Gábor Braun and Peter Leupold

SS 2013
Institut für Informatik
Universität Leipzig

Problem Set 6 for "Automata Theory"

Deadline: Monday, May 27, 13:15h

H 6-1: The *right syntactic congruence* of a language L in a monoid M is defined by

$$y \backsim_L y' :\Leftrightarrow \forall x \in M : yx \in L \Leftrightarrow y'x \in L.$$

Give the congruence classes of the language $\{a(ba)^i : i \geq 0\}$ for the syntactic congruence and also for the right syntactic congruence.

Give the minimal automaton that accepts L.

H 6-2: A rational language is called *star-free* if it can be described by a rational expression that does not use the Kleene-star but can additionally use the complement. Give a star-free expression for the following languages over the alphabet $\{a, b, c\}$:

(a) $a^* b^*$,

(b) $(abc)^*$,

(c) $\{w : w$ contains at most 3 $a\}$,

(d) $\{w : w$ has aba exactly twice as subword$\}$.

H 6-3: Determine the syntactic monoid of the language $\{a, b\}^* ab$.

The solution to the following problem should be prepared but is not handed in:

S 6-1: Let M be the syntactic monoid of the language $\{a^n b^n \mid n \geq 1\}$ over the alphabet $\{a, b\}$. Prove that the languages $\{[a]\}$ and $\{[b]\}$ in M are recognizable, but not their product.

S 6-2: Prove the following statement:

For every language over the free monoid the right syntactic congruence is of finite index if and only if the syntactic congruence is of finite index.

All answers must be proven.