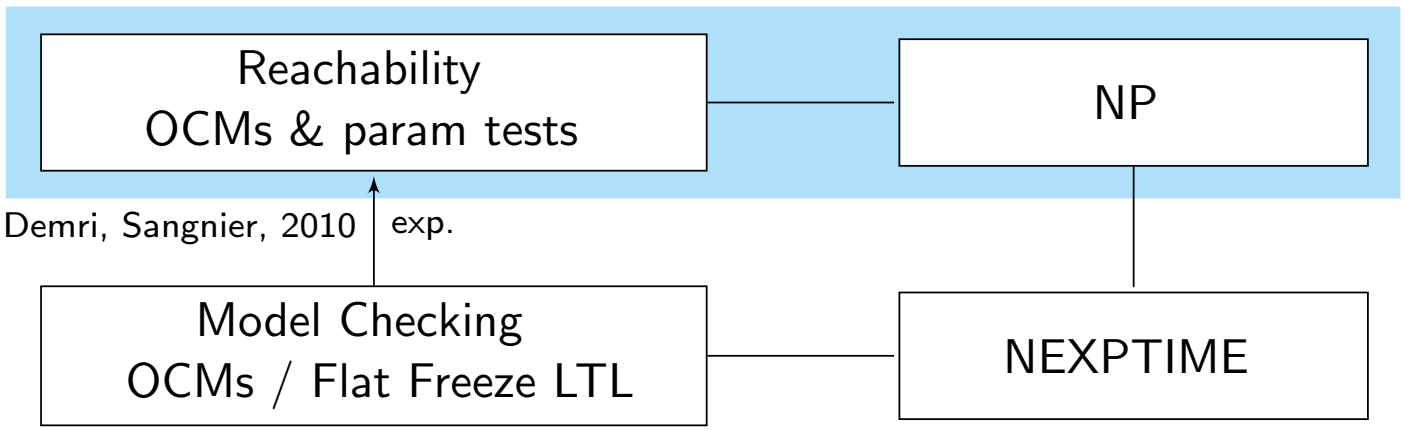
Our Contribution

Theorem

Model checking one-counter machines against formulas of Flat Freeze LTL is NEXPTIME-complete.

Proof outline



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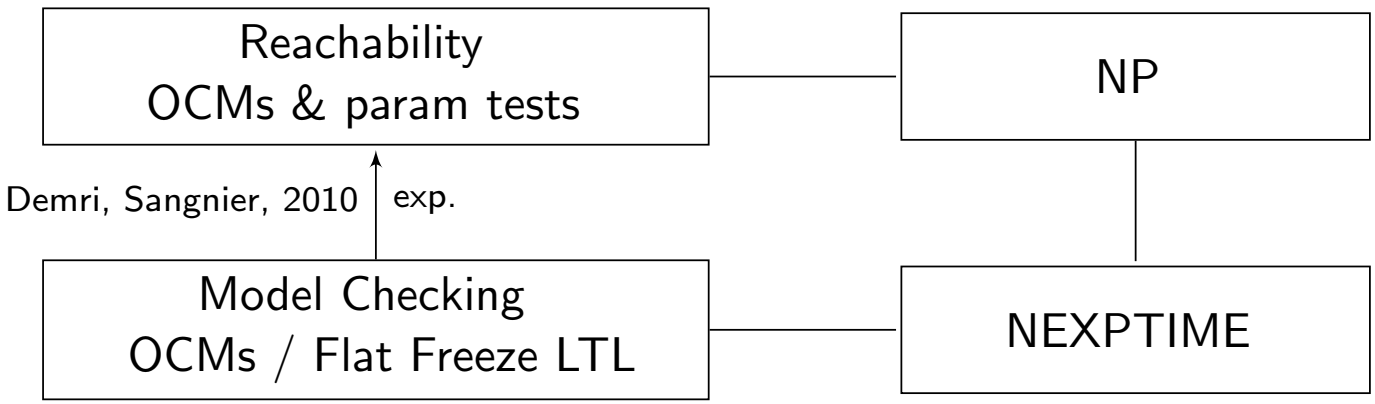

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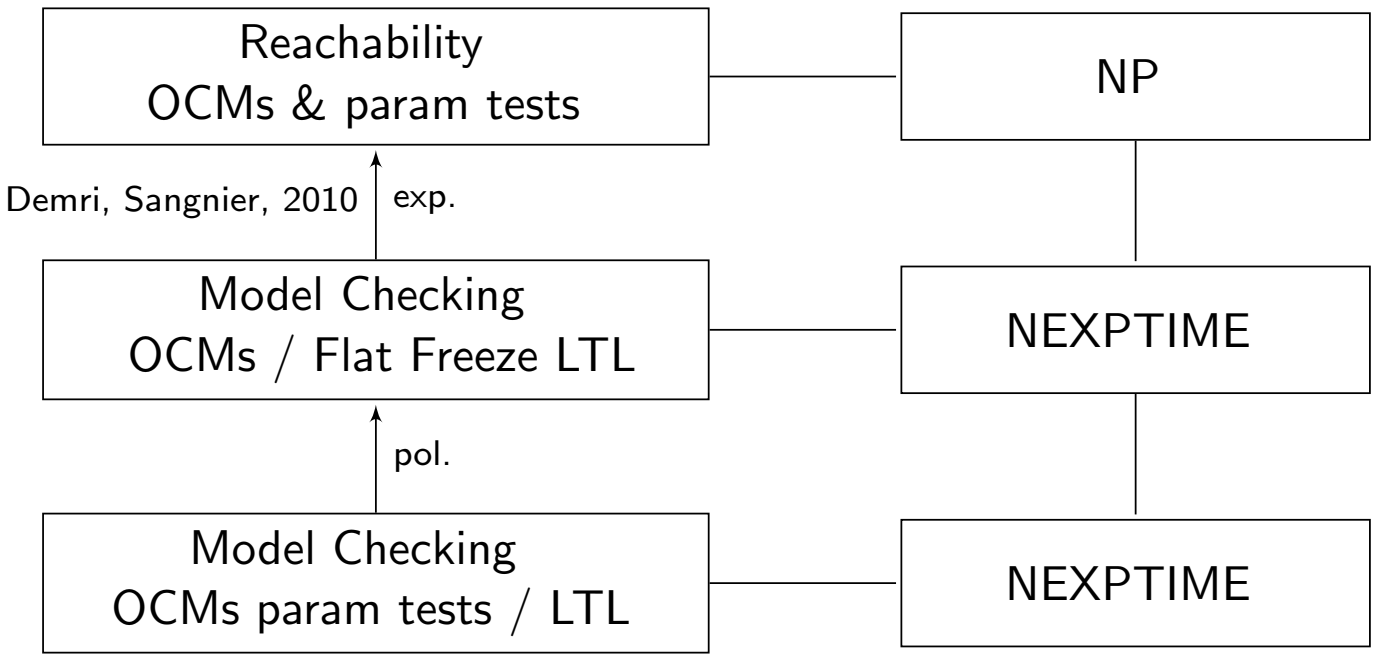

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- One-counter machines with parameterized tests and updates
- Parameterized timed automata (single clock case), where only a non-trivial 2NEXPTIME upper bound is known