The Marxian transformation problem revisited

Peter Fleissner, Vienna, Austria

Abstract

In the 20th century there was an unbridgeable gap between the "right" and "left" schools of economics with respect to the determination of value of commodities. While Marxian scholars insisted, that human labour being the essence of value to have a good argument for demonstrating exploitation, mainstream economists in the West focused on marginal utility theory to keeping up their basic axiom of methodological individualism. In contrast to this intellectual battlefield of the past century this paper demonstrates how the Marxian labour theory of value and neo-classical economic analysis can be used simultaneously to see the transformation problem under a new, and may be joint perspective.

Introduction

Many Marxists understand the transformation problem as the transformation of labour values into prices of production, but more appropriate and by contemporary scholars it is understood as the transformation of one system of relative prices into another where profit rates are equal in all industries of the economy.[1]

My point of view is a more comprehensive one. While I am in accordance with the position that the transformation done by Marx is dealing with two systems of relative prices, I still would like to link it to the realm of value. In my understanding one could start from a Gedankenexperiment, where we create ideal types (Max Weber) of economies controlled by different rules and try to compare them. But to be able to compare different systems we need a level of comparison where the same entity or indicator is used, and at the same moment there must be some difference between the systems. In the particular case of the transformation problem the difference can be found in the rules governing the behaviour of the enterprises in the economy.

In my opinion there are two different types of economies to be compared. The first one could be imagined as an economy of small commodity producers applying nothing more than their labour power to produce commodities for the market. Their system of relative prices is such that the individual producers receive a certain amount of currency units proportional to labour time they have spent directly and indirectly to produce the product. If they would receive more revenue from the market, other producers would enter the market and offer additional products for a cheaper price. Only if the prices of products are proportional to the labour time, this economy is assumed to be in equilibrium.

The term proportional is a crucial one. It allows us changing the units of measurement and to establish a system of relative prices (measured in currency units) instead of labour values (measured in labour time). While in labour theory of value (Volume 1 of Das Kapital) the universal unit is abstract labour time spent in the production process (more precisely socially necessary labour time needed for the production of a unit of output in the average), on the observable surface of the economy money represents labour values by a certain amount of currency units proportional to labour spent. If we assume the proportionality of labour values and prices, we are able to apply the Marxian concept of exploitation and profits as appropriation of labour time without compensation (beyond reproduction cost of labour).

The second type of the economy is an ideal type capitalist economy under perfect competition. The Marxian assumption for such an economy is that the system of relative prices allows the capitalists to gain equal profits relative to the capital they have advanced. Marx believed that there will no longer be any migration capital from one sector to the other because everywhere in the economy the profitability would be the same. He called the prices under this condition "prices of production" (Produktionspreise).

[1] For a recent review of the transformation problem see Kepa M. Ormazabal, The Transformation of Value into Competitive Price: rescuing Marx' Value Theory from Historical Misinterpretation, 3rd version: 13 February 2004, www.daskapital.org/files/04Ormanazabal.doc; for multi-level analysis see Fleissner, P. et al (1993) Input-Output-Analyse - Eine Einführung in Theorie und Anwendungen (Input-output analysis - an introduction into theory and applications) Vienna: Springer Verlag ISBN 3-211-82435-9

Marx' and Bortkiewicz solution

Marx solved the transformation problem by starting with an economy of small commodity producers. Their output is priced proportional to the content of labour spent. To approximate a capitalistic price system he determined first the overall surplus value of the economy, second the value of total capital advanced. The quotient of the two is the average rate of profit of the economy. To end up with capitalist mark-up pricing he defined the price of one unit of output by adding the average rate of profit per unit of capital advanced to the cost price of one unit of output. The problem with his solution is that the prices of the inputs of the commodities are different from the prices of the output. But in my opinion this does not create a big difficulty, because if one iterates the Marxian procedure by using output prices in a second round as input prices, determining the rate of profit resulting from the second iteration, fixing the new output prices in the same way as before, after some iterations one ends up at a solution where the prices and the rates of profit of the sectors will remain invariant. In a Leontief type economy with input-output matrices one can show that these prices represent the eigenvectors of certain matrices describing the economy, and the possible rate of accumulation is a function of the eigenvalue associated to the eigenvector of relative prices. This solution is identical with the solution that Ladislaus von Bortkiewicz[2] who criticized Marx for his "error" found exactly 100 years ago. In my understanding one can easily correct the Marxian solution of the transformation problem by applying his own method iteratively.

Nevertheless, famous mainstream economists still are not convinced that there is any link between labour values and prices. To illustrate this, I quote the cynical comment by Nobel Laureate Paul Samuelson:[3]

"The traditional transformation problem (...) has frequently been regarded as a vindication of Marx's Volume I analysis. However, direct and simple substitution (…) shows that (…) Bortkiewicz algorithm (…) can be described logically as the following procedure: "(1) Write down the value relations; (2) take and eraser and rub them out; (3) finally write down the price relations –thus completing the so-called transformation process"."

This gives us evidence that for one century there was and still is an irreconcilable contradiction between the marginalist school and Marxian economists.[4] But is this mutual opposition really justified at the level of the transformation problem?

As we have learned from Marx and also from philosophers of science abstraction is the most important tool we can apply to end up with a scientific description of our world. An example should illustrate this idea. Evidently, when we observe the fall of a leaf of a tree or a stone, there is a big difference in the velocity to fall to earth. While in reality the leaf is floating in the air and moving randomly in various directions, Newton's mathematical formulation of the law of gravitation teaches us – contradicting empirical evidence - that a "tenuous feather and solid gold fall with equal velocity". To end up with the law of gravitation in physics we have to think away the specific conditions of aerodynamics and friction by the method of abstraction. It needed the experiments of Robert Boyle to practically showing the correctness of Galileo's law on falling bodies in 1659. Boyle could do it by evacuating a tube to get rid of air resistance. This was a practical move following theoretical abstraction.

In social sciences in most cases it is not possible to practically get rid of side conditions of economies or specific societies. Therefore the only possibility available to us is to do abstraction on the level of thought. Marx has taught us that abstract human labour is the essence of the value of commodities. Volume 1 of Das Kapital is full with arguments about that. But reality consists of more than essence only. It shows us a surface – the level of appearance - which we can investigate empirically. The full research process is not completed by understanding the essence. It needs a step by step enrichment of the abstract essence up to the surface of observable appearance. Essence is only a skeleton, while the visible surface is carrying flesh and skin. Theory allows us to look through the surface and to

[2] There are three consecutive papers by Ladislaus von Bortkiewicz on this issue on "Wertrechung und Preisrechung im Marxschen System": The first one 1906 in Archiv für Sozialwissenschaft und Sozialpolitik, Vol. XXIII, Heft 1, 1906, pp 1-50, the last two in 1907, op. cit, Vol. XXV, pp 10-51 and pp 445-488. See also an English version: Bortkiewicz, Ladislaus von (1907): "*On the Correction of Marx' Fundamental Theoretical Construction in the Third Volume of* Capital", in Sweezy, Paul M. (ed.) (1949): "*Karl Marx and the Close of His System*", Augustus M. Kelley: New York.

[3] Samuelson, Paul A. (1970): "*The Transformation from Marxian "Values" to Competitive "Prices": A Process of Rejection and Replacement*", in "*The Collected Scientific Papers of Paul A. Samuelson. Volume III*", edited by Robert C. Merton, The M.I.T. Press: Cambridge, MA and London, England.

[4] Stephen A. Resnick and Richard D. Wolff (eds): New Departures in Marxian Theory (in the series: Economics As Social Theory), Routledge 2006, pp 253-305

identify main governing principles behind it, but to understand observable phenomena thoroughly we have to add non-essential, but necessary features.

What has this excursion to do with the transformation problem? In my understanding Marxian analysis is located on the level of the essence, and it is right to do so. But if we want to apply it to actually existing economies empirically, we have also to add some non-essentials. And here we can include marginalist theories of supply and demand, frequently connected with the notions of utility and marginal utility. This enrichment of the essence is necessary to end up on a level closer to empirical reality. Nevertheless, even with this extension we are far away from the surface we actually can see. In this paper we still do not include financial markets, we neglect the function of credits, we do not cover monopolistic power etc. Much additional work can be done to enrich the essence for a more comprehensive reconstruction of the surface and by that to complete the cycle from surface down to essence and back to surface again.

In this paper I add only one additional feature to Marx' essence of labour values: Changes in the demand of physical goods caused by price variations. Before the transformation, the system of relative prices is proportional to labour values. After the transformation the relative prices represent a competitive price system where profit rates are equalized, and simultaneously effects on the demand of consumer goods will be determined.

Example

I will illustrate this idea by a small mathematical example in a Leontief economy using an input-output matrix of dimension 2. Let us start with the description of the economy in input-output terms with a price system proportional to labour values: The elements of the matrices **A** diag(**x**), **c**, **inv**, and **x are** have the dimension of goods produced by industry, while the elements of **w**, **m**, **L** and **p** diag(**x**) are given in labour values.

A diag(**x**)

a_{11}	a_{12}
a_{21}	a_{22}

c

c_1
c_2

inv

inv_1
inv_2

x

x_1
x_2

w

w_1	w_2

L (labour input)

70	70

m

m_1	m_2

p diag(**x**)

$p_1\ x_1$	$p_2\ x_2$

We assume prices before the transformation to be proportional to labour values **p**

$$\mathbf{p} = f\,\mathbf{l}\,(\,\mathbf{E} - \mathbf{A}\,)^{-1},$$

where $\mathbf{l} = \mathbf{L}\ \mathrm{diag}(\mathbf{x})^{-1}$ is the direct labour input per unit of output and f is a constant scalar to illustrate the different dimension of **l** and **p** (we assume it f = 1). Wages shall be able to buy all the consumer goods available:

$$\mathbf{w1} = \mathbf{p}\ \mathbf{c}\ .$$

It is assumed that workers produce more than they consume, i.e. $wj_i \leq l_j$.

In numerical terms we assume

A diag(**x**)

1	0
20	30

c

1,5
21

inv

7,5
29

x

10
100

w

20	16

50	54

L (labour input)

70	70

m

p diag(**x**)

100	100

I define matrices **C** and **S** in analogy to matrix **A**, the matrix of technical coefficients. The elements of the matrices give the amount of physical goods related to one unit of output. The relative prices are given by a row vector **p**, the volumes by a column vector **x**. E.g. a consumption matrix **C** can be constructed by

C = (**c w**) **/ w1** . diag(**x**)$^{-1}$**.** Consumption by workers of one industry divides available consumption **c** proportional to the wage fraction **w / w1** available to the workers of that industry.

Multiplication by diag(**x**)$^{-1}$ transforms consumption to unit levels. **1** means the column vector of ones and is just used for summation.

A

a_{11}	a_{12}
a_{21}	a_{22}

C

c_{11}	c_{12}
c_{21}	c_{22}

S

s_{11}	s_{12}
s_{21}	s_{22}

In numerical terms we get

A

0,1	0
2	0,3

C

0,08333333	0,00666667
1,16666667	0,09333333

S

0,36057692	0,03894231
1,39423077	0,15057692

Marx' and Bortkiewicz' solutions

Marx' solution of the transformation problem is well known. He started with prices proportional to labour values (we denote them by $\mathbf{p}_0$ on unit level) and multiplied capital advanced by the average rate of profit increased by 1.

In our notation we can write for the resulting prices of production, $\mathbf{p}_1$:

$$\mathbf{p}_{Marx} = \mathbf{p}\ (\ \mathbf{A} + \mathbf{C}\ )\ (\ 1 + \pi\ )\ ,$$

with

$$(\ 1 + \pi\ ) = \mathbf{p}\ \mathbf{x}\ /\ \mathbf{p}\ (\ \mathbf{A} + \mathbf{C}\ )$$

If we apply Marx' method iteratively,

$$\mathbf{p}_{i+1} = \mathbf{p}_i\ (\ \mathbf{A} + \mathbf{C}\ )\ (\ 1 + \pi_i\ )\ ,$$

the prices of production converge to Bortkiewicz' solution, $\mathbf{p}_\infty$, which is identical to the left eigenvector of the matrix ($\mathbf{A}$+$\mathbf{C}$).

The following tables illustrate the result of the iteration process for pi and π_i

$\mathbf{p}_i$

Iteration	10	1
1	10,4166667	0,95833333
2	10,5107765	0,94892235
3	10,5325658	0,94674342
4	10,5376394	0,94623606
5	10,5388223	0,94611777
6	10,5390982	0,94609018
7	10,5391626	0,94608374
8	10,5391776	0,94608224
9	10,5391811	0,94608189
10	10,5391819	0,94608181
11	10,5391821	0,94608179
12	10,5391822	0,94608178
13	10,5391822	0,94608178
14	10,5391822	0,94608178
15	10,5391822	0,94608178

π_i

1,08333333
1,12577502
1,13560156
1,13788966
1,13842315
1,13854758
1,1385766
1,13858337
1,13858495
1,13858531
1,1385854
1,13858542
1,13858542
1,13858543
1,13858543
1,13858543

We can easily show that this method automatically keeps the value of total turnover invariant. By substituting π we get

$$1 + \pi_i = \mathbf{p}_i\ \mathbf{x}\ /\ \mathbf{p}_i\ (\ \mathbf{A} + \mathbf{C}\ )\ \mathbf{x}$$

If we right multiply the following equation

$$\mathbf{p}_{i+1} = \mathbf{p}_i\ (\ \mathbf{A} + \mathbf{C}\ )\ \mathbf{p}_i\ \mathbf{x}\ /\ \mathbf{p}_i\ (\ \mathbf{A} + \mathbf{C}\ )\ \mathbf{x}\ ,$$

by $\mathbf{x}$, we arrive at

$$\mathbf{p}_{i+1}\ \mathbf{x} = \mathbf{p}_i\ (\ \mathbf{A} + \mathbf{C}\ )\ \mathbf{x}\ [\ \mathbf{p}_i\ \mathbf{x}\ /\ \mathbf{p}_i\ (\ \mathbf{A} + \mathbf{C}\ )\ \mathbf{x}\ ]$$

and

$$\mathbf{p}_{i+1}\ \mathbf{x} = \mathbf{p}_i\ \mathbf{x}.$$

q.e.d.

A "more concrete" transformation problem

After having repeated the basics let us go on one step further and implement demand functions. For reasons of simplicity I assume a change in demand only for consumer goods. The consumer demand functions may have the following form:

$$C_{ij} = v_j\, x_j\, b_{ij} / p_i = \mathrm{diag}^{-1}(\mathbf{p})\, \mathbf{B}\, \mathrm{diag}(\mathbf{v})\, \mathrm{diag}(\mathbf{x})$$

where the b_{ij}'s are constants.

If one believes in utility functions one could derive the demand functions also from logarithmic utility functions N_j for each sector of production. One could maximize N_j w.r.t. a budget constraint (spending in one sector is restricted by wages w_j).

$$N_j = d_{1j} \log(C_{1j})+ d_{2j} \log(C_{2j})+ \mathrm{lamda}_j (w_j – p_1 C_{1j} – p_2 C_{2j}),\ j = 1,2$$

Because by any transformation of prices final demand **y** will be affected, we apply the Leontief inverse to determine **x***, the output needed to produce **y** (**inv** is the given and constant column vector of capital investment goods)

$$y_i = C_{i1} + C_{i2} + \mathrm{inv}_i,\ i = 1,2$$

$$\mathbf{x} = (\mathbf{E} – \mathbf{A})^{-1}\, \mathbf{y}$$

To perform the transformation we look for new relative prices **p*** and modified values of output **x*** that fulfil the following conditions:

The first two equations for the vector variables **x*** and **p*** are described by

$$\mathbf{x} = (\mathbf{E} – \mathbf{A})^{-1} [\, \mathrm{diag}^{-1}(\mathbf{p^*})\, \mathbf{B}\, \mathrm{diag}(\mathbf{v})\, \mathrm{diag}(\mathbf{x^*})\, \mathbf{1} + \mathbf{inv} \,],$$

where **B** is a matrix of constants that determine consumer demand. **inv** is the column vector of capital investment goods.

The third equation equalizes the two industrial rates of profit. Capital advanced (including wages) per sector can be described by a row vector **K**

$$\mathbf{K} = \mathbf{p^*} (\mathbf{A}\, \mathrm{diag}(\mathbf{x^*}) + \{ C_{ij} \}) = = \mathbf{p^*} [\mathbf{A}\, \mathrm{diag}(\mathbf{x^*}) + \mathrm{diag}^{-1}(\mathbf{p^*})\, \mathbf{B}\, \mathrm{diag}(\mathbf{v})\, \mathrm{diag}(\mathbf{x^*})] =$$

$$= \mathbf{p^*}\, \mathbf{A}\, \mathrm{diag}(\mathbf{x^*}) + \mathbf{1'}\, \mathbf{B}\, \mathrm{diag}(\mathbf{v})\, \mathrm{diag}(\mathbf{x^*})$$

By division of the elements of the row vector of the value of output

$$\mathbf{1'}\, \mathrm{diag}(\mathbf{p^*})\, \mathrm{diag}(\mathbf{x^*})$$

by the respective elements of capital advanced, **K**, we get the industrial rates of profit, π_j, + 1, or the growth of capital advanced, g_j. With these definitions we can write the third equation which might be simplified by right-multiplication of the vectors **K** and the turnover $\mathbf{1'}\, \mathrm{diag}(\mathbf{p^*})\, \mathrm{diag}(\mathbf{x^*})$ by $\mathrm{diag}(\mathbf{x^*})^{-1}$ as

$$g_1 = g_2,$$

or explicitly

$$p_1 / [\, p_1 a_{11} + p_2 a_{21} + v_1 (b_{11} + b_{21})] = p_2 / [\, p_1 a_{12} + p_2 a_{22} + v_2 (b_{12} + b_{22})]$$

The fourth and last equation assures the equality of the total value of output before and after the transformation

$$\mathbf{p\, x} = \mathbf{p^*\, x^*}.$$

Now we should be able to compute the values of **x*** and **p***.

We applied the open source software Maxima (you can download it from http://maxima.sourceforge.net/) to find the solution of the resulting polynomial of 4th order. The program came up with the following four solutions:

Solution 1:	p1 = 10.494,	p2 = 0.941,	x1 = 9.928,	x2 = 101.780
Solution 2:	p1 = 3.641,	p2 = 0.308,	x1 = 21.835,	x2 = 390.685
Solution 3:	p1 = 26.977,	p2 = - 0.848,	x1 = 8.750,	x2 = 42.538
Solution 4:	p1 = 0,	p2 = - 1,	x1 = 16,	x2 = - 200.

Only the first two solutions are economically feasible. The two others have either negative volumes or prices, and are therefore economically meaningless.

The following table allows to compare five different systems of relative prices, (1) prices proportional to labour values (as in an ideal type economy of small commodity production), (2) Marx' solution of the transformation problem, Bortkiewicz' solution, and my own proposal with two different solutions (4 and 5) for a competitive capitalist economy. All solutions except the first represent competitive capitalist economies.

	Labour values Marx		**Production prices Marx**		**Production prices Bortkiewicz**		**Two solutions of the "concrete" transformation problem**			
	(1)	(1)	(2)	(2)	(3)	(3)	(4)	(4)	(5)	(5)
	sector1	sector2	sector1	sector2	sector1	sector2	sector1	sector2	sector1	sector2
unit prices	10	1	10,417	0,958	10,539	0,946	10,494	0,941	3,641	0,308
volumes	10	100	10	100	10	100	9,928	101,78	21,835	390,69
turnover	100	100	104,17	95,83	105,39	94,61	104,18	95,82	79,50	120,50
profit rates	1,000	1,174	1,083	1,083	1,139	1,139	1,128	1,128	0,221	0,221
labour	70	70	70	70	70	70	69,49	71,25	152,84	273.48
wages	20	16	20	16	20	16	19,85	16,28	43,63	62,46
consumptn. matrix	0,833 11,67	0,667 9,333	0,833 11,67	0,667 9,333	0,833 11,67	0,667 9,333	0,788 12,305	0,647 10,093	4,998 82,589	7,15 118,22
utility[5]	1,357	1,134	1,357	1,134	1,357	1,134	1,365	1,167	3,245	3,604

It is interesting to compare the levels of utilities: After "concrete" transformation, all the utility levels for all groups of workers have increased. The first solution of the "concrete" problem, rather close to prices proportionate to labour values, and also close to production prices of Marx and Bortkiewicz, is associated with only slightly increased utility values, while the second solution more or less doubles them. For a regulated economy, both solutions, (4) and (5), represent a kind of economic equilibrium, because there is no need for capital to move to the other sector, as rates of profit are already equal everywhere.

The second solution comes with low prices and a rather low profit rate (only about 20% of the rate of the first "concrete" solution). It represents a solution where consumption levels are high, but because of a low rate of profit the potential for growth is limited compared with the first "concrete" solution. Although all the unit wages (2 resp. 0,16 units) and also the wage per worker in the resp. industry (0,286 resp. 0,228 units) remain invariant over the "concrete" transformation, one can see from solution (5) that much more labour is needed for the economy than in all other cases.

It is also possible to arrive at these two solutions by iteration, starting from labour values. As shown above for the solution of the classical transformation problem we can establish similar iteration processes. In the case of the concrete transformation we have to define two different ways how the iterations are defined. The first solution, close to the one by von Bortkiewicz can be found by firstly determining the overall rate of profit expressed at labour values. Secondly, multiplying cost prices at labour values by the factor (1 + rate of profit – at labour values) results in a first approximation of prices of production $\mathbf{p}_1$. With these prices consumption levels $\mathbf{c}_1$ are computed, which will change overall final demand $\mathbf{y}_1$. Right-multiplying the Leontief inverse $(\mathbf{E} - \mathbf{A})^{-1}$ by $\mathbf{y}_1$ results in changed output

[5] For both industries we used the same utility function: $N = N_j = 0.41667 \cdot \log(C_{1j}) + 0.58333 \cdot \log(C_{2j})$

$\mathbf{x}_1$. To keep the total turnover at the same level we have to standardize $\mathbf{p}_1 \mathbf{x}_1 = \mathbf{p}_0 \mathbf{x}_0$. Applying the same steps again on the basis of $\mathbf{p}_1$ and $\mathbf{x}_1$, and so on, we finally reach equilibrium with equal rates of profit in both sectors, and supply equals demand for consumer markets. The iteration process illustrates that starting in an economy with small commodity producers a new rule of price formation is established, leading step by step to prices of production in a capitalist economy.

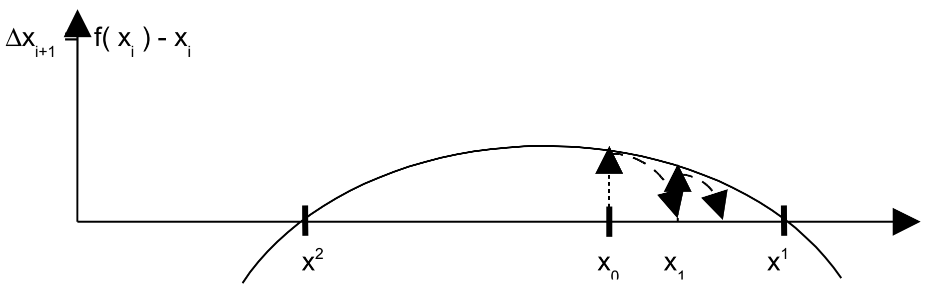
But what about the second solution? We have to show two aspects: First, we have to demonstrate that there is an iterative process, and second, we have to explain the economic meaning of it. Let us respond to the first challenge. To illustrate the idea we start with a one dimensional case. Let us assume the iteration process is applied to a function

$$x_{i+1} = f(x_i), \ \ x_0 = x^*$$

or

$$\Delta x_{i+1} = x_{i+1} - x_i = f(x_i) - x_i, \ x_0 = x^*$$

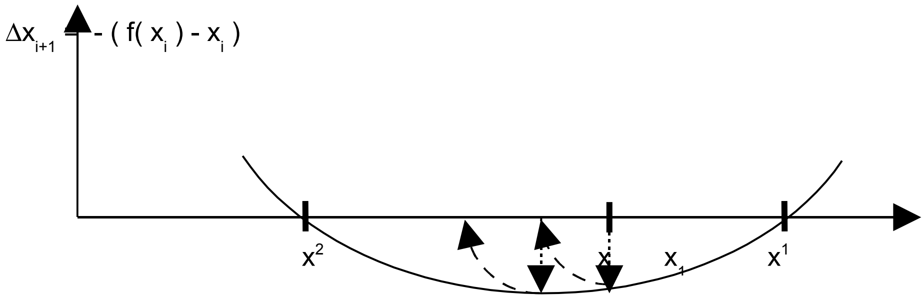
If we assume the function f has more than one solution, we can illustrate this situation in the following way:



x^1 and x^2 represent the two solutions (where $\Delta x = 0$), x_0 is the starting value for the iteration. To find the fixed point, in a first step to get x_1 we add Δx_1 (represented by the dotted line) to x_0. Graphically, we have just to rotate the dotted line by 90 degrees (see dashed arcs). In this way we will finally arrive at the first solution x^1. From the graph it becomes evident what we have to do to find the second solution x^2. We have just to change the sign of Δx. This will reduce x_0 step by step up to the moment when the fixed point

$$x^2 = f(x^2)$$

is reached.



For the more-dimensional case of more than one variable we have more than one option. All the combinations of inverting or not inverting the sign of the change of the variables are possible in principle and could lead to different solutions of the equations. Some of them might not be feasible. If we already know the solution in advance (by means of numerical methods) an indication for the correct choice of signs is the necessary change of the start value towards the fixed point. In case the first iteration results in an increase of the variable, but the variable at the fixed point is smaller, then one should apply the following rule:

$$x_{i+1} = x_i - \Delta x_{i+1} = x_i - (f(x_i) - x_i)$$

To adjust the speed of the process one can also choose an additional parameter q > 0 weighting the difference between successive iterations of a certain variable x:

$$x_{i+1} = x_i - q\, \Delta x_{i+1} = x_i - q\, (f(x_i) - x_i) = (1 + q)\, x_i - q\, f(x_i)$$

In our illustrative numerical example with output vector **x** and price vector **p** we have chosen a negative sign of the difference of the output of the first sector x_1 and also for its unit price p_1, while the iteration process of x_2 and p_2 was kept without any change. This procedure leads to the second solution of the transformation problem.

If we assume that the second solution is the one with lower prices and higher levels of output, related to a higher living standard, while still the rates of profit are equal, but lower than in the first solution, we can think about how such a solution could iteratively be brought into existence in a real economy. Of course, I am aware of the still very abstract level of the model applied and the artificiality of this argumentation. If we would be able to regulate the price levels – what in the numerical example in fact means to lower them step by step, we would end up with a situation where output is high, prices are low and profit rates are equal. The open question remains how this can be done against the resistance of capitalist managers.

Final remarks

How does the more concrete view of the transformation problem presented here fit into the framework of Marxian theory? What will a Marxian scholar do with such considerations? Does it violate assumptions backed by Marxian tradition or does it enrich the understanding of the economy in Marxian terms? In my perspective, the above proposal allows for more flexibility and widens the scope of solutions traditionally covered. Although starting from a position of equilibrium, it gives room for the variation not only of prices but simultaneously also of volumes of commodities produced. Realistically, it accepts explicitly the influence of supply and demand – although the theoretical background of utility theory is not necessarily needed.

It keeps up the Marxian axiom that market prices are nothing else than modified labour values. One can still identify surplus value as source for profits, and necessary labour time corresponding to wages. The concept of exploitation can be still applied in the realm of competitive prices.

What is different to the Marxian approach then? Although theoretically clearly described by Marx (see e.g. Chapter 3 of Volume 1 of Das Kapital) that a commodity has to do a somersault (Purzelbaum) to the market and that labour value needs the market to transform itself from individual into social labour,[6] he did not include the effects of supply and demand in his numerical examples. In my opinion, the reason for this was that he wanted to bring more basic and essential features of capitalist economy to the attention of the reader than mechanisms of the surface.

But if we allow for the simultaneous determination of volumes and prices, we are able to create a more realistic model of observable processes. A second effect comes to the fore: We have shown that more than one solution of the "more concrete" transformation problem is possible. If we assume that transformation was an actual process in history, in my understanding this property is very much in agreement with another philosophical postulate, that the historical process is an open one and is not completely predetermined.

[6] See a recent paper by Marc-André, GAGNON, Measuring Exchange-Value; Evaluation of the Ricardian and Aristotelian Traditions at www.er.uqam.ca/nobel/ieim/IMG/pdf/NoteE_2006-12-07-Gagnon.pdf (21/01/07)