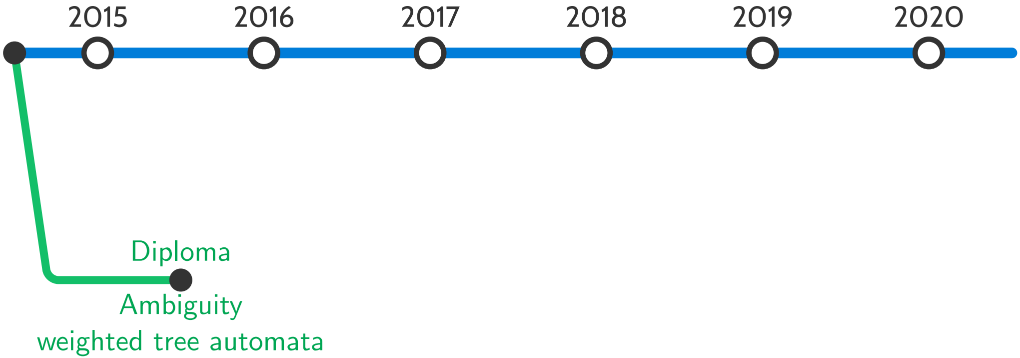
Trees, Logics, and Automata

My Time in QuantLA

Erik Paul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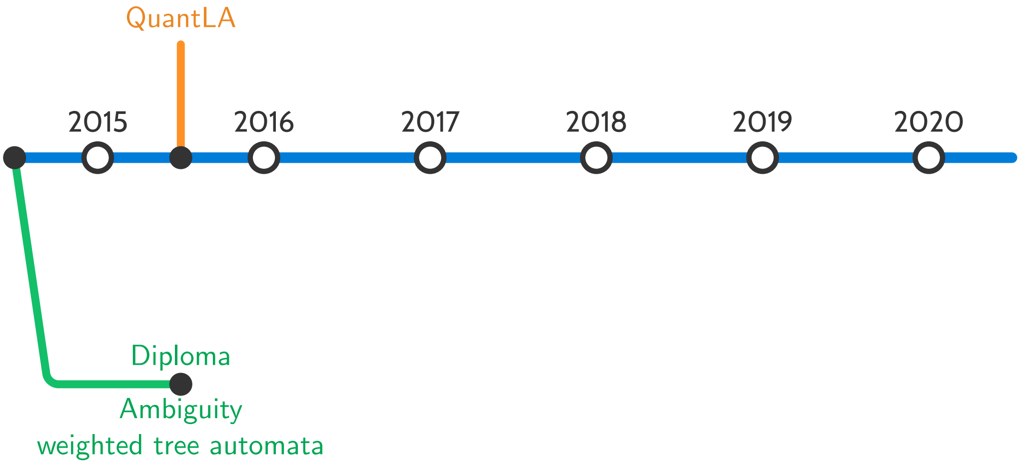
Ambiguity
weighted tree automata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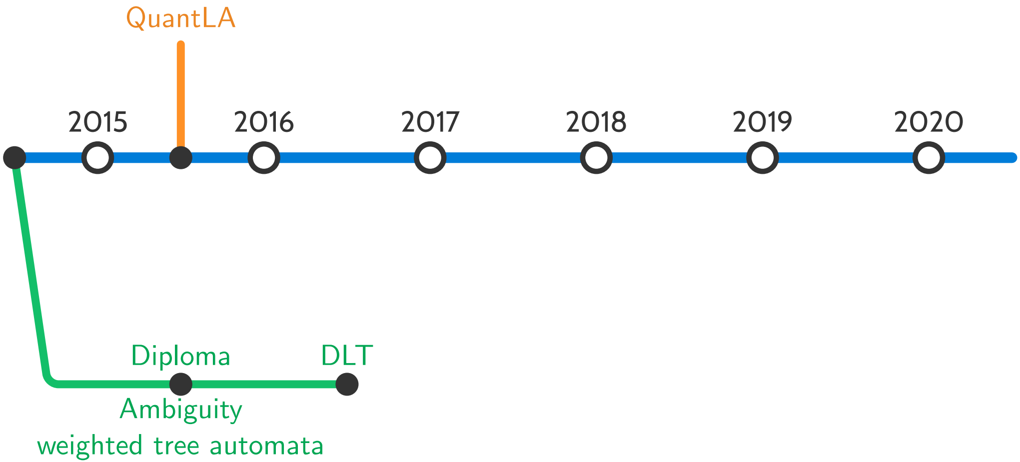
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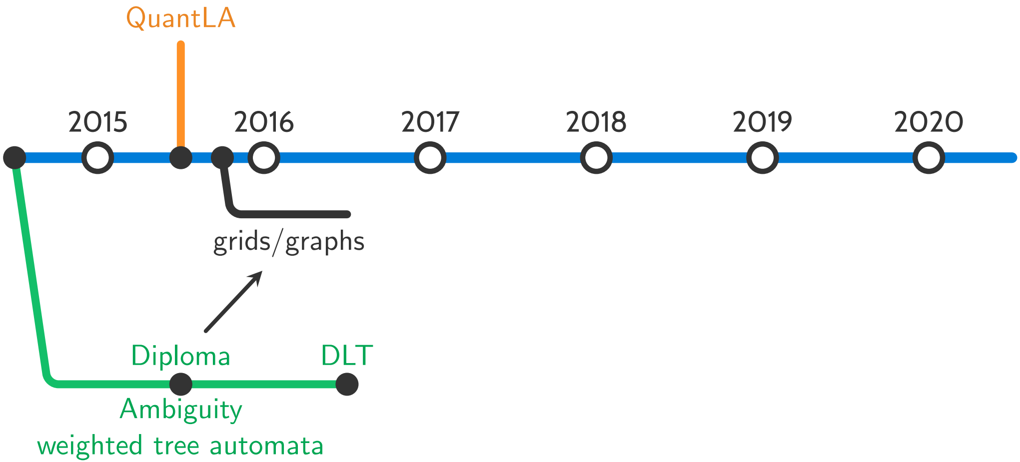
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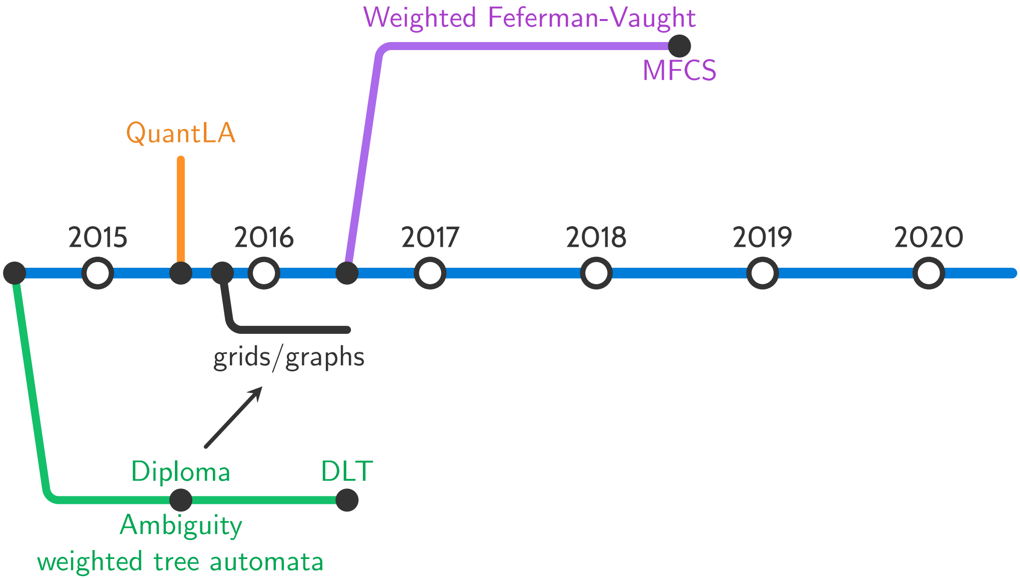

Weighted Feferman-Vaught
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formula β $\xleftrightarrow{\text{satisfaction}}$ structure $\mathcal{A}$

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question about union of structures $\mathcal{A} \sqcup \mathcal{B}$

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$\Uparrow$

combine answers

$\nearrow$ $\nwarrow$

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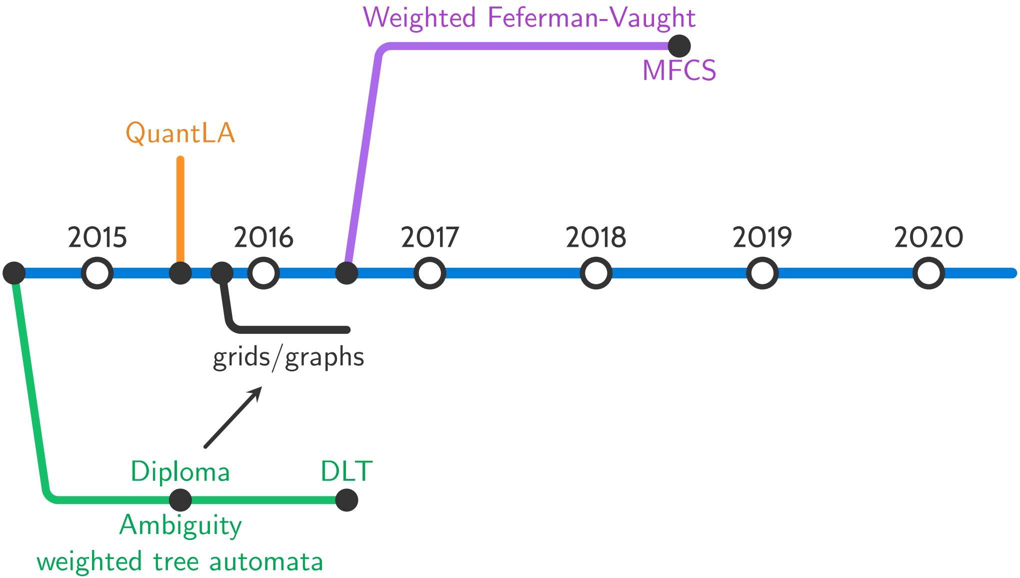
Example $\bigoplus_x \text{label}_a(x)$ graph $\mapsto$ number of a-nodes



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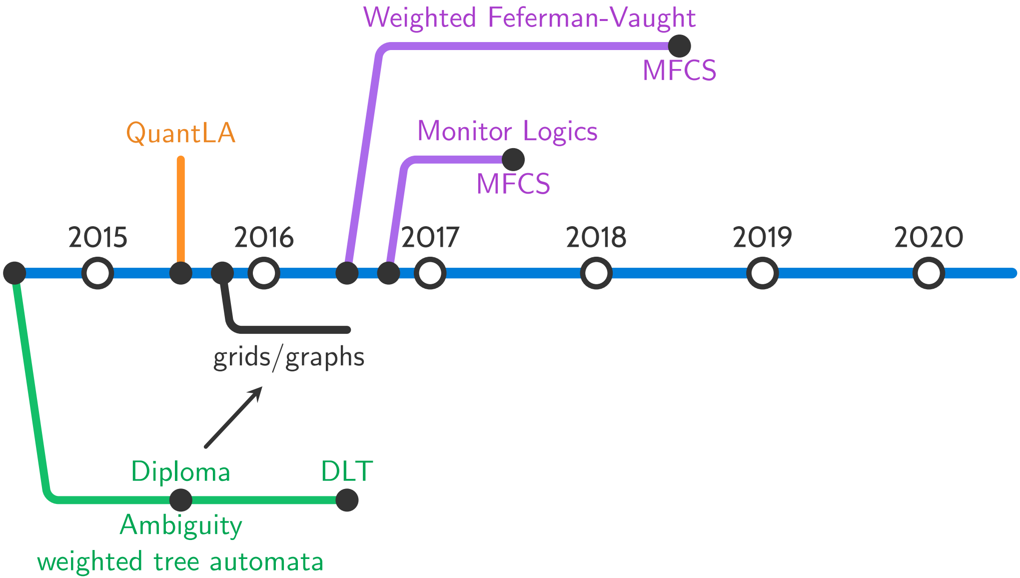
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Quantitative Monitor Automata [Chatterjee, Henzinger, Otop '16]

- infinite words
- words $\to \mathbb{R}$
- passive ("monitor") counters

Monitor Logics

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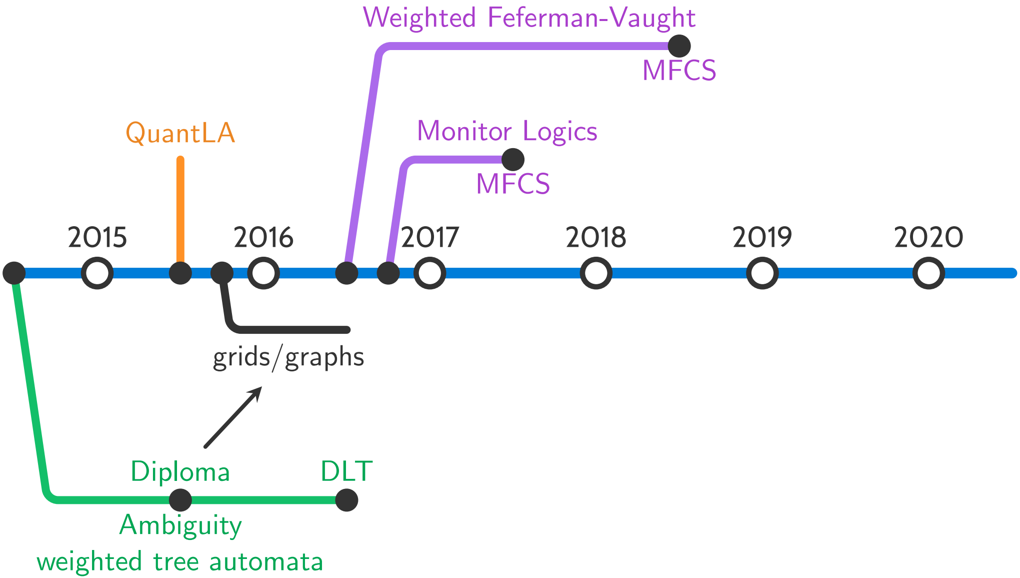
Logical characterization

- Quantitative Monitor Automata $\leftrightarrow$ Monitor Logic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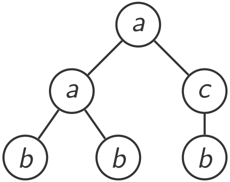
Max-Plus Tree Automata



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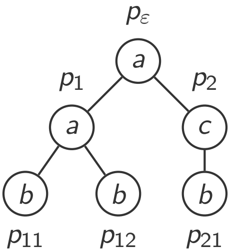
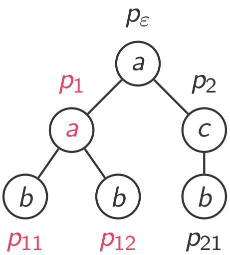
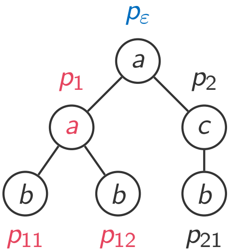




$$(p_{11}, p_{12}, a, p_1)$$

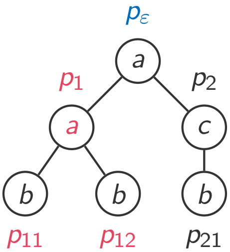


final weights

transition weights

in $\mathbb{R} \cup \{-\infty\}$

(p_{11}, p_{12}, a, p_1)



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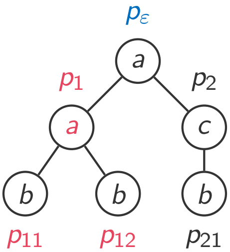
transition weights $+$ final weight



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weight of run =
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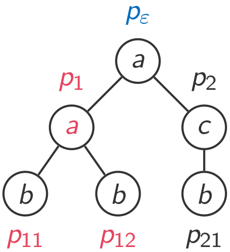
weight of tree =
maximum over runs on tree



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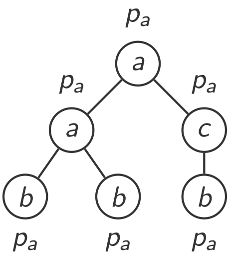
Example tree $\mapsto$ $\max\{a\text{-nodes},\ b\text{-nodes}\}$

Max-Plus Tree Automata



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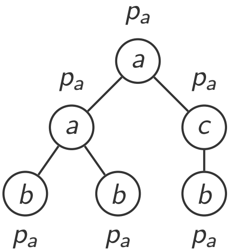
Example

tree $\mapsto$ $\max\{a\text{-nodes},\ b\text{-nodes}\}$
states $\{p_a, p_b\}$

run valid $\leftrightarrow$ only p_a or only p_b



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Example

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$(p_a, p_a, a, p_a) \mapsto 1$ $\quad$ $(b, p_b) \mapsto 1$ $\quad$ rest $\mapsto \{0, -\infty\}$

Ambiguity

sequential / deterministic

bottom-up determinism

$(p_1, \ldots, p_m, a, q)$

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sequential / deterministic

bottom-up determinism

$(p_1, \ldots, p_m, a, q)$

$$\mathrm{Run}(t) = \{\text{Runs } r \text{ on } t \text{ with weight}(r) \neq -\infty\}$$

Ambiguity

sequential / deterministic — bottom-up determinism

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unambiguous — $|\text{Run}(t)| \leq 1$

Ambiguity

sequential / deterministic	bottom-up determinism $(p_1, \dots, p_m, a, q)$

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Ambiguity

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Diploma

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AMBIGUITY

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Diploma

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Diploma

- finitely ambiguous = union of unambiguous
- structure of polynomially ambiguous tree automata
- logical characterization of ambiguity

Four Decision Problems

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Equivalence problem

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Finite Sequentiality problem

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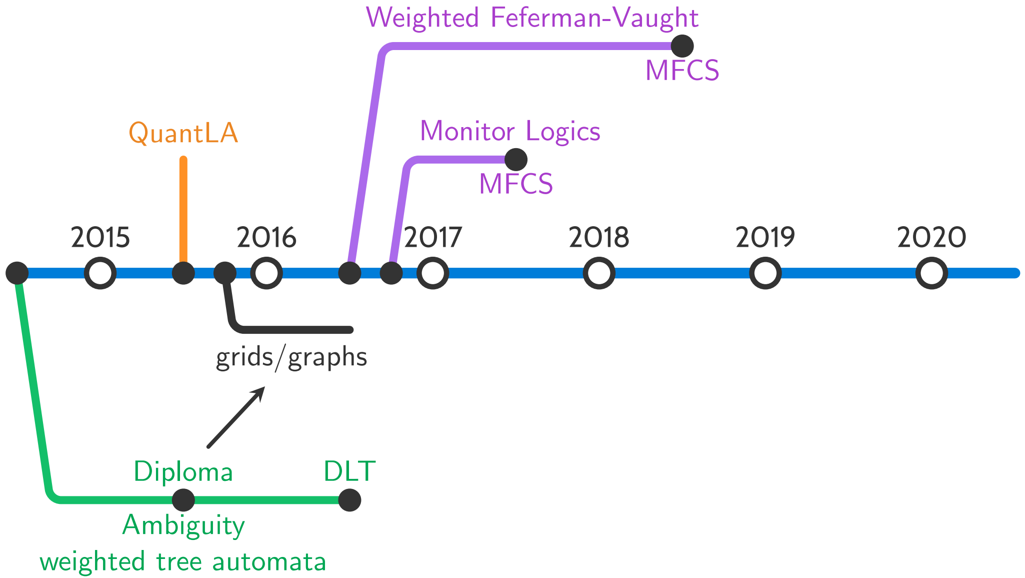
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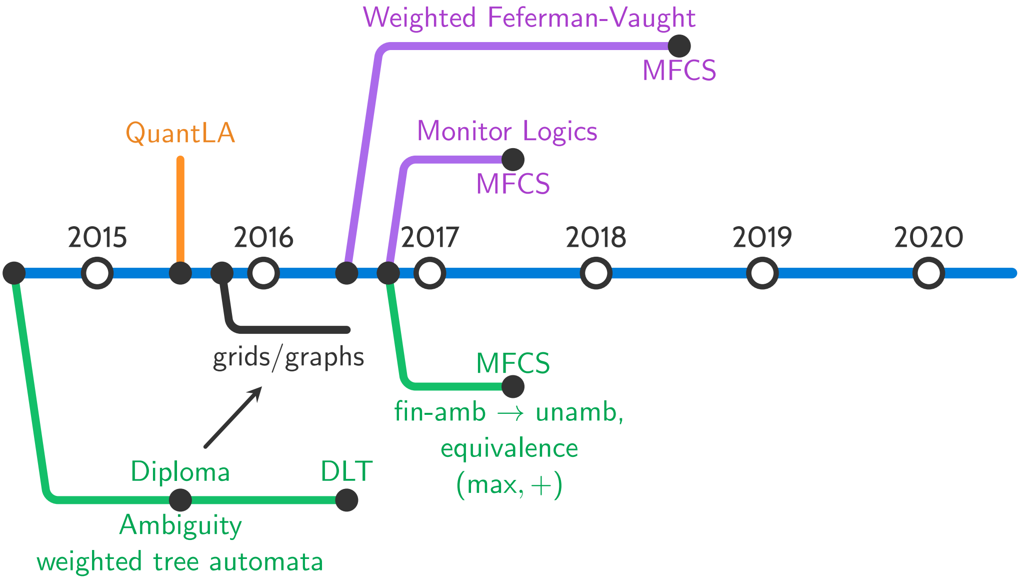
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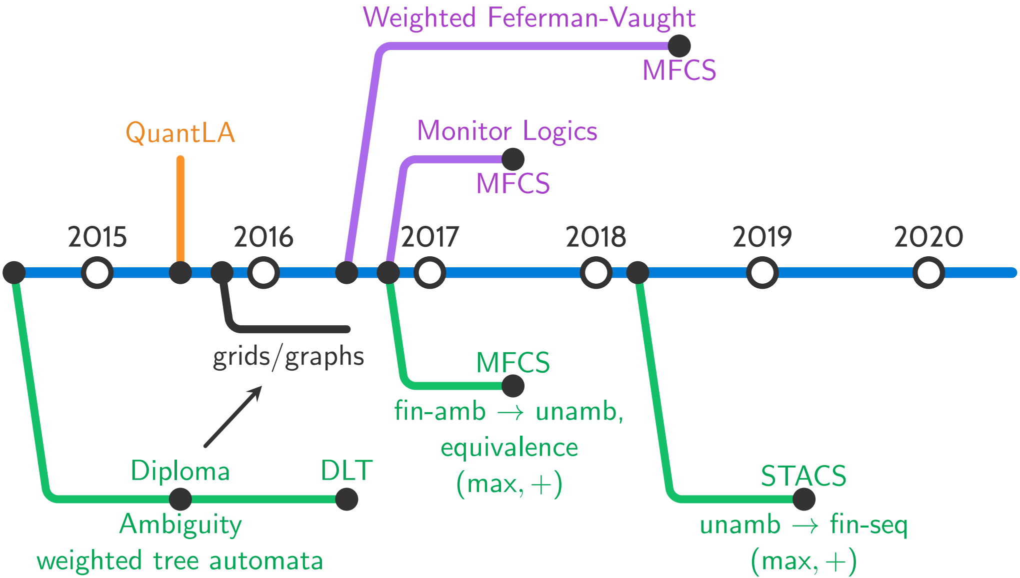
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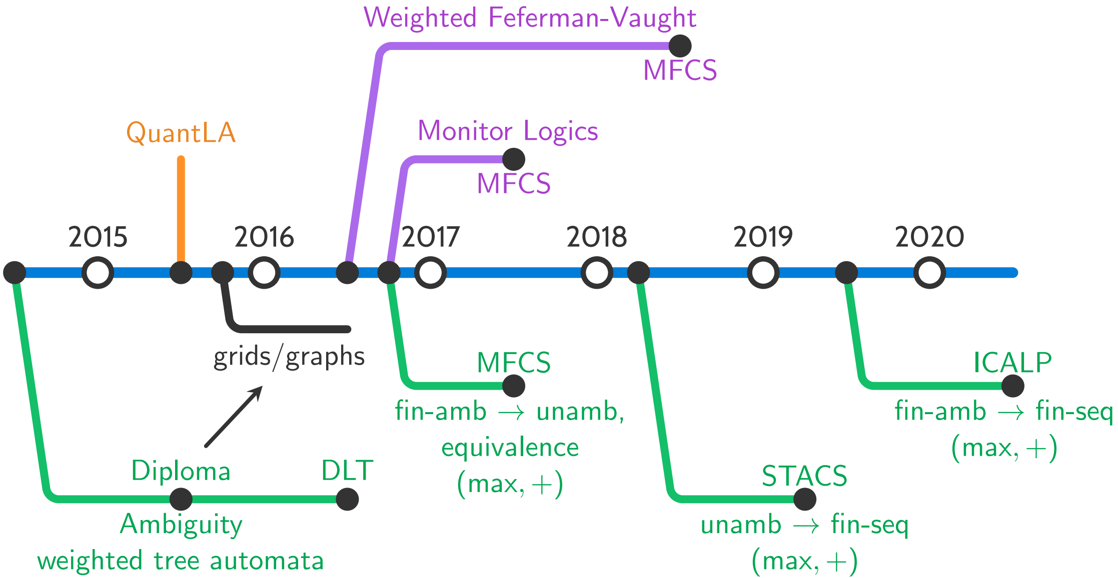
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	Equivalence	Unambiguity	Sequentiality	Finite Sequentiality
fin-amb	**yes**	**yes**	**yes**	**yes**
poly-amb	**no**	**?**	**?**	**?**
general	**no**	**?**	**?**	**?**

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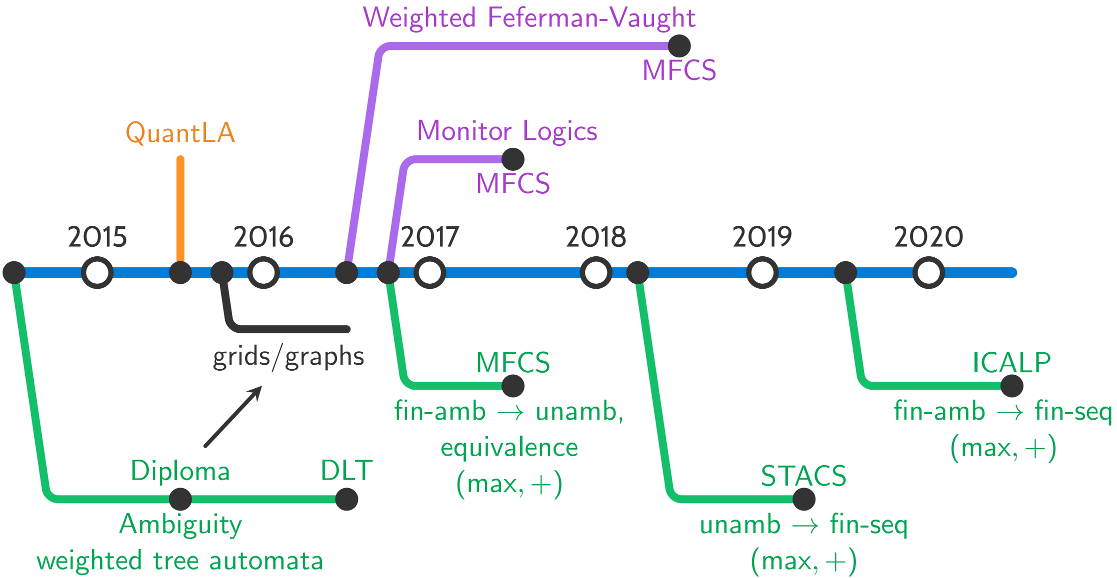
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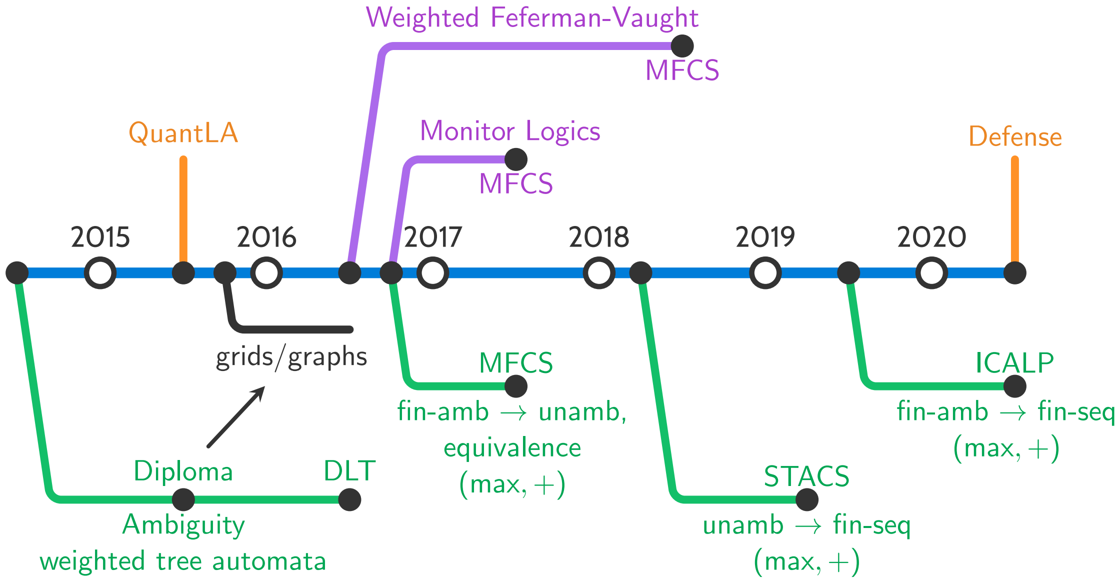
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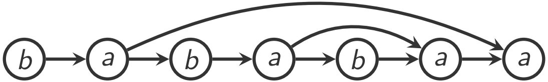
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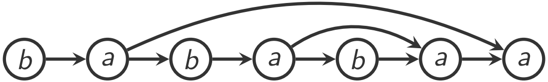
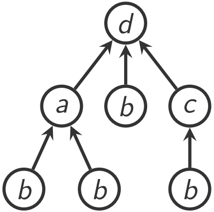

b a b a b a a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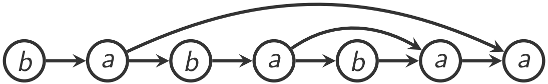
a b c
b b b


a b b a a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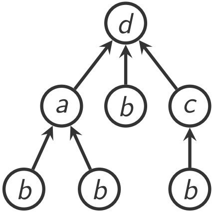
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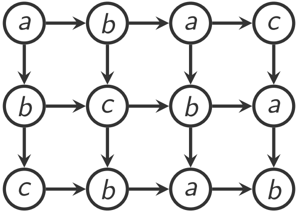
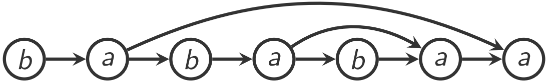
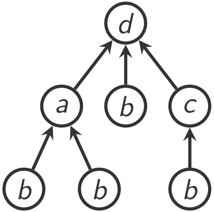

a b a c
b c b a
c b a b


a b b a a


b a b a b a a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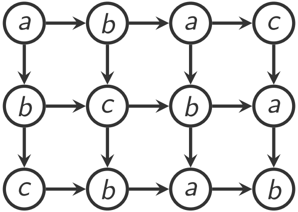
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b c b a
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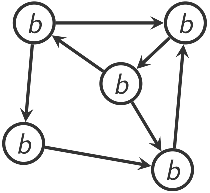
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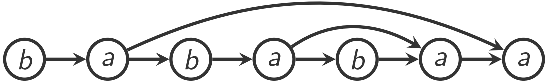
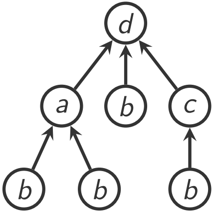
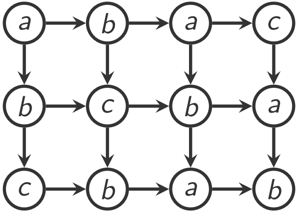
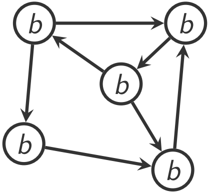
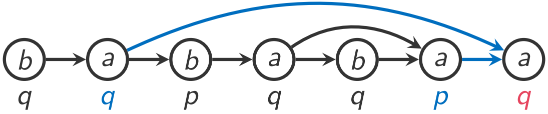


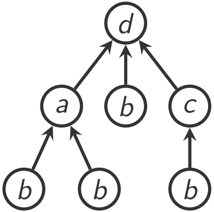


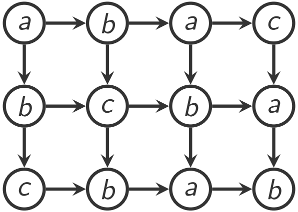


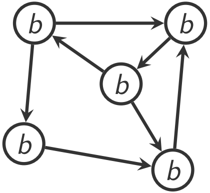


vertex marking automaton: states on nodes $(q_1, \ldots, q_0) \xrightarrow{a} q$









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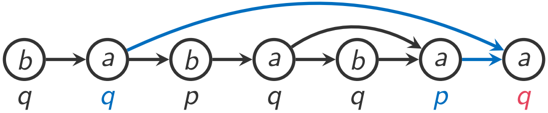
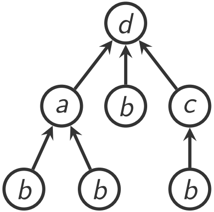
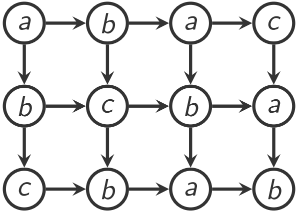
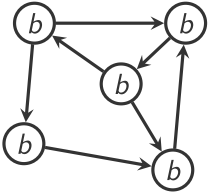


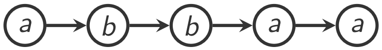


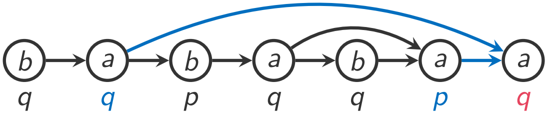


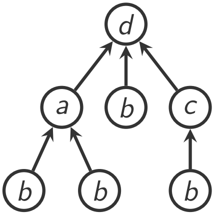


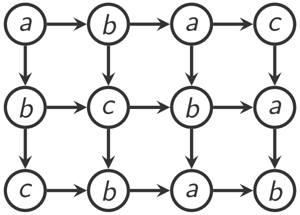
vertex marking automaton: states on nodes $(q_1, \dots, q_0) \xrightarrow{a} q$

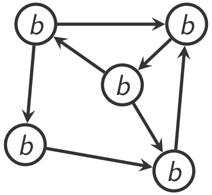
determinize? powerset construction what states could I be in?







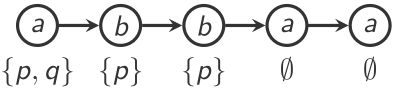


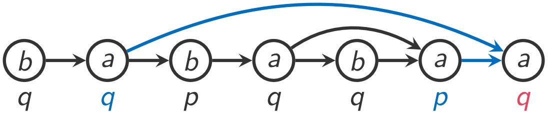


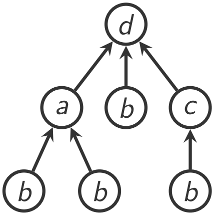
vertex marking automaton: states on nodes $(q_1, \ldots, q_0) \xrightarrow{a} q$

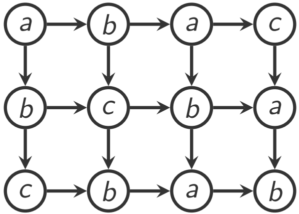
determinize? powerset construction what states could I be in?

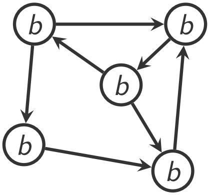
word automaton: $\xrightarrow{a} p$ $\xrightarrow{a} q$ $p \xrightarrow{b} p$











vertex marking automaton: states on nodes $(q_1, \ldots, q_0) \xrightarrow{a} q$

determinize? powerset construction what states could I be in?

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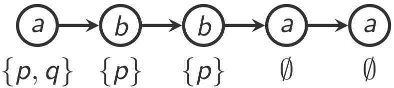

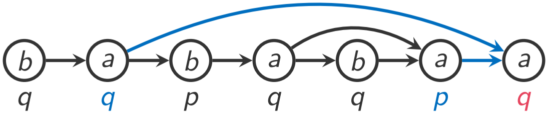

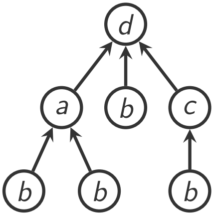

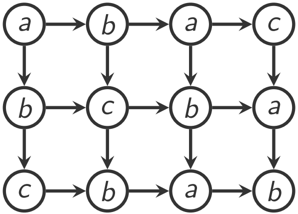

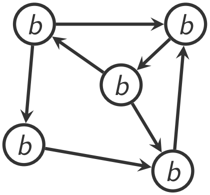

vertex marking automaton: states on nodes $(q_1, \ldots, q_0) \xrightarrow{a} q$

determinize? powerset construction what states could I be in?

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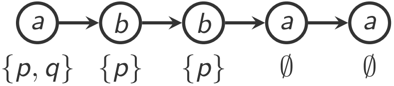
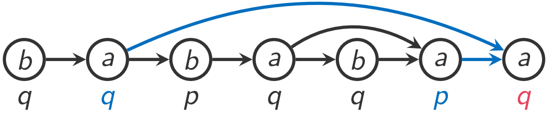
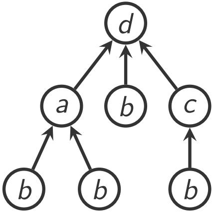
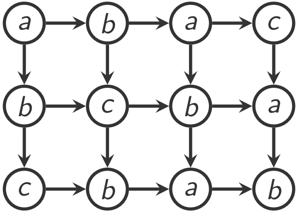
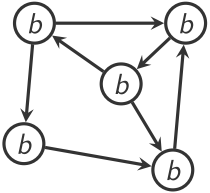
→ fails on top-down trees, nested words, pictures synchronization after split











vertex marking automaton: states on nodes $(q_1, \ldots, q_0) \xrightarrow{a} q$

determinize? powerset construction what states could I be in?

nested word automaton: $\xrightarrow{a} p$ $p \xrightarrow{a} p$ $\xrightarrow{a} q$ $q \xrightarrow{a} q$ $(p, q) \xrightarrow{a} p$

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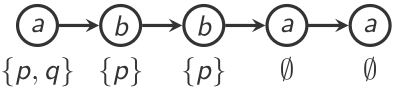

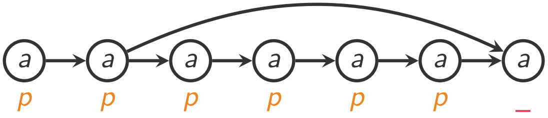

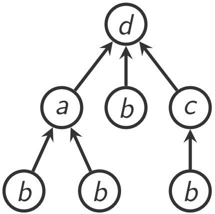

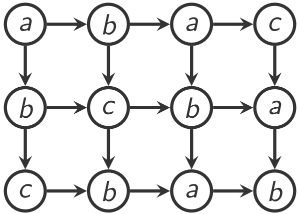

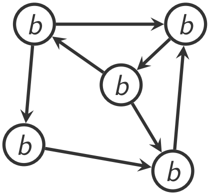

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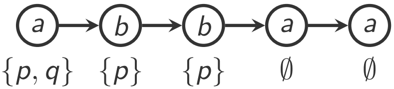

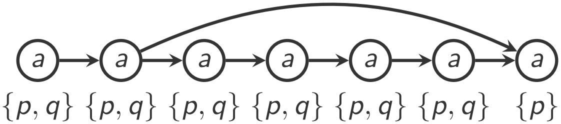

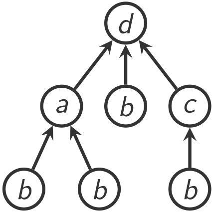

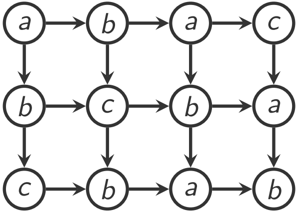

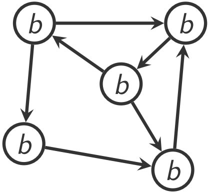

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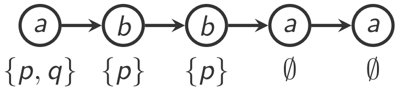

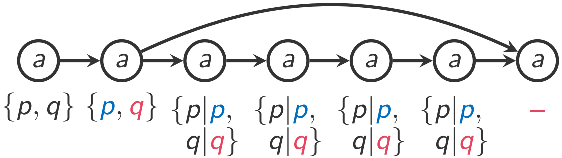

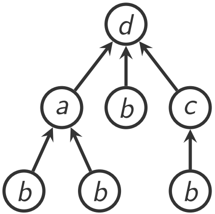

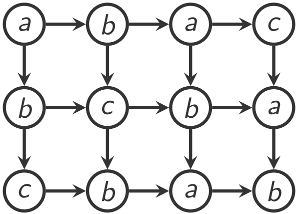

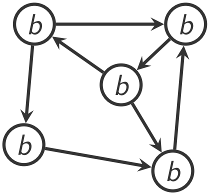

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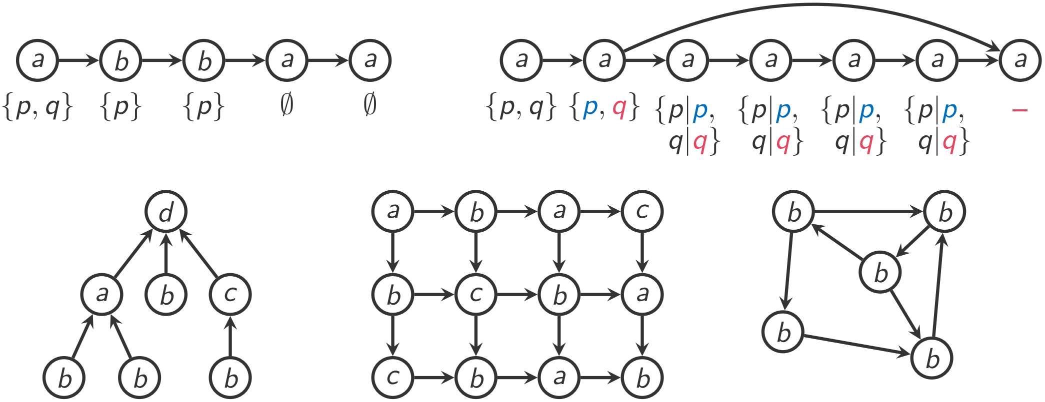
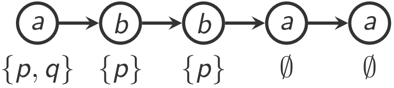
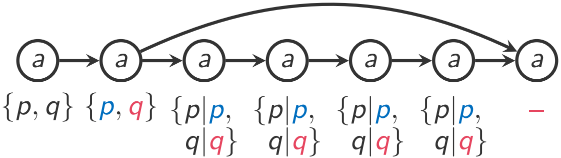
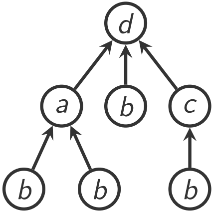


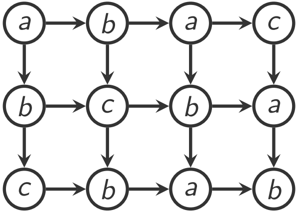
When does modified powerset construction work?

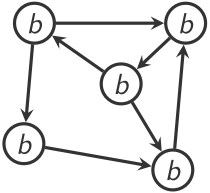
Characterization of determinizable graph classes?











When does modified powerset construction work?

Characterization of determinizable graph classes?

- DAGs with single sink
- Tree width?