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Problem Set 7 for "Automata Theory"

Deadline: Monday, June 3, 13:15h

H 7-1: As you have shown in Problem H 6-2, the following languages are star-free:

(a) a^*b^*,

(b) $(abc)^*$,

(c) $\{w : w$ contains at most 3 $a\}$,

(d) $\{w : w$ has aba exactly twice as subword$\}$.

Thus, according to Theorem 3.5 they are also aperiodic. Find the index of each of these languages.

H 7-2: Which of the following monoids are aperiodic?

(a) $(\mathbb{N}, +)$.

(b) $(\mathbb{N}, max)$.

(c) $(\{0, 1, 2, \ldots, n\}, +_n)$ where $x +_n y = n$ if $x + y > n$; else $x +_n y = x + y$.

(d) The flip-flop monoid. This is the smallest monoid with two right-zero elements. Thus it"remembers" the last non-neutral of two possible inputs like an electronic flip-flop. In detail, $(\{1, A, B\}, \Diamond)$ with neutral element 1 and $A \Diamond B = B \Diamond B = B$ and $B \Diamond A = A \Diamond A = A$.

H 7-3: Let L and K be aperiodic languages with $n = i(X) + i(Y) + 1$. Further let x, y, and z be words such that $xy^{n+1}z \in LK$. Show that also $xy^n z \in LK$. (this completes the proof of Theorem 3.5)

The solution to the following problems should be prepared but is not handed in:

S 7-1: Which of the following languages are star-free?

(a) $\{ab, ba\}^*$.

(b) $\{ab, bc, ca\}^*$.

(c) $\{ab, bc, ac\}^*$.

Give a star-free expression for the ones that are.

S 7-2: The *shuffle* of two languages L_1 und L_2 is the language

$$shuffle(L_1, L_2) := \{u_1 v_1 \ldots u_n v_n : u_1 \cdots u_n \in L_1, v_1 \cdots v_m \in L_2, n \geq 1\},$$

where $u_1, v_1, \ldots, u_n, v_n$ can be arbitrary words.

Show that the shuffle of two regular languages is regular. Show that $(aba)^*$ and $(cbc)^*$ are star-free, but their shuffle is not.

All answers must be proven.