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SS 2013
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Problem Set 4 for "Automata Theory"

Deadline: Monday, May 13, 13:15h

H 4-1: Prove Theorem 2.4. : (a) Let A be a set, M a monoid, and $f : A \mapsto M$ a mapping. Then there exists a morphism $g : A^* \mapsto M$ with $g|_A = f$. Moreover, g is determined uniquely by these properties.

(b) Let M be any monoid. Then there exist a set A and an epimorphism $g : A^* \mapsto M$.

H 4-2: Find a finite (N, max)-automaton recognizing the language $\{2, 5\}$.

H 4-3: Give the syntactical monoid of the following languages:

(a) $\{a, aaa\}^*$ in the monoid $\{a\}^*$.

(b) $\{ab\}^*$ in the monoid $\{a, b\}^*$.

(c) $\{w : |w|_a = |w|_b\}$ in the monoid $\{a, b\}^*$.

(d) $\{5\}$ in the monoid $(\mathbb{Z}, +)$.

(e) $\{(n, n) : n \in \mathbb{N}\}$ in the monoid $(\mathbb{N}, +)^2$.

The solution to the following problems should be prepared but is not handed in:

S 4-1: How many submonoids does the monoid $(\{0, 1, 2, 4\}, max)$ have?

How many submonoids does the monoid $(\mathbb{N}, +)$ have?

S 4-2: Consider the mapping $|\dots|_a$ that gives the number of occurrences of the letter a in a word over the alphabet $\{a, b\}$. Show that it is a morphism. Which is the congruence it induces on $\{a, b\}^*$?

Is the mapping $|\dots|_{ab}$ that gives the number of occurrences of factors ab in a word over the alphabet $\{a, b\}$ a morphism, too?

S 4-3: (a) Let $A = \{a, b, c, d\}$, M the monoid $(\mathbb{N}, +)$, and f the mapping given by $a \mapsto 1$, $b \mapsto 1$, $c \mapsto 2$, and $d \mapsto 3$. What is the morphism g specified in Theorem 2.4.(a)?

(b) Let $A = \{a, b, c\}$, M the monoid $(\mathbb{N}, \cdot)$, and f the mapping given by $a \mapsto 2$, $b \mapsto 3$, and $c \mapsto 5$. What is the morphism g specified in Theorem 2.4.(a)?

Also prove that your morphisms are really morphisms.

All answers must be proven.