Gábor Braun and Peter Leupold

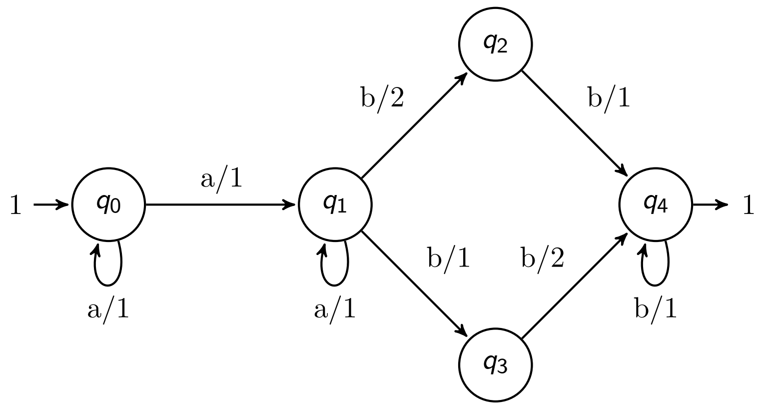
SS 2013
Institut für Informatik
Universität Leipzig

Problem Set 11 for "Automata Theory"

Deadline: Monday, July 1, 13:15h

H 11-1: Prove that the structure $(K^{n\times n}, +, \cdot, 0, E)$ is a semiring. Here $K^{n\times n}$ is the set of all $n \times n$ matrices over an arbitrary semiring K. The operations $+$ and $\cdot$ are the common matrix addition and multiplication; E is the unit matrix.

H 11-2: Determine the behavior of the following weighted automaton over the alphabet $\{a, b, c\}$ and the semiring $(\mathbb{N}, +, \cdot, 0, 1)$:



H 11-3: Construct weighted automata $\mathcal{A}$ with the following behaviors over the alphabet $\{a, b\}$:

(a) $(||\mathcal{A}||, w) = |w|_{ba}$.

(b) $(||\mathcal{A}||, b) = 5$, $(||\mathcal{A}||, bb) = 3$, and $(||\mathcal{A}||, b^k) = 4$ for $k > 2$; $(||\mathcal{A}||, w) = 0$ else.

Choose appropriate semirings and construct two automata over different semirings for each behavior.

The solution to the following problems should be prepared but is not handed in:

S 11-1: Show that for any given weighted automaton over the alphabet A and the semiring $(\mathbb{N}, +, \cdot, 0, 1)$ the following language is recognizable:

$$\{w : w \in A^* \ \wedge\ (||A||, w) \neq 0\}.$$

All answers must be proven.