Weighted Bottom-up and Top-down Tree Transformations are Incomparable

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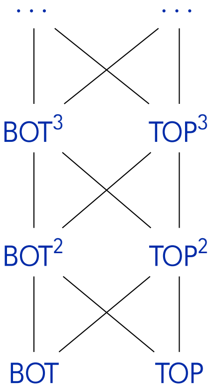
Bottom-up and Top-down Composition Hierarchy

Composition hierarchy [Engelfriet 1982]

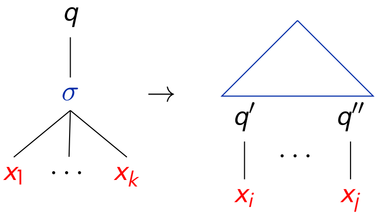


BOT — Bottom-up Tree Translations TOP — Top-down Tree Translations

Top-down Tree Transducer

Rule shape TOP

[Rounds 1968], [Thatcher 1970]



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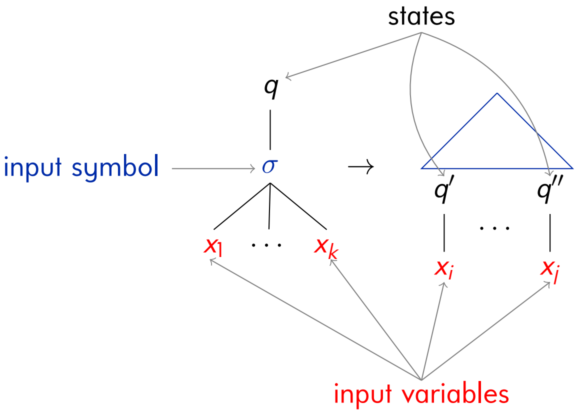

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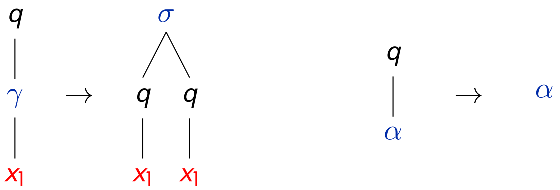

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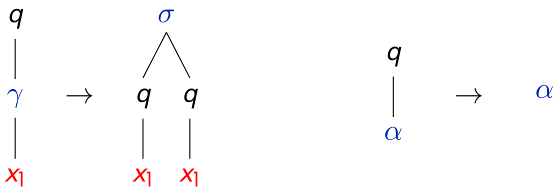
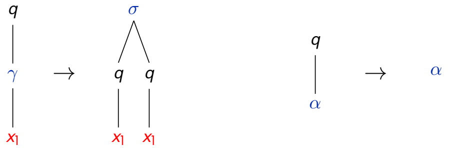


Notes

- 1st rule copies input tree
- Copies of input tree are independently processed

Top-down Tree Transducer



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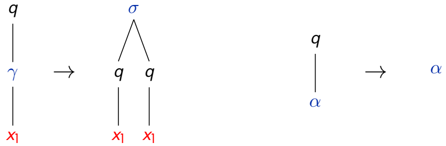

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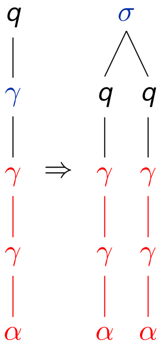

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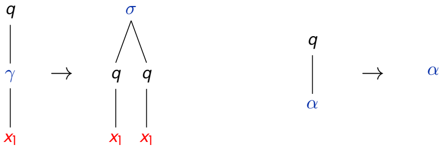

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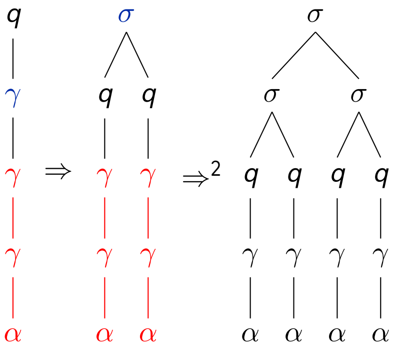

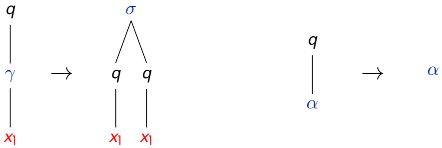

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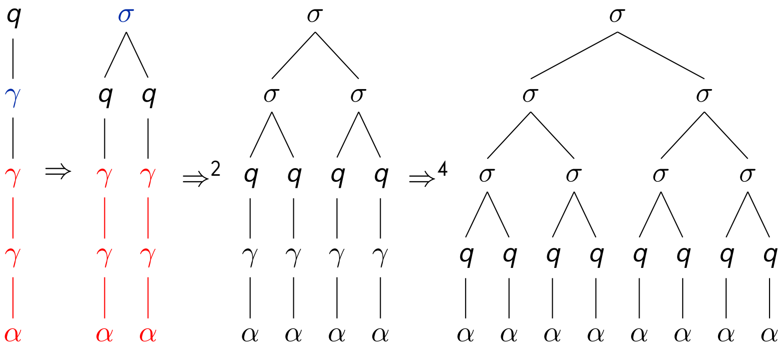

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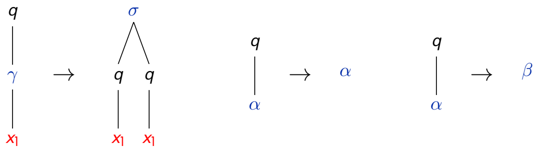

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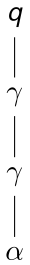

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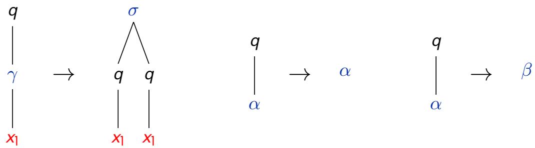

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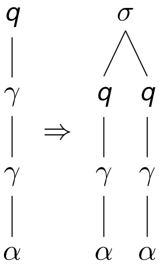

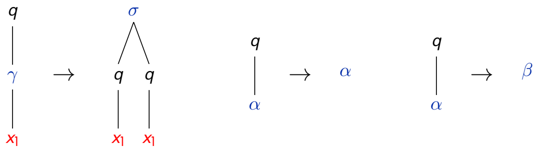

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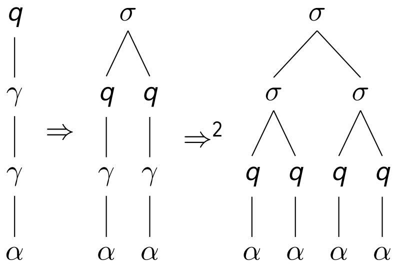

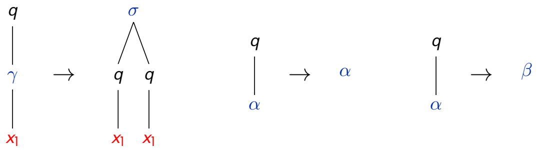

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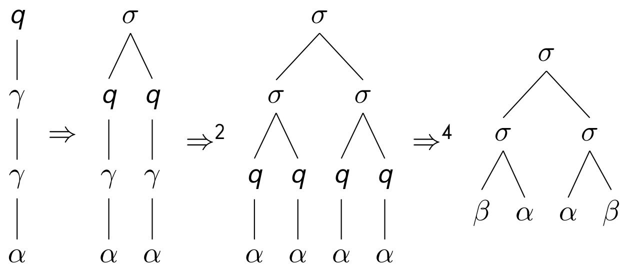

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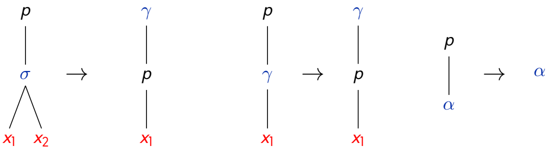

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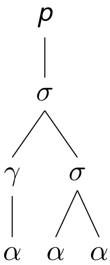

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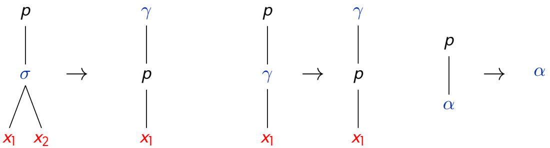

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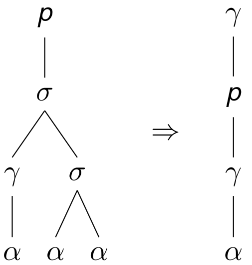

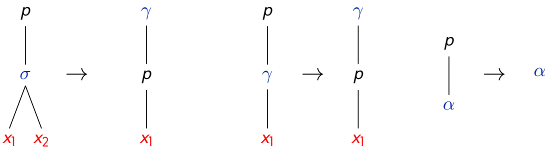

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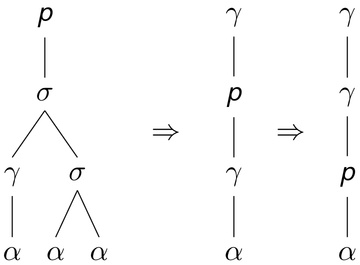

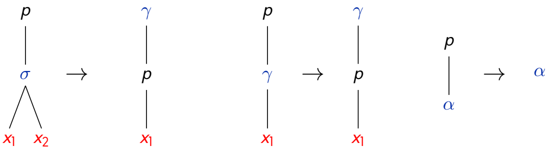

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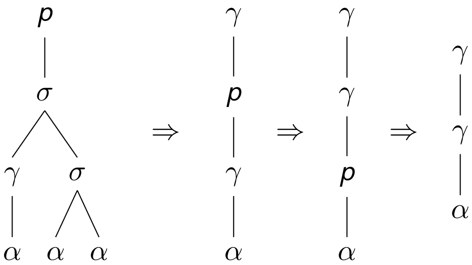

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General features TOP [Engelfriet 1975]

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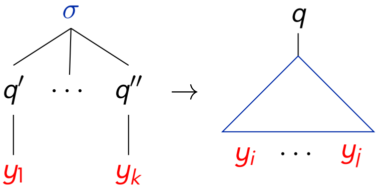
- ✓ Can copy and nondeterministically process copies
  (copies input subtrees)
- ✗ Cannot output identical trees for nondetermininstically processed inputs (except by chance) (due to independent processing of copies)
- ✗ Cannot restrict subtrees before deletion
  (deletes by ignoring input subtrees)

Rule shape BOT

[Rounds 1968], [Engelfriet 1975]



Bottom-up Tree Transducer

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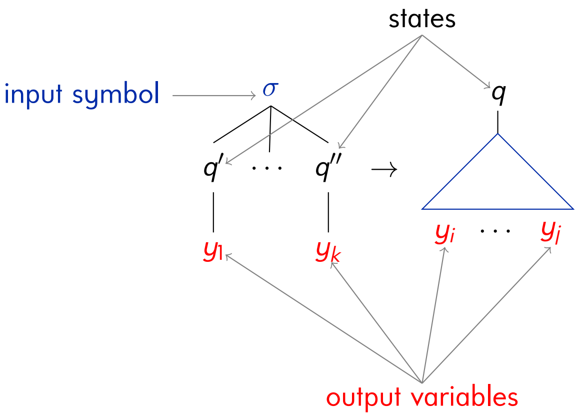

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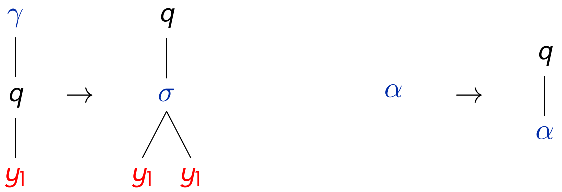

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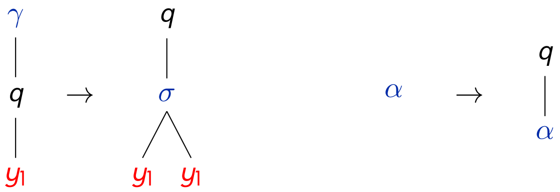

Notes

- 1st rule copies output tree
- Results in exact copies of output

Bottom-up Tree Transducer

BOT example derivation

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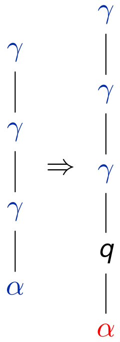

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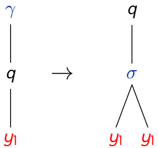

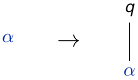

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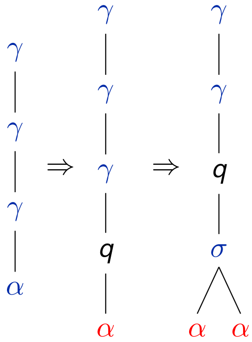

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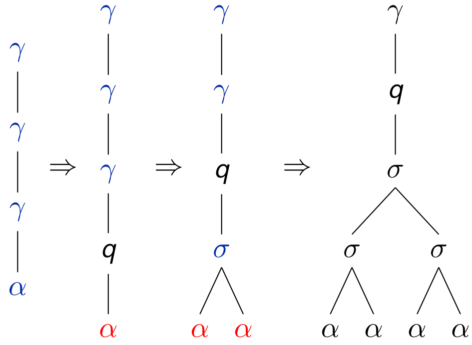

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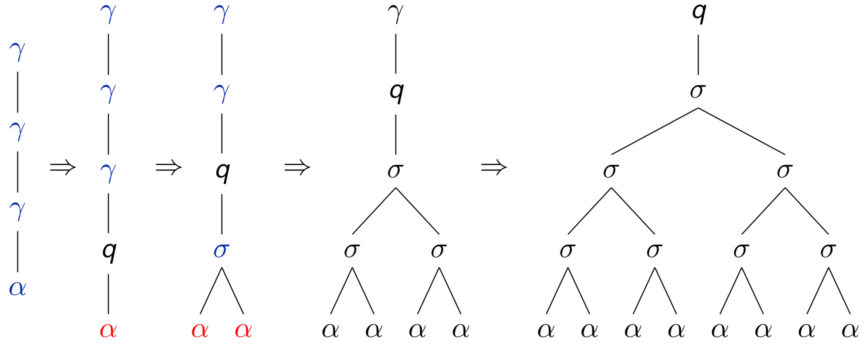

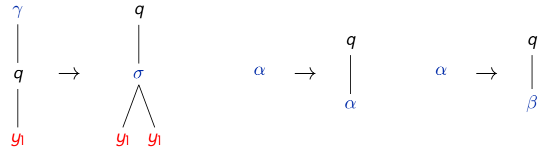

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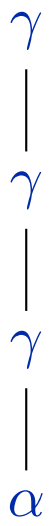

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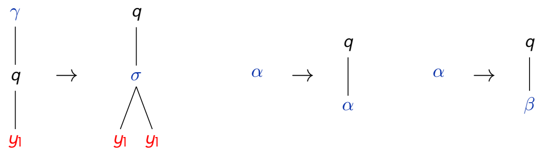

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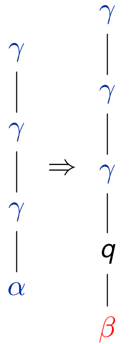

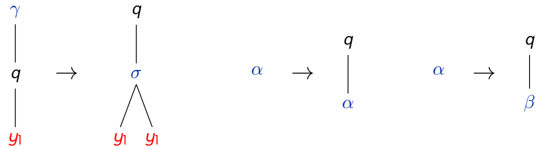

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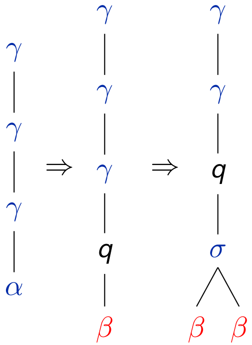

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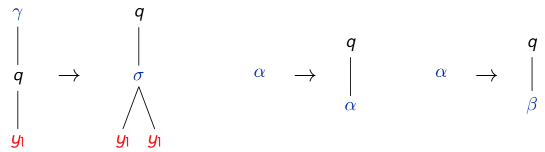

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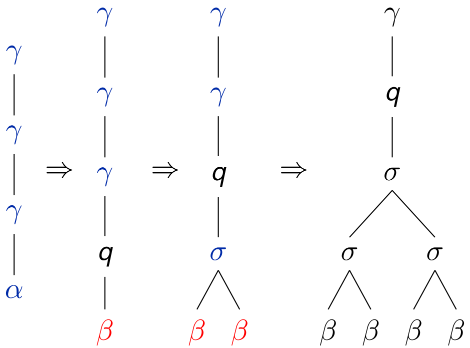

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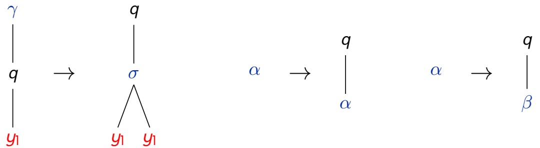

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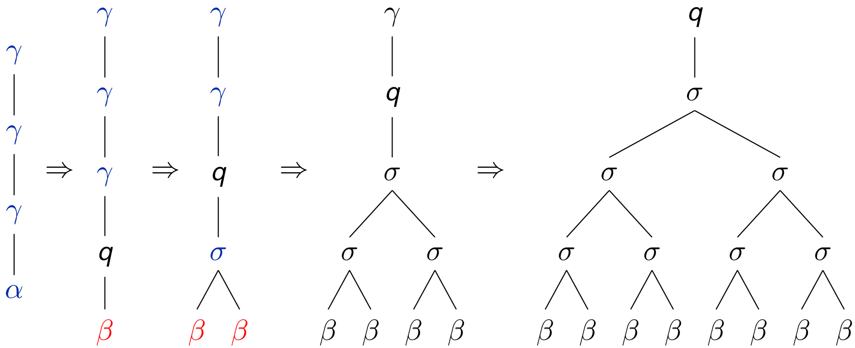

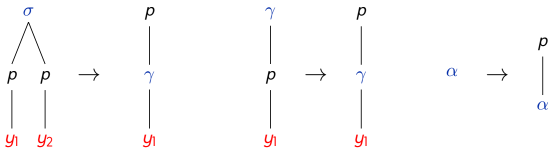

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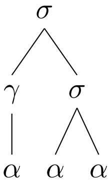

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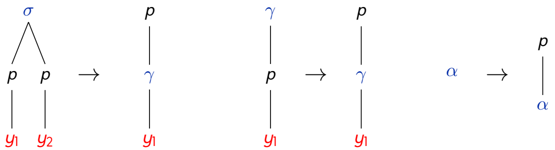

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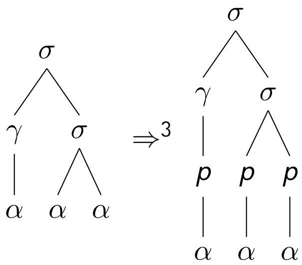

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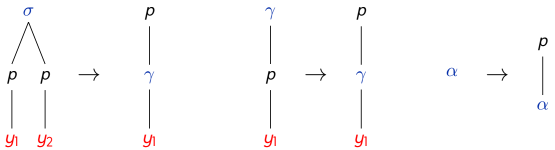

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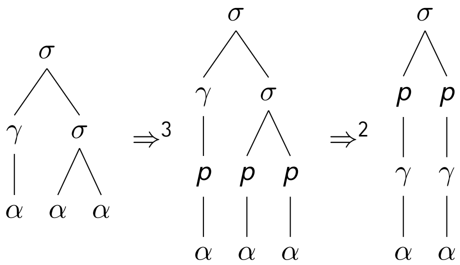

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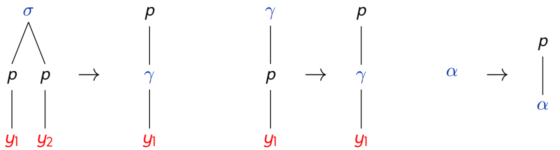

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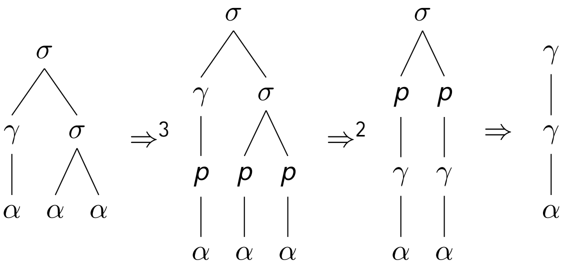

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- ✗ Cannot output different translations of same input subtree
  (cannot copy input subtrees)

Weights

Extension to TOP and BOT [Kuich 1999], [Engelfriet, Fülöp, Vogler 2002]

- Weights used in practice to resolve nondeterminism
- Each rule is assigned weight
- Weights multiplied along derivation
- Weights of alternatives are added

Typical weights

- Probabilities
- Costs
- Flows
- Profits

Weight structure

Definition

Commutative semiring $(C, +, \cdot, 0, 1)$ if

- $(C, +, 0)$ and $(C, \cdot, 1)$ commutative monoids
- $\cdot$ distributes over finite (incl. empty) sums

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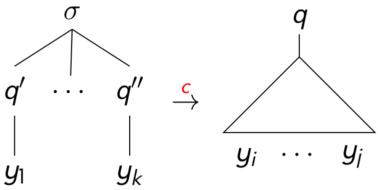
Example

- Boolean semiring $(\{0, 1\}, \max, \min, 0, 1)$ (unweighted case)
- semiring $(\mathbb{N}, +, \cdot, 0, 1)$ of nonnegative integers
- field $(\mathbb{Q}, +, \cdot, 0, 1)$ of rational numbers
- any field, ring, etc.

Weighted Bottom-up Tree Transducer

Rule shape Weighted BOT [Engelfriet, Fülöp, Vogler 2002]



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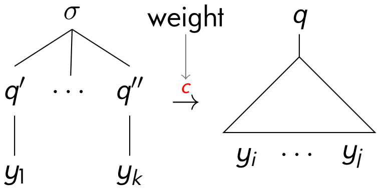

Weighted Extended Top-down Tree Transducer

Adjustment of semantics

- Weight of derivation is product of rule weights
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- Weight of translation is sum of all derivations for that translation
  (actually only left-most derivations to normalize rewrite order)

Weighted Composition

Given $\tau_1 \colon T_\Sigma \times T_\Gamma \to C$ and $\tau_2 \colon T_\Gamma \times T_\Delta \to C$

$$(\tau_1 \,;\, \tau_2)(s, u) = \sum_{t \in T_\Gamma} \tau_1(s, t) \cdot \tau_2(t, u)$$

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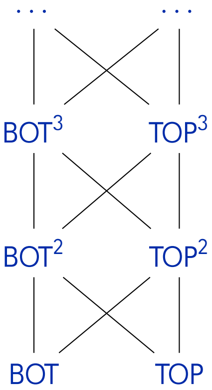
Note

- Weighted composition is standard matrix product
  (identify $\tau_1 \in C^{T_\Sigma \times T_\Gamma}$ and $\tau_2 \in C^{T_\Gamma \times T_\Delta}$)

Weighted Composition Hierarchy

Existing hierarchy [Fülöp, Gazdag, Vogler 2004]



Weight structure **must not** be ring

(Non-rings permit semiring homomorphism into Boolean semiring)

Weighted Composition Hierarchy

Notes

- Results for rings or fields sometimes “better” than in unweighted case
- Automata (NFA) minimization
  - in P for fields [Flouret, Laugerotte 1997]
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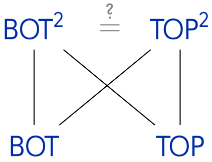
  - Unweighted: PSpace-complete [Jiang, Ravikumar 1993]
- HOM-problem
  - in NL for certain semirings that embed into field [M., Nász, Paul 2023]
  - Unweighted: ExpTime-complete [Creus, Gascón, Godoy, Ramos 2016]

Weighted Composition Hierarchy

Main result



for all commutative semirings

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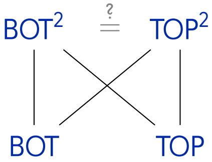


for all commutative semirings

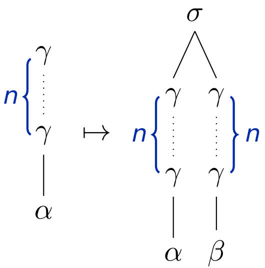
Main proof obligations

TOP $\not\subseteq$ BOT   BOT $\not\subseteq$ TOP

Weighted Composition Hierarchy

Obligation TOP $\not\subseteq$ BOT

- Straightforward due to BOT's structural inability to output different trees for same input subtree
- Support (nonzero-weighted pairs) cannot be mapping



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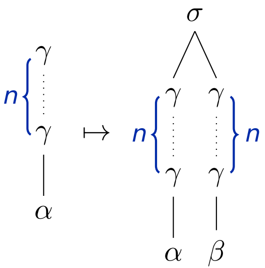

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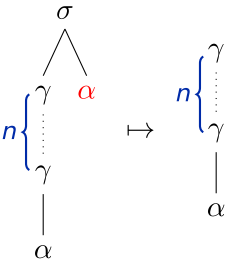

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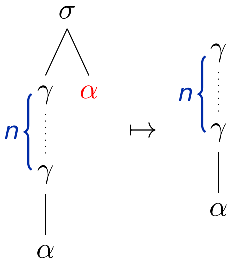

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Weighted Composition Hierarchy

Proof.

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- Let n be maximal size of rhs

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$$(s, t) \in f = \text{supp}(\tau) \qquad\qquad (s', t) \notin f = \text{supp}(\tau) \tag{1}$$

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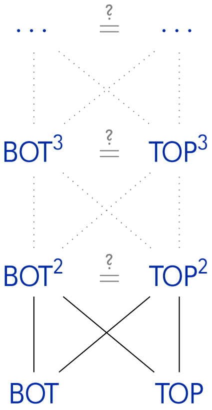
- $\tau(s, t) = \tau(s', t)$ contradicts (1) $\square$

Summary

Status quo



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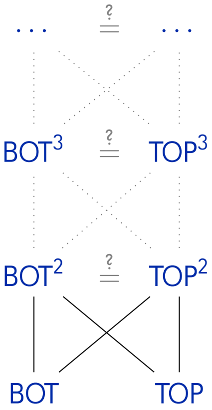

Thank you for your attention!