Universität Leipzig
Institute of Computer Science
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Exercises for the lecture

Foundations of Complexity Theory

Exercise Sheet 5

for the exercise on November 25, 2026

Exercise 5.1

Prove that the function $f\colon \mathbb{N} \to \mathbb{N}$, which is given by $f(n) = \lceil \log_2 n \rceil$ for all $n \in \mathbb{N}$, is space-constructible.

Only provide the general working principle of the required Turing machine.
You need not prove its correctness.

Exercise 5.2

Show that the following divisor problem is in **NP** $\cap$ co-**NP**.

Divisor problem: Given $n, k \in \mathbb{N}$, does n have a divisor t such that $2 \leq t \leq k$?

Hint: You can assume that primality testing is in **P**.

Exercise 5.3

Show that propositional formulas can be transformed into negation normal form in **L**.

Exercise 5.4

Prove: If **P** $=$ **NP**, then **E** $=$ **NE** where

$$\mathbf{E} = \mathsf{DTIME}\big(2^{O(x)}\big) \qquad \text{and} \qquad \mathbf{NE} = \mathsf{NTIME}\big(2^{O(x)}\big)$$

(Hint: translation technique)