The Equivalence, Unambiguity and Sequentiality Problems of Finitely Ambiguous Max-Plus Tree Automata are Decidable

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QuantLA

Max-Plus Automata

$$q_0 \xrightarrow{a} q_1 \xrightarrow{a} q_2 \xrightarrow{b} q_3 \xrightarrow{a} q_4 \xrightarrow{b} q_5 \xrightarrow{a} q_6$$

Weights in $\mathbb{R} \cup \{-\infty\}$

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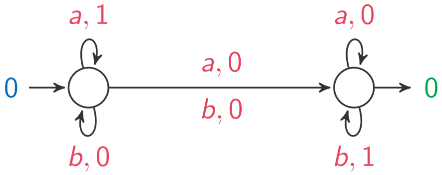

	one "initial state"
sequential / deterministic	no two valid $p \xrightarrow{a} q_1,\ p \xrightarrow{a} q_2$

$$\text{Run}(w) = \{\text{Runs } r \text{ on } w \text{ with weight}(r) \neq -\infty\}$$

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Decidability for max-plus automata on words

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Krob

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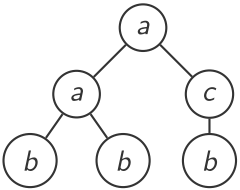

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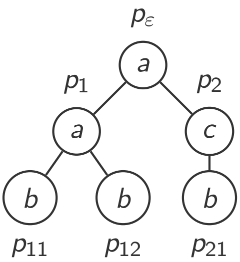

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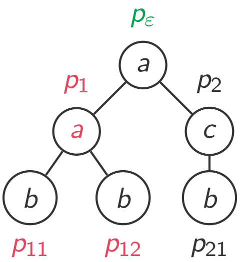
(p_{11}, p_{12}, a, p_1)



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all runs of $\mathcal{A}_1$, one of $\mathcal{A}_2$ in parallel

$$\begin{matrix} p_1 & & p_2 & & p_2 & & p_1 & & p_3 & & p_2 & & p_2 \\ p_1 & a & p_3 & b & p_3 & a & p_1 & b & p_2 & b & p_3 & a & p_4 \\ p_2 & & p_4 & & p_3 & & p_1 & & p_1 & & p_3 & & p_3 \\ q_1 & & q_2 & & q_1 & & q_3 & & q_2 & & q_1 & & q_1 \end{matrix}$$

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p_1	a	p_3	b	p_3	a	p_1	b	p_2	b	p_3	a	p_4
p_2		p_4		p_3		p_1		p_1		p_3		p_3
q_1		q_2		q_1		q_3		q_2		q_1		q_1

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$$
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Cycle Decomposition

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x_i, y_i short: $|x_i|, |y_i| \leq |\text{states}(\mathcal{A}_1)|^3 \cdot |\text{states}(\mathcal{A}_2)|$

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Vectors of weights

$$\begin{matrix} \mathrm{wt}_1 \\ \mathrm{wt}_2 \\ \mathrm{wt}_3 \\ \mathrm{wt}_4 \end{matrix} = \begin{matrix} 7 \\ 11 \\ 4 \\ 8 \end{matrix} + \begin{matrix} 13 \\ 8 \\ 6 \\ 19 \end{matrix} + \begin{matrix} 7 \\ 3 \\ 1 \\ 9 \end{matrix} + \begin{matrix} 2 \\ 3 \\ 15 \\ 4 \end{matrix} + \begin{matrix} 12 \\ 10 \\ 9 \\ 4 \end{matrix} + \begin{matrix} 3 \\ 7 \\ 5 \\ 14 \end{matrix} + \begin{matrix} 18 \\ 2 \\ 5 \\ 1 \end{matrix}$$

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satisfiability decidable! (linear Diophantine inequalities)

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2. "Long words": one cycle two times $\implies$ cut

We show: $\mathcal{A}_1$ fin-amb $\Longrightarrow$ $\mathcal{A}_1 \geq \mathcal{A}_2$ decidable

Cycle Decomposition

$$\vec{P}_1 \quad x_1 \quad \vec{P}_2 \quad y_2 \quad \vec{P}_2 \quad x_3 \quad \vec{P}_3 \quad y_4 \quad \vec{P}_3 \quad x_5 \quad \vec{P}_2 \quad y_2 \quad \vec{P}_2 \quad x_7 \quad \vec{P}_5$$

Vectors of weights

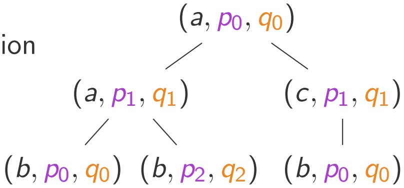
$$\begin{matrix} \mathrm{wt}_1 \\ \mathrm{wt}_2 \\ \mathrm{wt}_3 \\ \mathrm{wt}_4 \end{matrix} = \begin{matrix} 7 \\ 11 \\ 4 \\ 8 \end{matrix} + \begin{matrix} 13X_1 \\ 8X_1 \\ 6X_1 \\ 19X_1 \end{matrix} + \begin{matrix} 7 \\ 3 \\ 1 \\ 9 \end{matrix} + \begin{matrix} 2X_2 \\ 3X_2 \\ 15X_2 \\ 4X_2 \end{matrix} + \begin{matrix} 12 \\ 10 \\ 9 \\ 4 \end{matrix} + \begin{matrix} 13X_3 \\ 8X_3 \\ 6X_3 \\ 19X_3 \end{matrix} + \begin{matrix} 18 \\ 2 \\ 5 \\ 1 \end{matrix}$$

$\mathrm{wt}_1, \mathrm{wt}_2, \mathrm{wt}_3 < \mathrm{wt}_4$? for some choice of $X_1, X_2, X_3 \in \mathbb{N}$?

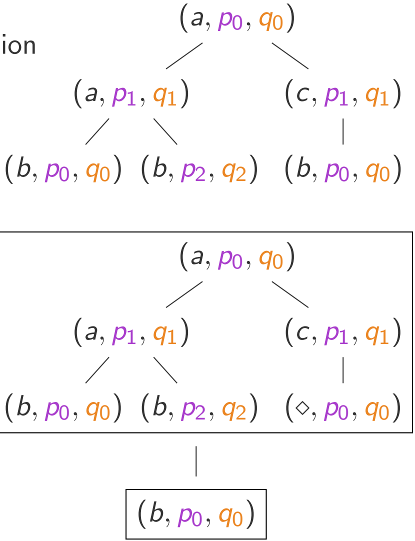
1. Check satisfiability for all cycle decompositions of “short” words
2. “Long words”: one cycle two times $\Longrightarrow$ cut

The Equivalence Problem on Trees

Cycle Decomposition

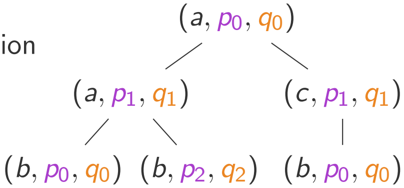


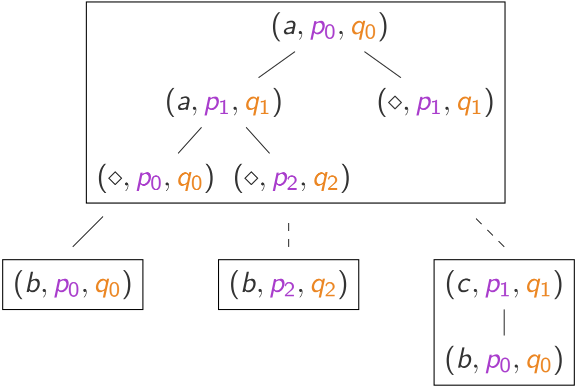
Cycle Decomposition



The Equivalence Problem on Trees

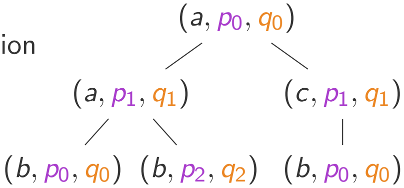
Cycle Decomposition





The Equivalence Problem on Trees

Cycle Decomposition



Removing Cycles?