

Modelling Sustainable Systems and Semantic Web

Immersive and Submersive System Theories

Lecture in the Module 10-202-2309
for Master Computer Science

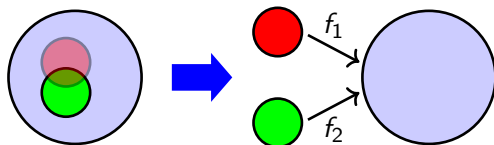
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Relation Between two Systems in a Supersystem

How can the relationship between two systems be conceptualised in their environment?

According to our systems approach, only if we describe *a larger system* (supersystem) and the relation of the two subsystems to the supersystem as *components*.



Mathematical formulation of the question

We look for functions $f_1 : S_1 \rightarrow S$, $f_2 : S_2 \rightarrow S$ with certain properties.

For a "generic solution" we ask if for such a constellation exists a **Universal Categorical Object**, i.e. a universal U and universal maps

$$p_1 : S_1 \rightarrow U, \quad p_2 : S_2 \rightarrow U,$$

such that for each triple (f_1, f_2, S) the above constellation may be written as

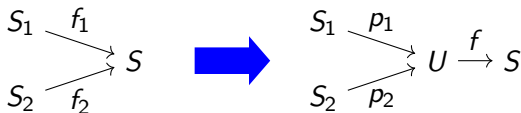
$$f_1 = f \circ p_1 : S_1 \rightarrow U \rightarrow S, \quad f_2 = f \circ p_2 : S_2 \rightarrow U \rightarrow S$$

for a suitable $f = f_1 \oplus f_2 : U \rightarrow S$.

Immersive Concept

U is in some sense the "most general system" that combines the systems S_1 and S_2 without "further effect".

U in such a case is called a **direct sum** and we write $U = S_1 \coprod S_2$.



Mathematical Categories

Most mathematical models live in concrete **categories**, for example, the category of sets, vector spaces, fibre bundles, algebraic varieties, and so on.

Each such category is characterised by the fact that the terms **object** and **morphism** have a clear meaning there.

Morphisms between vector spaces, for example, are operationally faithful mappings, i.e. linear mappings that can be described by matrices for finite-dimensional vector spaces.

Such universal objects do not exist in every category.

Remark: The construction can easily be extended to finitely many S_i and even to infinitely many $S_i, i \in I$, and so it is defined in mathematics.

Category of Sets

In this category direct sums U exist for both finite and infinite index sets I . This is just the **disjunct union** of the sets S_i .

The maps p_i are just the embeddings $p_i : S_i \rightarrow U$ of the partial sets in their disjunct union.

The map $f : U \rightarrow S$ works as follows: For each $a \in U$ exists exactly one i and one $a' \in S_i$ with $a = p_i(a')$. Put $f(a) = f_i(a')$.

If $|S_1| = a, |S_2| = b$, then $|S_1 \coprod S_2| = a + b$.

The whole is no more than the sum of its parts.

Invert all Arrows (TRIZ Prinzipie 13)

We look for functions $f_1 : S_1 \leftarrow S$, $f_2 : S_2 \leftarrow S$ with certain properties.

Does for such a constellation exists a **Universal Categorical Object**, i.e. a universal U and universal maps

$$p_1 : S_1 \leftarrow U, \quad p_2 : S_2 \leftarrow U,$$

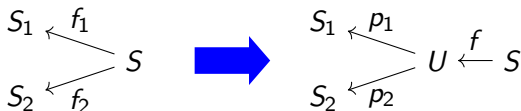
such that for each triple (f_1, f_2, S) the above constellation may be written as

$$f_1 = p_1 \circ f : S_1 \leftarrow U \leftarrow S, \quad f_2 = p_2 \circ f : S_2 \leftarrow U \leftarrow S$$

for a suitable $f = f_1 \otimes f_2 : S \rightarrow U$.

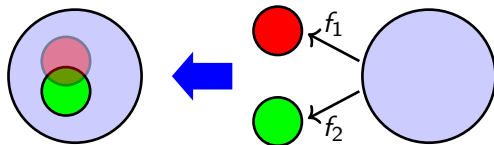
Submersive Concept

U in such a case is called a **direct product** and we write $U = S_1 \amalg S_2$.



Submersive Concept

How does this change the perspective on the concept of system?



Category of Sets

In this category direct products U exist for both finite and infinite index sets I . This is just the **cartesian product** of the sets S_i .

The maps p_i are just the projections $p_i : U \rightarrow S_i$ of the product to the individual components.

The map $f : S \rightarrow U$ works as follows: For each $a \in S$ we set $f(a) = (f_i(a)) \in U$.

If $|S_1| = a, |S_2| = b$, then $|S_1 \amalg S_2| = a \cdot b$.

The whole is clearly more than the sum of its parts, most of the "information" is of relational nature.

Submersive and Immersive System Theories

System theories rarely make a distinction between between these two approaches.

To distinguish between the approaches, system theories in which the first modelling principle dominates, are called **immersive system theories**. They can be recognised their constructions are essentially based on embeddings (immersions).

System theories that are based on the second modelling principle are called **submersive system theories**. They can be recognised by the fact that their constructions are essentially based on projections (submersions) and thus on processes of staggered complexity reduction.

The Theory of Dynamical Systems is a submersive system theory.