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SS 2013
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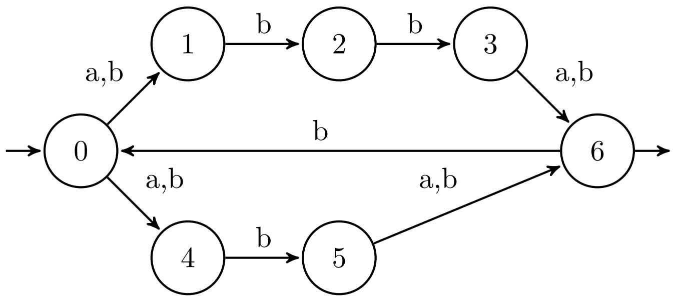
Problem Set 5 for "Automata Theory"

Deadline: Tuesday, May 21, 9:15h

H 5-1: Consider the morphism $f : (\{a, b, c\}^*, \cdot) \mapsto (\{ab, aab\}^*, \cdot)$ defined by $f(a) = ab$, $f(b) = aab$ and $f(c) = ab$. Show that f is a morphism. What is the congruence that f induces on $(\{a, b, c\}^*, \cdot)$ (cf. Example 2.3)? Prove that this is a congruence without referring to results from the lecture.

H 5-2: Determine the minimal automaton of the following automaton:



Note that the automaton above is not complete.

H 5-3: Let M be a monoid. Show that for L, L_1, and L_2 from $Rec(M)$ also $L_1 \cap L_2 \in Rec(M)$ and $L^c \in Rec(M)$. Use automata constructions that provide an alternative to the proof of Corollary 2.15 in the lecture notes.

The solution to the following problems should be prepared but is not handed in:

S 5-1: Determine $Rec(\mathbb{Z}, +)$.

Find a monoid M such that $Rec(M) = \{\emptyset, M\}$.

S 5-2: Consider the language $L = \{(a^n b^n, c^n) : n = 0, 1, ...\}$ over the monoid $\{a, b\}^* \times c^*$ with component-wise concatenation. Write it as an intersection of rational languages.

Prove that the complement of L is rational, however L itself is not rational. Note that this shows that the statement from Problem H 5-3 does not hold for rational languages.

All answers must be proven.