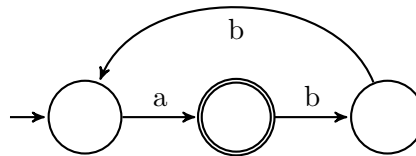


Problem Set 9 for “Automata Theory”

Deadline: Monday, June 17, 13:15h

H 9-1: Construct the MSO sentence according to the proof of Theorem 4.4 that describes the language that is recognized by the following automaton:



H 9-2: We use the notations of the excursion on pages 47/48 of the lecture notes. Let

$$\psi : [A \rightarrow [B \rightarrow C]] \rightarrow [(A \times B) \rightarrow C]$$

with $f \mapsto \bar{f} : A \times B \rightarrow C$ such that $\bar{f}(a, b) := f(a)(b)$.

Show that ψ is bijective.

H 9-3: Let $\mathcal{V} = \{x, y, X\}$ as in the examples on page 49 of the lecture notes. For how many words $v \in (A_{\mathcal{V}})^4$ is there an assignment σ such that v corresponds to some $(abac, \sigma)$?

The solution to the following problems should be prepared but is not handed in:

Recall from Problem Set 7 that the *shuffle* of two languages L_1 and L_2 is the language

$$\text{shuffle}(L_1, L_2) := \{u_1 v_1 \dots u_n v_n : u_1 \dots u_n \in L_1, v_1 \dots v_n \in L_2, n \geq 1\},$$

where $u_1, v_1, \dots, u_n, v_n$ can be arbitrary words.

S 9-1: Let φ_1 and φ_2 be MSO sentences over the same alphabet. Find an MSO sentence that defines the shuffle of $L(\varphi_1)$ and $L(\varphi_2)$.

S 9-2: Prove Lemma 4.8 by constructing an automaton for $N_{\mathcal{V}}$.

S 9-3: Complete the proof of Lemma 4.9. Give an automaton recognizing $L(x \in X)$.

All answers must be proven.