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Problem Set 10 for "Automata Theory"

Deadline: Monday, June 24, 13:15h

H 10-1: Consider the following two structures:

(a) $(\mathbb{N} \cup \{\infty\}, \min, +)$.

(b) $(2^X, \cap, \triangle)$ for the power set 2^X of an arbitrary set X and the symmetric difference defined as $A \triangle B := (A \cup B) \setminus (A \cap B)$.

One is a semiring while the other is not. For the semiring, prove that it is a semiring. For the other structure, show why it is not a semiring.

H 10-2: Show that in a semiring there cannot exist any other element z besides 0 that satisfies

$$\forall x[xz = zx = z].$$

H 10-3: Give rational expressions for the following languages:

(a) $L(\forall x \exists y(P_a(x) \to y = x + 1 \wedge P_a(y))$

(b) $L(\exists x \forall y(P_a(x) \to y = x + 1 \wedge P_a(y)))$.

(c) $L(\forall x \forall y[y = x + 1 \to (P_a(x) \leftrightarrow P_b(y))])$.

S 10-1: Al Capone has a network of friends who help him launder his black money. They are connected by international money transfers, cash withdrawals etc. Every type of transaction has a limitation for the maximum amount that can pass through, for example the daily limit for cash withdrawal. Now Al Capone wants to know through which series of transactions he can bring the highest amount of money in one pass from his black money account to his regular account. Propose a semiring to model this with a discrete system.

All answers must be proven.