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SS 2013
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Problem Set 12 for "Automata Theory"

Deadline: Monday, July 8, 13:15h

H 12-1: Determine the behavior of the weighted automaton with the following representation over the semiring $(\mathbb{N} \cup \{-\infty\}, max, +, -\infty, 0)$ and the alphabet $\{a, b\}$:

$$\lambda = \begin{bmatrix} 0 & -\infty \end{bmatrix} \qquad \gamma = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\mu(a) = \begin{bmatrix} 0 & 1 \\ -\infty & 1 \end{bmatrix} \qquad \mu(b) = \begin{bmatrix} 0 & -\infty \\ -\infty & -\infty \end{bmatrix}.$$

H 12-2: Let $\{S_i\}_{i\in I}$ and $\{T_j\}_{j\in J}$ be locally finite families of formal power series over an arbitrary semiring. Prove that

$$\left(\sum_{i\in I} S_i\right) \cdot \left(\sum_{j\in J} T_j\right) = \sum_{\substack{i\in I \\ j\in J}} S_i \cdot T_j.$$

Also show that the right-hand side is a sum of a locally finite family.

H 12-3: Find rational expressions for the power series from **H 11-3** over the alphabet $\{a, b\}$ and over one of the semirings (each) which you used there:

(a) $(||\mathcal{A}||, w) = |w|_{ba}$.

(b) $(||\mathcal{A}||, b) = 5$, $(||\mathcal{A}||, bb) = 3$, and $(||\mathcal{A}||, b^k) = 4$ for $k > 2$; $(||\mathcal{A}||, w) = 0$ else.

The solution to the following problems should be prepared but is not handed in:

S 12-1: Prove the following: Let S_1 and S_2 be two formal power series where S_1 is proper. Then the equation $S = S_2 + S_1 S$ has the unique solution $S = S_1^* S_2$.

All answers must be proven.