Formal Semantics and Ontologies

Towards an Ontological Account of Formal Semantics

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Abstract. Formal ontology relies on representation languages for expressing ontologies. This involves the formal semantics of these languages which is typically based on a limited set of abstract mathematical notions. In this paper, we discuss the interplay between formal semantics and the intended role of ontologies as semantic foundation. In this connection a circularity is identified if ontologies are to determine the conceptual equivalence of expressions. This is particularly relevant for ontologies which are to be provided in multiple formalisms. In order to overcome this situation, *ontological semantics* is generally defined as a novel kind of semantics which is purely and directly based on ontological entities. We sketch a specific application of this semantics to the syntax of first order logic. In order to beneficially rely on theoretical results and reasoning systems, an approximation of the proposed semantics in terms of the conventional approach is established. This results in a formalization method for first order logic and a translation-based variant of ontological semantics. Both variants involve an ontology for their application. In the context of developing a top-level ontology, we outline an ontology which serves as a meta-ontology in applying ontological semantics to the formalization of ontologies. Finally, resolved and remaining issues as well as related approaches are briefly discussed.

Keywords. formal semantics, ontology, ontological semantics, first order logic

1. Introduction

The development and application of ontologies frequently involves their provision in several distinct formalisms, adopting an understanding of "ontology" as a "conceptualization" rather than its "specification" in a particular language, cf. Gruber's definition [1, p. 199]. Especially top-level ontologies must be available in multiple formalisms in order to facilitate their application in distinct areas like conceptual modeling, information integration, and the Semantic Web. The issues in this paper arise in the context of a long-term research project of developing a top-level ontology, the General Formal Ontology (GFO)[2] [2,3]. The most relevant formalisms in our project are primarily logical languages like first order logic (FOL) and description logics (DL), but also languages employed in conceptual modeling, especially the Unified Modeling Language (UML) [4]. Providing formalizations of GFO in several formalisms leads to the problem of how

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to justify that these formalizations capture the same contents, and can thus be used consistently. In a broader context, the same question is formulated in [5] for two arbitrary languages L_1 and L_2: "What exactly do we mean when we say that a set S_2 of L_2 sentences is a translation of a set S_1 of L_1 sentences?" This question is closely connected to the relationship between languages and their semantics, and we argue that this is still an open problem whose solution is intimately tied to ontologies.

In this paper, section 2 discusses interrelations of classical formal semantics and ontologies. We identify a specific circularity and argue that ontologies should play an important role for the foundation of *formal* semantics, in the context of ontology representation and meaning-preserving translations. Accordingly, section 3 introduces the notion of a formal, *ontological semantics*[3]. There we further outline an application of this semantics to the syntax of first order logic, and provide an approximation via classical FOL semantics in order to build on established work. The approximation results in a formalization method for FOL which itself requires an ontology to found syntactic compositions. Section 4 thus complements the approach by outlining an ontology which is proposed for applying ontological semantics to the formalization of ontologies. The motivating problems and the role of the approach in their regard are considered, accompanied by related work, in section 5, before we conclude and mention future directions.

2. Analysis of the Roles of Ontologies and Formal Semantics

Let us start with an ontology Ω and two representations of it, $R_1(\Omega) \subseteq L_1$ and $R_2(\Omega) \subseteq L_2$, in the languages L_1 and L_2 with distinct formal semantics. Since every communication about Ω must rely on representations, an important question arises: How to justify that $R_1(\Omega)$ and $R_2(\Omega)$ are representations of one and the same ontology? More generally, what does it mean to state that two expressions in distinct languages have the same meaning? Our first central claim in this respect is that the established types of formal semantics of languages are insufficient for answering these questions. Of course, this does not trivialize their value and adequacy for other tasks, e.g. theoretical analyses of mutual (formal) expressiveness, consistency, decidability, or complexity issues.

The claim of the inadequacy of formal semantics for meaning-preserving translations is based on a previously established meta-architecture for analyzing ontology constituents [7], which distinguishes the notions of *abstract core ontology* (ACO) and *abstract top ontology* (ATO). To avoid wrong intuitions about these and despite the terminological proximity, note that ACO and ATO do not immediately relate to the common ontology classification into top-level, core / generic domain, and domain ontologies [8, Sect. 1.4]. An ACO functions as an *ontology* for ontology constituents, i.e., it refers to the question of what ontological kind ontology constituents are (e.g., categories, relations, or attributes). This forms an (ontological) meta-level for languages, i.e., ontology constituents link to the ACO level via instantiation. ACOs relate closely to the abstract syntax categories of a language and correspond to an ontological understanding of knowledge representation ontologies in [8, Sect. 1.4]. For instance, for OWL's abstract syntax categories classID and individualvaluedPropertyID [9] one may postulate categories and (binary) relations, respectively, as appropriate ontological kinds in a suitable ACO. In

[3]Our work is not specifically related to and clearly differs from the equally termed approach in [6], which addresses natural language processing and semantics.

contrast, an abstract top ontology refers to the mathematical notions underlying the classical formal semantics assigned to a language, i.e., it captures the ontology of the formal semantics. In the case of OWL, this would be standard set theory based on the notion of sets and the membership relation. Accordingly, ontological constituents are *encoded* by means of instances of an ATO. For example, a unary FOL predicate $Lion$ (viewed syntactically) is interpreted by a set in the classical formal semantics (the abstract top view). This set encodes a category C within an animal ontology, i.e., C instantiates "category" with respect to an ACO.

Problems in ontology representation originate from (a) the lack of explicating the ACO view during formalization and (b) different choices for encodings in the formal semantics for the same syntax. As an example for (b), a FOL theory may encode category C by a functional constant $lion$ or a unary predicate $Lion$. Distinct encodings create formal differences originating from the same ontological entity by capturing different aspects of it. This in turn accumulates problems for language translations, even if a provably sound translation between the formal semantics of those languages is available. For instance, such translation exists for standard DLs and FOL [10, Sect. 4.2]. But translating a DL theory which encodes polyadic relations as DL concepts [11, use case 3] to FOL is hard if one expects for FOL that polyadic relations are expressed by polyadic predicates. That means, different encodings with respect to the ATO level may require "non-standard" translations between languages.

The justification for such "non-standard" translations lies outside of formal semantics alone. We believe that there is a kind of *conceptual* or *intensional* semantics which refers to the intensions of users of formal languages and is prior to formalization acts, cf. also [12]. *Ontologies* were "invented" in the context of knowledge-based systems research in order to tackle this problem, among others, cf. [13,14]. The basic idea is that different systems or languages *commit to* a common ontology Ω in order to share conceptual meaning, which should allow for a notion of meaning-preserving translations based on Ω. As an exemplary case of how this is frequently understood we formulate Def. 1, already taking into account that Ω can only be involved through a representation $R(\Omega)$ of it.

Definition 1 Let $R(\Omega) \subseteq L_\Omega$ be a representation of an ontology Ω in a logical formalism L_Ω, i.e., a theory. Let L_1 and L_2 be two arbitrary languages, and $\tau_i : L_i \to L_\Omega$ for $i \in \{1, 2\}$ be translations from L_i to L_Ω with respect to $R(\Omega)$. Two expressions $e_1 \in L_1$ and $e_2 \in L_2$ are said to be *conceptually equivalent* with respect to τ_1 and τ_2 iff their translations into L_Ω are $R(\Omega)$-equivalent:

- for terms $\tau_1(e_1)$ and $\tau_2(e_2)$: $R(\Omega) \models_{L_\Omega} \tau_1(e_1) = \tau_2(e_2)$
- for formulas $\tau_1(e_1)$ and $\tau_2(e_2)$: $R(\Omega) \models_{L_\Omega} \tau_1(e_1) \leftrightarrow \tau_2(e_2)$ □

The representation formalism L_Ω in this definition is a problematic parameter. Due to the above analysis we deny the common assumption that logical languages are "ontologically neutral" [15, p. 492] and could be used without an ontological bias. Instead there is a vicious circle in this approach. Ontologies are meant to overcome insufficiencies of formal semantics with respect to conceptual equivalence, which is itself based on the formal equivalence defined for L_Ω and the encoding of Ω into L_Ω – and thus on the formal semantics of L_Ω. This yields two problems that will be addressed subsequently.

Problem 1 How to assign a semantics to a language that is directly based on ontologies and avoids the just-mentioned circularity.

A solution would further clarify how to represent and interpret ontologies ontologically. For this purpose we introduce the notion of *ontological semantics* in the next section. The approach is applicable to arbitrary languages and is intimately tied to ontologies. In order to apply ontological semantics to the formalization of ontologies, it is necessary to provide suitable abstract core ontologies.

Problem 2 is to develop and specify suitable abstract core ontologies.

The plural form indicates that we expect multiple solutions for Problem 2. Sect. 4 outlines a proposal for a small yet powerful abstract core ontology. In general, the overall approach applied to ontologies can be understood as defining a semantics which is directly based on the abstract core level, or as one which combines the functions of an abstract core and an abstract top ontology [7].

3. Ontological Semantics

3.1. Ontological Structures and Ontological Semantics in General

A model theoretic semantics can abstractly be understood as a system $(L, M, \models)$ of a language L, a set of interpretation / model structures M, and a relation of satisfaction $\models \subseteq M \times L$, cf. [16, Ch. I.1, II.1]. We aim at a model theoretic approach for defining a formal semantics based on purely ontological entities. For this purpose, we establish a notion of *ontological structures* as an analogon to (mathematical) interpretation structures. These structures should avoid built-in ontological assumptions to the greatest possible extent. In order to achieve this and to draw an appropriate analogy to classical model theory, the set-theoretic background of model theory must first be explicated.

Consider a typical FOL-structure (restricting the signature to predicates and functional constants for simplicity) $\mathcal{A} = (A, R_1, \ldots, R_m, c_1, \ldots, c_n)$, where constants form elements of the logical universe A (a set), and predicates are interpreted as relations over A (as mathematical relations, i.e., as sets of tuples). This logical universe A does not cover all those entities which appear as interpretations of symbols in the formal semantics, in particular, it does not cover the interpretations of predicates. That means, relations over A are assumed silently based on standard set theories. The latter typically allow for constructing tuples and power sets over a given set, hence the structure $\mathcal{A}^{\mathcal{P}^{\omega}_{\text{fix}}} = \left(A^{\mathcal{P}^{\omega}_{\text{fix}}}, r_1, \ldots, r_m, c_1, \ldots, c_n\right)$ can be derived from $\mathcal{A}$, with $\mathcal{P}$ denoting the power set operator and $A^{\mathcal{P}^{\omega}_{\text{fix}}}$ being defined by:

$$A^{\mathcal{P}^{0}_{\text{fix}}} = A \tag{1}$$

$$A^{\mathcal{P}^{n}_{\text{fix}}} = \bigcup_{k<\omega} \left(\mathcal{P}\left(\left(A^{\mathcal{P}^{n-1}_{\text{fix}}} \right)^{k} \right) \right) \quad \text{for } n > 0 \tag{2}$$

$$A^{\mathcal{P}^{\omega}_{\text{fix}}} = \bigcup_{n<\omega} A^{\mathcal{P}^{n}_{\text{fix}}} \tag{3}$$

In $A^{\mathcal{P}^{\omega}_{\text{fix}}}$, all symbols of a logical language can be considered as constants, i.e., they are interpreted by elements of $A^{\mathcal{P}^{\omega}_{\text{fix}}}$. Some of them are in parallel subsets of (tuples over)

$A^{\mathcal{P}^{\omega}_{\text{fix}}}$. Hence, the elements of $A^{\mathcal{P}^{\omega}_{\text{fix}}}$ are interrelated according to the underlying set theory (i.e., set theory functions as an abstract top ontology here) – except for A-members. Only the elements c_i which interpret FOL constants are unconstrained by the set theory and may be related in arbitrary ways.

$\mathcal{A}^{\mathcal{P}^{\omega}_{\text{fix}}}$ appears as an adequate "template" for our intended ontological structures, in contrast to the classical structure $\mathcal{A}$. An ontological structure $\mathcal{O}$ is meant to provide constant-like interpretations for all symbols in terms of appropriate "members" of some "universe" O of $\mathcal{O}$. Technically, we say that those "members" are *associated with* O. No hidden assumptions are to be made on interrelations within O – if there are interrelations, they should be captured in an axiomatization using the symbols of the language. As *ontological* structures, neither O nor what is associated with O can generally be constrained to refer to mathematical notions. Altogether, this leads us to the following definition:

Definition 2 An ontological structure $\mathcal{O}$ can be described as $\mathcal{O} = (O, c_1, c_2, \ldots)$ where O is an arbitrary entity and the c_i are entities associated with O. □

Rendering O as an arbitrary entity may sound problematic. In other attempts to characterize O one may state that it is a "structure of intended semantics" (intended by the user of the language), a part of the world, or a state of affairs. In terms of situation theory [17], O corresponds closest to a "union" of *real* situations and events (but integrating individuals and categories). For a simplistic example, assume someone watching lion Leo in chasing some other animal. The observer may thus claim the existence of a corresponding part of the world O which would have associated with it the actual process / event, Leo, the categories of lion and chasing, Leo's participation in that process, etc. Referring to a single observer as well as to "reality" are problematic issues themselves, but cannot be discussed here. We just note that our view is more similar to that in [18, p. 13, Sect. 3.1, §2] which allows for a cognitive bias with respect to reality than to a purely objective and subject-independent view on reality. It must further account for entities in the widest sense, e.g. including hypothetical and fictitious entities, as well.

From the point of view of the c_i, O forms a kind of "aggregate" which comprises at least the c_i (and possibly further entities). The relation *associated with* must likewise be understood ontologically. In particular, we see this as a basic relation which generalizes e.g. part-of and inherence (which connects qualities with their bearers), and might include set membership. It is necessary that O offers counterparts for *all* basic syntactic entities of a language in the form of the c_i (see the treatment of FOL predicates below). Therefore, the c_i may be of arbitrary ontological kinds. All c_i and O coexist legitimately, without assuming reductions among them. For instance, we see no reductions among lion Leo, the category of lions, the chasing, and an ontological structure all of those entities are associated with. In addition, more entities may be associated with O than only the interpretations of the constants of a language. E.g. one can expect many, more or less detailed ontological structures comprising Leo. To emphasize the fact again, in general, O is not considered a set.

Definition 3 An *ontological semantics* for an arbitrary language L is a model theoretic semantics whose interpretation structures are ontological structures. □

This definition is clearly a very general characterization. For ontological structures as introduced in Def. 2 only constants can be interpreted immediately. In order to establish an ontological semantics for a declarative language L directly, more complex syntactic constructions must be assigned ontological interpretations in terms of those structures.

We indicate this direct approach for FOL in Sect. 3.2. Beforehand, note a difference in assigning an ontological semantics to a language L and common definitions of formal semantics. Given L in terms of a grammar G, G usually bottoms out with *identifiers*, i.e., symbols for which no further distinctions are made in the semantics. For example, in FOL, a non-terminal 'predicate' would have a range of admissible predicate-identifiers (terminals) assigned, but those do not influence the standard semantics of FOL. The latter is usually defined by non-terminal syntactic categories and a few fixed terminal symbols (or *keywords*), like '$\wedge$' and '$\rightarrow$'. In contrast, ontological semantics / interpretations must ensure an appropriate interpretation for each single terminal in each particular use of a language. Since the number of non-terminal syntactic categories is limited, languages can be used very differently, which refers to their syntactic constructs and thus indirectly to the resulting formal semantic counterparts. It corresponds to our prior analysis that these distinct forms of using a language are only remotely dependent on classical formal semantics, and should be explicated by an ontological semantics.

3.2. Application to FOL Syntax

We sketch a definition of ontological semantics for FOL syntax, following classical definitions in a rather straightforward way in most cases, cf. [19, Sect. 2.2].[4] We assume appropriate valuations ν for variables in addition to an ontological structure $\mathcal{O}$ under consideration, where variables are assigned to entities associated with O. For a valuation ν, $\nu[\frac{x}{e}]$ refers to any valuation which agrees with ν on all assignments, yet only that of x is e in $\nu[\frac{x}{e}]$. $\mathcal{O} \models_\nu \phi$ means that $\mathcal{O}$ satisfies a formula ϕ for ν, $\mathcal{O} \models \phi$ that $\mathcal{O} \models_\nu \phi$ for every ν.

FOL logical constants do not have entities associated with O as semantic counterparts, corresponding to the case of set-theoretic interpretations. Rather, they manipulate or determine the combination of structures based on expressions and sub-expressions. They are defined for ontological structures in strict analogy to the standard definitions, e.g. for conjunction and negation:

$$\mathcal{O} \models \neg\alpha \text{ iff } \mathcal{O} \not\models \alpha\ . \tag{4}$$

$$\mathcal{O} \models \alpha \wedge \beta \text{ iff } \mathcal{O} \models \alpha \text{ and } \mathcal{O} \models \beta\ . \tag{5}$$

For quantification there are several options. Here, we adopt a variant that is equivalent with the classical definition and maintains the duality between existential and universal quantification.

$$\mathcal{O} \models_\nu \exists x\, .\, \phi \text{ iff there is an } e \text{ associated with } O \text{ s.t. } \mathcal{O} \models_{\nu[\frac{x}{e}]} \phi\ . \tag{6}$$

$$\mathcal{O} \models_\nu \forall x\, .\, \phi \text{ iff } \mathcal{O} \models_{\nu[\frac{x}{e}]} \phi \text{ for every } e \text{ associated with } O\ . \tag{7}$$

Notably and despite of adopting the same definitions, the nature of quantification changes considerably due to O being the domain of quantification (cf. the relations of A and $A^{\mathcal{P}^\omega_{\text{fix}}}$ above). Further definitions, e.g. of the validity of formulas regarding a structure, of logical validity, etc. also strictly follow their set-theoretic equivalents.

[4]Here we neglect epistemological issues as well as discussions on truthmakers, different degrees of conviction like beliefs, assumptions, truths, etc. Currently, we assume as a simplification only that all formulas share a common degree of conviction.

The major difference to the set-theoretic approach refers to predication. Classical semantics provides a uniform account in terms of set-membership: $P(x_1, \ldots, x_n)$ is true in interpretation I iff $(x_1^I, \ldots, x_n^I) \in P^I$. It is hard to provide a uniform ontological account of (syntactic) predication, because an arbitrary ontological interconnection among the arguments may be chosen for each individual predicate. For example, an atomic sentence $\mathsf{part\text{-}of}(x, y)$ may be read semantically as "x is a part of y" (intensionally, in contrast to $(x^I, y^I) \in \mathsf{part\text{-}of}^I$ which reduces "part of" to an extensional set of argument tuples). This would require a semantic condition like:

$$\mathsf{part\text{-}of}(x, y) \quad \text{iff} \quad \text{"}x \text{ is a part of } y\text{"}. \tag{8}$$

It is clearly undesirable to introduce every predicate in terms of informal phrases. Moreover, such definition would not profit from the requirement for an ontological semantics for a syntax with $\mathsf{part\text{-}of}(x, y)$ to contain an entity for the symbol part-of in its ontological structures. In this connection abstract core ontologies (ACOs) become relevant. An ACO should comprise a few basic entities and should be capable of classifying arbitrary entities (at a very abstract level). With sufficiently rich logical connectives, this allows for formally defining common predication patterns with respect to arbitrary entities (see Sect. 4 for sample patterns based on our proposed abstract core ontology). But before an actual formal proposal is to be made, let us consider to what extent classical interpretations can be "reused".

3.3. A Formalization Method based on Approximations of Ontological Models of FOL

The direct approach to defining an ontological semantics is not favorable due to the weak theoretical basis of such semantics. A better approach, especially for FOL, would be to build on classical theoretical results and to use established theorem provers for reasoning over ontological interpretations. Therefore, a major question concerns the relationship of ontological and set-theoretic interpretations, and whether the latter may be used for simulating or approximating ontological structures. For the present discussion we avoid interferences among the two types of semantics by restricting every ontological structure $\mathcal{O} = (O, c_1, c_2, \ldots)$ such that neither sets nor representational entities / syntactic elements like symbols are associated with O. Otherwise, one would have to take e.g. relationships between fore- and background membership relations into account.

Starting from an ontological structure $\mathcal{O}$ with its "universe" O, our approximation is initiated by an algebraic structure $\mathcal{A}(\mathcal{O}) = (U(O), c_1, c_2, \ldots)$ where $U(O) = \{x \mid x \text{ is associated with } O\}$ is a set of urelements (entities which are not sets). Consequently, every $c_i \in U(O)$ is the very same entity as considered ontologically (which connects to the relation between A and $A^{\mathcal{P}^{\omega}_{\text{fix}}}$ in Sect. 3.1). Next, we enrich these structures such that FOL formulas under a set-theoretic interpretation can be given an ontological interpretation, as well. Based on Sect. 3.2 the syntactic constructions which concern logical constants and quantification are directly transferable. It remains to accommodate the semantics of predication, now continuing the argumentation of Sect. 3.2. We aim at linking predication with intensions (via constants for the c_i) by explicit definitions:

$$\forall \bar{x} \,.\, P(\bar{x}) \leftrightarrow \phi(\bar{x}) \,. \tag{9}$$

Above we have indicated the potential diversity of interpreting (syntactic) predication ontologically, and we have argued that abstract core ontologies can be used to "bootstrap" such definitions. This leads us to the following method of formalizing ontologies.

Formalization Method. For an abstract core ontology Ω_{base}, we introduce a basic signature Σ_{base} for expressing relations of Ω_{base} by predicate symbols. Moreover, interconnections within Ω_{base} are specified axiomatically in a theory $Ax_{base} \subseteq L(\Sigma_{base})$. The major guideline of the method is to represent entities of *every* ontological kind, e.g. including categories and (ontological) relations, first as a functional constant in FOL. The introduction of new predicates (beyond Σ_{base}) must then be accompanied by a definition which involves previously introduced syntactic elements, most reasonably those functional constants which are understood to represent the intension of a predicate. Moreover, the new symbol(s) for representing an entity can be characterized axiomatically. $\square$

FOL theories resulting from this method and their set-theoretic models can easily be related to an ontological semantics and ontological models. The main idea is that variables, functional constants and the Σ_{base} predicates are interpreted intensionally (or ontologically), whereas all other predicates are conceived extensionally, e.g., for $P \notin \Sigma_{base}$ the expression $P(x)$ is ontologically interpreted as $x \in P^{\in}$, which is adequate from a classical and an ontological point of view.[5] Moreover, an intensional specification of each predicate is available due to the required definitions and possibly additional axioms. Those definitions must ultimately rely on Σ_{base}-predicates, i.e., some intensionally interpreted relations from an abstract core ontology. Illustrations of the approach are presented in Sect. 4.

3.4. Ontological Usage Schemes

In terms of the approximation proposed for FOL, we can define a translational variant for ontological semantics.

Definition 4 Let L be an arbitrary language, and let Ω be an ontology for L with a FOL representation $R(\Omega) \subseteq L_\Omega$, $L(\Sigma_{base}) \subseteq L_\Omega$ according to the formalization method. An *ontological usage scheme* of L is a translation $\tau : L \rightarrow L_\Omega$.
For a set $S \subseteq L$ of L-expressions, its *ontological image* is the deductive closure of $R(\Omega) \cup \{\tau(s) \mid s \in S\}$. $\square$

Ontological usage schemes have the intended advantage compared to the direct approach of defining an ontological semantics: one can rely on theoretical results established against the background of classical semantics, as well as on corresponding theorem proving algorithms and software. Nevertheless, the resulting classical models remain approximations of possible ontological models. In particular, given the exclusion of sets and symbols, there are far less ontological models of a theory T than there are set-theoretic approximations, because for the latter e.g. distinct, but isomorphic structures are also models of T. The existence of ontological models based on the existence of classical models must thus be justified individually. Moreover, the relation between set theory (as an ontology of sets) and the remaining ontological theory must be clarified. Altogether we think that a careful reuse of existing work clearly outweighs those approximation effects.

[5] There is one ontological concession, namely to grant sets an ontological status. This is applicable in GFO.

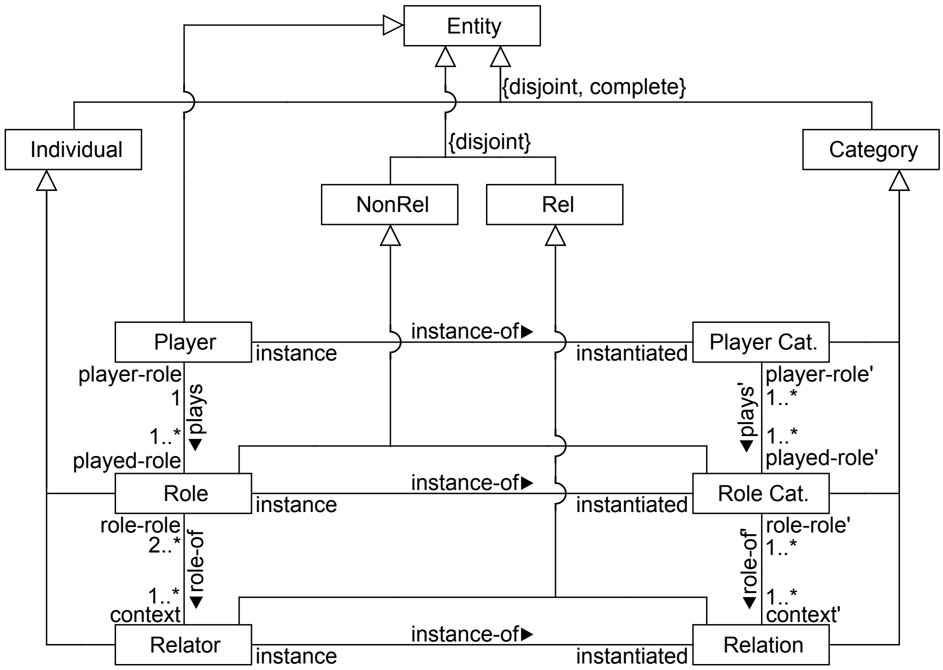

Figure 1. Overview of the ontology of categories and relations. The upper part shows two major distinctions of Entity, the first into categories and individuals based on instantiation, the second into relational (Rel) and non-relational (NonRel) entities. The lower part illustrates major categories relevant for relations, and their mediation between entities (note that Player and PlayerCat are extensionally equivalent with Entity and Category, respectively). Primed relations originate from their counterparts by lifting those to the level of categories, whereas primed roles are roles of primed relations.

4. Ontology of Categories and Relations

As a suitable abstract core ontology for formalizing top-level ontologies, we advocate an ontology of categories and (ontological) relations based on [20,21,7,2], among others. Fig. 1 outlines its major constituents in UML notation [4]. The most general notion for anything which exists is entity in GFO. Categories are those entities which can be instantiated (instance-of), in contrast to individuals. For example, a particular lion leo is an individual whereas $lion$ is a category[6]. Relations are granted an ontological status as categories of relators, specific entities which mediate between other entities. Relators are composed of (relational) roles[7], which appear like parts of a relator (role-of) and which "link" it to one of its arguments, cf. [20]. Roles are doubly dependent entities. Firstly, an entity plays a role (plays), which is a dependence on that player. Secondly, a role depends on other roles appearing in a relator, which must consist of at least two roles. Fig. 2 illustrates a part-of relator mediating between lion Leo and its head.

[6] We use the term "category" in accordance with GFO for anything which can be instantiated or predicated of something else. This is a much less specific use than the common philosophical reading of category as "highest kinds or genera" [22].

[7] This notion of roles differs from the notion of roles in description logics. The latter would typically correspond to relators connecting two arguments, hence being composed of two roles in the present sense.

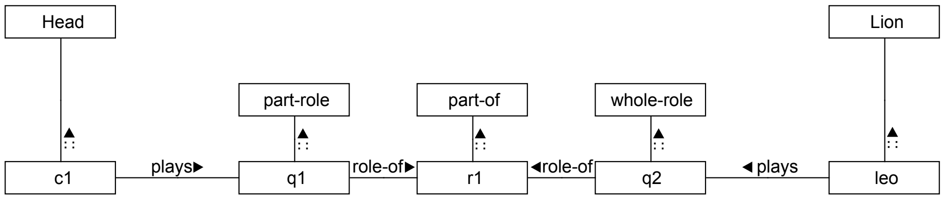

Figure 2. An example of relation analysis in GFO based on a phrase like "Lion Leo's head is a part of Leo". The relation part-of is a category of relators, instantiated (::) by r1. The relator r1 mediates between Leo's head c1 and leo via its two roles q1 and q2, determined by the given role categories as roles of part and whole, respectively.

This ontology is adopted due to (a) the fundamental nature of categorization and (b) the possibility of relating two entities which appears as soon as they are distinguished from each other. The constituents of the approach further suffice to analyze themselves. Moreover, we see similarities with the most abstract levels in meta-modeling approaches which reinforces this position, cf. the root diagram of the Kernel package in the UML Superstructure Specification [23, fig. 7.3, p. 25].

The above formalization method can now be illustrated in connection with the ontology. Let $\Sigma_{base} = (::, \rightsquigarrow, \multimap)$ comprise only three binary predicates for our basic relations: $x :: y$ for "x instantiates y", $x \rightsquigarrow y$ for "x plays role y", and $x \multimap y$ meaning "x is a role of y". To distinguish symbols from their denotations, P^{ι} refers to the intended denotation of a symbol P. If P^{ι} has an extension, i.e., a set of instances or argument tuples, this is denoted by $P^{\in}$.

We introduce exemplary definition patterns which capture common types of predication, starting with the unary case. Unary predicates typically refer to non-relational categories. Following our method we introduce a functional constant ${}_{c}P$ and a unary predicate P. The following definition is added to bind P to its counterpart ${}_{c}P$.

$$\forall x \,.\, P(x) \leftrightarrow x :: {}_{c}P \,. \tag{10}$$

Classically, a FOL model interprets P and :: as sets over some universe which has an element interpreting ${}_{c}P$. The ontological interpretation of this formula (or of a corresponding classical model) is that ${}_{c}P$ captures an intension ${}_{c}P^{\iota}$ directly, and – as (10) states – P captures ${}_{c}P^{\in}$, the extension of ${}_{c}P^{\iota}$. Apart from this definition, appropriate axioms involving ${}_{c}P^{\iota}$ via P or ${}_{c}P$ should be stated.

For relations, there are several options of how n-ary predicates, $n \geq 2$, can be understood to abbreviate the linking of arguments via relators. A weak form (for a binary predicate R) is "there is a relator r which is an instance of the relation ${}_{c}R$ with respect to which x and y play roles in r":

$$\begin{aligned} \forall xy \,.\, R(x,y) \leftrightarrow \exists r q_1 q_2 (& \\ & q_1 \neq q_2 \wedge r :: {}_{c}R \wedge \\ & x \rightsquigarrow q_1 \wedge q_1 \multimap r \wedge \\ & y \rightsquigarrow q_2 \wedge q_2 \multimap r \;\;) \,. \end{aligned} \tag{11}$$

This form entails symmetry of R, which may be counterintuitive for ${}_{c}R^{\iota}$. It can be strengthened to specify the instantiated role categories, cf. also Fig. 2. Moreover, assume that R is based on two intensionally distinct role categories ($Q_1 \neq Q_2$), each with exactly one role individual per R-relator, following the "closed world" intuition that a tuple (x, y) contains exactly x and y. This case concludes our sample patterns.

$$
\begin{aligned}
\forall xy \,.\, R(x,y) \leftrightarrow \exists r q_1 q_2 Q_1 Q_2 (& \\
& q_1 \neq q_2 \wedge Q_1 \neq Q_2 \wedge r :: {}_c R \wedge \\
& x \rightsquigarrow q_1 \wedge q_1 \multimap r \wedge q_1 :: Q_1 \wedge \\
& y \rightsquigarrow q_2 \wedge q_2 \multimap r \wedge q_2 :: Q_2 \wedge \\
& \forall q' (q' \multimap r \rightarrow (q' = q_1 \vee q' = q_2)) \;) \,.
\end{aligned}
\tag{12}
$$

5. Discussion

5.1. Reconsideration of the Motivating Problems

The overall purpose of this paper is to propose a theoretical foundation for explaining "meaning-preserving" translations among languages, which maintain the declarative contents among the different expressions of those languages. This differs from simulations among the dynamics of the languages, e.g. encodings of reasoning problems of one logic into another. One may argue that logical formalisms can be used with intensional interpretations independently of or in addition to their set-theoretic model theory. Even if this is case, those intensional interpretations remain implicit and thus cannot be used e.g. for translations among languages.

The general approach to ontological semantics clearly adopts ontological entities as its foundation and thus avoids Problem 1, the circular interplay between ontologies and formal language semantics based on mathematical notions. The same applies indirectly to ontological usage schemes and ontological images of arbitrary expressions, i.e., translations of those expressions into FOL formalizations constructed according to the presented method. Classical models of ontological images are potential approximations which can help in determining ontological models. Moreover, ontological images comprise explicit ontological explanations for each predicate, which binds their extensional interpretation to complex expressions with intensional components (through the intensionally understood functional constants and basic relations). This suggests the need for refining Def. 1 of conceptually equivalent expressions in our initial analysis. For instance, two predicates are conceptually equivalent by Def. 1 if they originate from two intensionally distinct, but extensionally equivalent categories. Due to the formalization method, a suitable refinement of that definition can be based on the identity of functional constants and of compositions via basic relations. A related aspect is the dependence of Def. 1 on the chosen logic. For example, assume that an ontology is represented (i) in a monotonic logical language and (ii) a nonmonotonic language. In its present form, there will be immediate differences in the resulting notions of conceptual equivalence. We see a need for further investigations in this respect, which may involve different types of categories.

Allowedly, the formalization method is rather simple and could be adopted on an *ad hoc* basis. Readers who share our analysis and / or who take a purely proof-theoretic point of view may thus miss benefits of the approach, e.g. computational ones. Conceptually, we believe that the theory will prove useful in some respects. Deriving a novel definition for conceptual equivalence is one candidate for this. Another is the provision of justifications or rejections of certain formalizations. To name an example, if one were to formalize a category *entity* as the category which classifies everything (including sets), a predicate Entity could not be introduced meaningfully in line with formula (10) since this would contradict the well-foundedness of standard set theories.

The second problem of providing an abstract core ontology has been addressed in the previous section by outlining an ontology of categories and relations. This is a proposal of one potentially suitable ontology rather than its "unique solution", and different such proposals should be compared and evaluated. In general, ontological semantics leads to the fact that comparisons of two ontology representations $R(\Omega_1)$ and $R(\Omega_2)$ with an ontological semantics must determine one of the compared systems as a point of reference. Considering the use of a third ontology $R(\Omega_{ref})$ in this respect does not differ significantly, because then embeddings of the $R(\Omega_i)$ into $R(\Omega_{ref})$ are required – which is a case of the first kind.

Another important aspect of the general approach is its non-reductionism. In particular, we consider everything in an ontological model to be on a par with each other. The use of an abstract core ontology as a means to *analyze* entities and to initiate formalizations is not to be understood as a *reduction* to notions in the abstract core ontology, neither for its categories nor relations. Metaphorically, it is not sufficient to think of leo as an individual (at the abstract core level), nor as a lion (at a domain level), but leo is only fully recognized as leo, and is analyzable and related to other entities.

5.2. Related Work

There is an overwhelming amount of broadly related work, e.g. meta-modeling in conceptual modeling, works based on situation theory [17,24] and information flow theory [25,26] as well as approaches to intensional logics like Montague's, Tichý's, cf. [27], and George Bealer's [28]. From the perspectives of these fields, our approach originates from "practical" concerns in representing foundational ontologies like GFO, whereas establishing detailed connections to them is an ongoing effort. Situation theory in its origins is currently the most promising candidate regarding a tight linkage and particular aspects of its motivations. Nevertheless, basic differences remain there, as well, e.g., a built-in "ontology" of primitives (individuals, relations, space-time locations) plus situations and their construction from these primitives, and a set-theoretic metatheory.

Concerning knowledge representation, we focus on the approach of *Ontology-Based Semantics* by Ciocoiu and Nau [5]. It shares its motivation and goals with ours, and we agree with most of the analysis in its Sections 1 and 2, leading to the use of an ontology as a common semantic foundation. However, Problem 1 (circularity) is not identified in [5], but a classical FOL representation $R(\Omega) \subseteq \mathcal{L}_\Omega$ of the ontology is used. For defining ontology-based models of a language L, the authors use a two-step translation, (i) from L into a FOL language $\mathcal{L}$, and (ii) an interpretation in the sense of [19, Sect. 2.7] from $\mathcal{L}$ into $\mathcal{L}_\Omega$. We have reservations about both steps. For (i), this requires an encoding of ontological notions into set theory, which may differ for the same ontological notions contained in distinct languages. Some of these encodings cannot be unified in the second step, because those interpretations maintain the number of free variables in interpreted predicates, which prevents switching from a constant $lion$ to a predicate $Lion$, for instance. On the other hand, (ii) may allow for too strong encodings in other respects.

Finally, note that recent language proposals (e.g. Common Logic [29]; [30] in description logics) relate to our approach. They allow for syntactic expressions which seemingly require a classical higher order semantics, like in the theory $\{P(R), R(x, y)\}$. These languages have a non-standard set-theoretic semantics with parallels to our FOL approximations. It is promising to use the syntax of these languages with an ontological semantics, or to build approximations due to their semantics. However, the method of linking all predicates (more generally, composed syntactic expressions) to "intensional specifications" is not enforced elsewhere. It should be added since it is of major relevance for appropriate definitions of conceptual equivalence.

6. Conclusions

In this paper we have argued that it is insufficient to rely on conventional formal semantics when representing ontologies, due to their foundational role with respect to semantics. We proposed a new type of model theoretic semantics called *ontological semantics* together with an approximation for FOL syntax in order to utilize existing work, resulting in an additional, translation-based definition. Complementarily, we have outlined an ontology of categories and relations as one meta-ontological option for applying this semantics to the formalization of ontologies.

Essentially, our approach for formalizing ontologies should add a level of explanation to the encoding of ontological into abstract mathematical notions. We expect that this leads to "meaning-preserving translations" which may appear conceptually more adequate than conventional formal reductions (which are undoubtedly very valuable in other respects). This should influence working with ontologies, in particular ontology matching and integration. Moreover, the resulting theories offer specific properties which may be exploited, e.g. for modularly structured ontologies.

Future work comprises further studies of the proposed structures and their approximations. Furthermore, the general approach requires to explore its application based on different logics and the use of different abstract core ontologies. Instead of unique final solutions, for all parameters we expect a plurality of options which should compete with respect to practical applications.

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