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SS 2013
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Problem Set 2 for "Automata Theory"

Deadline: Monday, April 29, 13:15h

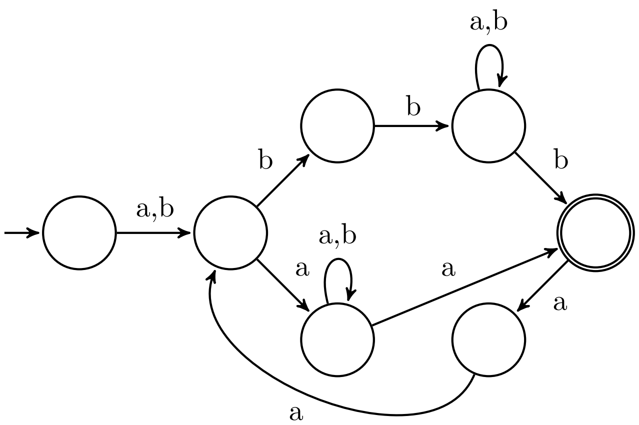
H 2-1: Let L_1, L_2 and L_3 be languages that do not contain the empty word. Show that the following statements are equivalent:

$$L_1 = L_2 \cup L_1 L_3 \quad \text{und} \quad L_1 = L_2 L_3^*$$

Further, show that these similar statements are equivalent, too:

$$L_1 = L_2 \cup L_3 L_1 \quad \text{und} \quad L_1 = L_3^* L_2$$

H 2-2: Give a rational expression for the language recognized by the following automaton:



H 2-3: Prove the Pumping Lemma:

Let A be a finite automaton. Then there exists a number n such that every $w \in L(A)$ with $|w| > n$ has a factorization $w = xyz$ with $|xy| < n$ and $y \neq \epsilon$ such that $xy^k z \in L(A)$ for all $k \geq 0$.

Note: The Lemma's wording here is slightly different from the one in the Lecture (Lemma 1.11); proving either one is fine.

The solution to the following problems should be prepared but is not handed in:

S 2-1: Use the Pumping Lemma (Problem H 2-3) to show that the language $\{a^n b^n : n > 0\}$ is not recognizable.

S 2-2: Regular expressions in programming languages are derived from rational expressions, but often have more operators like the complement. Give rational expressions (without using the complement) for the following sets over the alphabet $\{a, b\}$:

a) $\overline{\{a\}}$ b) $\overline{\{ab\}}$ c) $\overline{\{ab\}^*}$

All answers must be proven.