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SS 2013
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Problem Set 13 for "Automata Theory"

Deadline: Monday, July 15, 13:15h

H 13-1: Find a weighted automaton with only two states over the semirings $(\mathbb{Z}, +, \cdot, 0, 1)$ and $(\mathbb{Z} \cup \{-\infty\}, max, +, -\infty, 0)$ whose behavior S is:

$$(S, a^{n_1} b a^{n_2} b \cdots b a^{n_1}) = \sum_{i=1}^{k} (-1)^i n_i,$$

where n_1, n_2, ...,$n_k \geq 0$ and $k \geq 1$.

Also find rational expressions for S over each of the semirings.

H 13-2: Recall that the Hadamard product of two formal power series S_1 and S_2 is the pointwise product: $(S_1 \odot S_2, w) = (S_1, w) \cdot (S_2, w)$. For two weighted automata $\mathcal{A}_1$ and $\mathcal{A}_2$, construct an automaton with the behavior $||\mathcal{A}_1|| \odot ||\mathcal{A}_2||$.

H 13-3: The sequence of Fibonacci numbers is defined recursively as follows: $F_0 := 0$, $F_1 := 1$ and $F_k := F_{k-1} + F_{k-2}$ for $k \geq 2$. So the start of the sequence is $0, 1, 1, 2, 3, 5, 8, 13, 21$.

Find a weighted automaton $\mathcal{A}$ with the behavior

$$(||\mathcal{A}||, a^k) = F_{k+1}$$

over the semiring $(\mathbb{N}, +, \cdot, 0, 1)$ and the unary alphabet $\{a\}$.

The solution to the following problem should be prepared but is not handed in:

S 13-1: A *palindrome* is a word that is the same read from the left or from the right, for example "lagerregal" or "racecar." Formally $w = w^R$, where w^R denotes the reverse of w. Show that the language of all words that are not palindromes over $\{0, 1\}$ is the support of a rational series over the semiring $(\mathbb{Z}, +, \cdot, 0, 1)$.

Hint: First show that

$$\sum_w [w]_{bin} w$$

is rational, where $[w]_{bin}$ denotes the natural number corresponding to the interpretation of the word w as a binary number.

All answers must be proven.