QuantLA

The Structure of Weighted Automata on Trees and Tree-like Graphs

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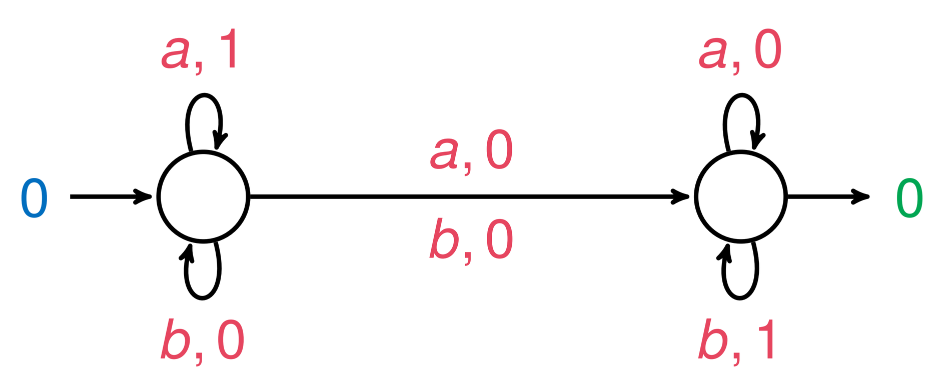
Decidability of Max-Plus Tree Automata

Max-Plus Automata

$(\mathbb{R} \cup \{-\infty\}, \max, +, -\infty, 0)$



Weight of run:
initial + transition + final weights

Weight of word / tree:
maximum over all runs

Ambiguity

For all trees t	$\lvert\text{Run}(t)\rvert \leq 1$	$\rightarrow$	unambiguous
	$\lvert\text{Run}(t)\rvert \leq M$	$\rightarrow$	finitely ambiguous

Decidability Problems

Equivalence problem

Given $\mathcal{A}_1, \mathcal{A}_2$ — Is $[\![\mathcal{A}_1]\!](t) = [\![\mathcal{A}_2]\!](t)$ for all trees t?

Unambiguity problem

Given $\mathcal{A}$ — Is there an unambiguous $\mathcal{A}'$ with $[\![\mathcal{A}]\!] = [\![\mathcal{A}']\!]$?

Sequentiality problem

Given $\mathcal{A}$ — Is there a deterministic $\mathcal{A}'$ with $[\![\mathcal{A}]\!] = [\![\mathcal{A}']\!]$?

Finite Sequentiality problem

Given $\mathcal{A}$ — Are there deterministic $\mathcal{A}_1, \dots, \mathcal{A}_n$ with $[\![\mathcal{A}]\!] = \max_{i=1}^{n} [\![\mathcal{A}_i]\!]$?

Results

Equivalence, Unambiguity, and Sequentiality decidable for
finitely ambiguous max-plus tree automata

Finite Sequentiality decidable for
unambiguous max-plus tree automata

Erik Paul. "The Equivalence, Unambiguity and Sequentiality Problems of Finitely Ambiguous Max-Plus Tree Automata are Decidable". in: Proc. MFCS, 2017

Erik Paul. "Monitor Logics for Quantitative Monitor Automata". in: Proc. MFCS, 2017

A Weighted Feferman-Vaught Theorem

MSO

$\beta ::= R(x_1, \dots, x_n) \mid x \in X \mid \neg\beta \mid \beta \vee \beta \mid \exists x.\beta \mid \exists X.\beta$

almost boolean wMSO

$\psi ::= \beta \mid s \mid \psi \oplus \psi \mid \psi \otimes \psi$

wMSO

$\varphi ::= \beta \mid s \mid \varphi \oplus \varphi \mid \varphi \otimes \varphi \mid \bigoplus_x \varphi \mid \bigoplus_X \varphi \mid \bigotimes_x \psi$

Expressions

$E ::= x_i \mid y_i \mid E \oplus E \mid E \otimes E$

Theorem

Given
signature σ — commutative semiring S — $\varphi \in$ wMSO

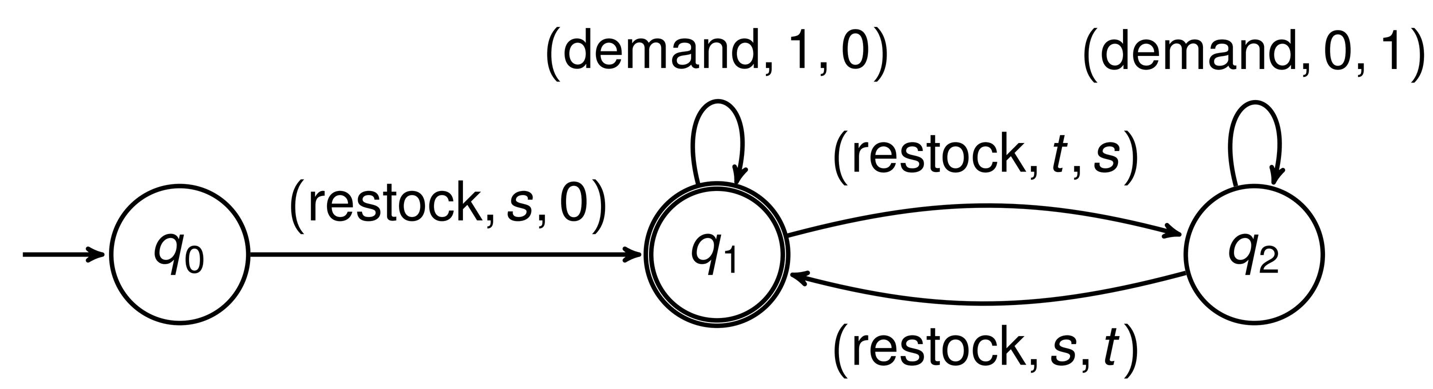
there exist
$n \geq 1$ — $\bar{\varphi}^1, \bar{\varphi}^2 \in \text{wMSO}^n$ — $E \in$ Exp

such that for all σ-structures $\mathfrak{A}$, $\mathfrak{B}$

$$[\![\varphi]\!](\mathfrak{A} \sqcup \mathfrak{B}) = \langle\!\langle E \rangle\!\rangle([\![\bar{\varphi}^1]\!](\mathfrak{A}), [\![\bar{\varphi}^2]\!](\mathfrak{B}))$$

Monitor Logics

Quantitative Monitor Automata $(\Sigma, Q, I, F, n, \delta, \textbf{Val})$



$\beta ::= P_a(x) \mid x \leq y \mid x \in X \mid \neg\beta \mid \beta \vee \beta \mid \exists x.\beta \mid \exists X.\beta$

$\psi ::= k \mid \beta \, ? \, \psi : \psi$

$\zeta_x ::= \bot \mid \beta \, ? \, \zeta_x : \zeta_x \mid \bigotimes^{x,Z} y.\psi$

Monitor Logics

$\varphi ::= \beta \, ? \, \varphi : \varphi \mid \min(\varphi, \varphi) \mid \inf x.\varphi \mid \inf X.\varphi \mid \text{Val}\, x.\zeta_x$

Theorem — Monitor Logics expressively equivalent to Quantitative Monitor Automata

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