

Problem Set 8 for “Automata Theory”

Deadline: Monday, June 10, 13:15h

H 8-1: Give MSO formulas for the following languages:

- (a) $\{w : aba \text{ is a factor of } w\}$.
- (b) $\{w : \text{every factor } bb \text{ is followed by } a \text{ before the next occurrence of } bb\}$.
- (c) $\{w : |w| \text{ is a multiple of } 3\}$.

H 8-2: Give rational expressions for the following languages:

- (a) $L(\forall x \forall y [(P_a(x) \wedge x = y + 1) \rightarrow P_b(y)])$.
- (b) $L(\exists x \exists y [P_a(x) \wedge P_b(y) \wedge x = y + 1 \wedge \forall z (z \neq x \rightarrow x \leq z)])$.
- (c) $L(\forall x \exists y [P_a(y) \wedge \forall z (z \in X \rightarrow y \leq z)])$.

H 8-3: Give FO sentences that describe the same languages as the following MSO sentences:

- (a) $\forall X \forall x \forall y \forall z [(x \in X \wedge y \in X \wedge z \in X \wedge x \neq y \wedge x \neq z \wedge z \neq y) \rightarrow (P_a(x) \vee P_a(y) \vee P_a(z))]$.
- (b) $\exists X \exists Y \forall x \forall y [(x \in X \wedge y \in Y) \rightarrow (x \leq y \wedge P_b(x) \wedge P_a(y))]$.
- (c) $\forall X \forall x [(x \in X \rightarrow P_a(x)) \rightarrow \exists y (P_b(y) \wedge \forall z (z \in X \rightarrow P_b(z)))]$.

The solution to the following problems should be prepared but is not handed in:

S 8-1: Let φ be a sentence in MSO logic. Find a sentence in MSO logic for $L(\varphi)^*$.

S 8-2: Let φ_1 and φ_2 be two FO sentences. Define the language $L(\varphi_1) \cdot L(\varphi_2)$ by an FO sentence.