Finite Sequentiality of Unambiguous Max-Plus Tree Automata

Erik Paul

Leipzig University

Max-Plus Automata

$$q_0 \xrightarrow{a} q_1 \xrightarrow{a} q_2 \xrightarrow{b} q_3 \xrightarrow{a} q_4 \xrightarrow{b} q_5 \xrightarrow{a} q_6$$

Max-Plus Automata

Weights in $\mathbb{R} \cup \{-\infty\}$

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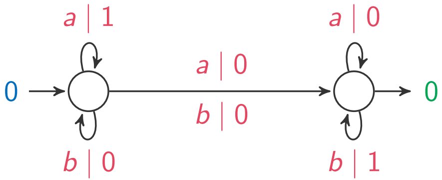

Max-Plus Automata: Ambiguity

sequential / deterministic	one "initial state" no two valid $p \xrightarrow{a} q_1$, $p \xrightarrow{a} q_2$

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unambiguous — $|\mathrm{Run}(w)| \leq 1$

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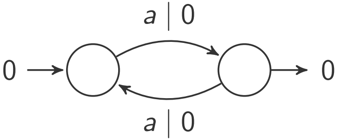
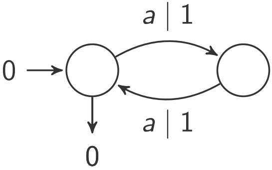
decidable on words for unamb $\mathcal{A}$ — [Mohri]





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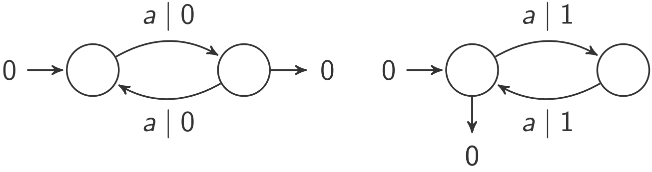
$\mathcal{A}$ max-plus automaton p, q states



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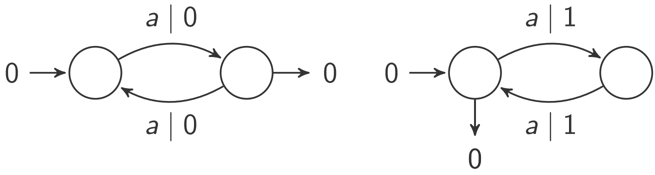

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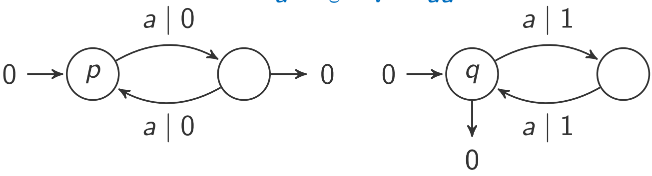

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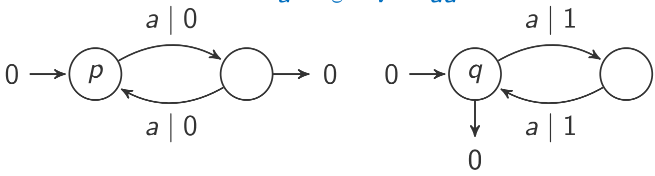

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THM $\mathcal{A}$ unamb $\Rightarrow$ $\quad$ $\mathcal{A}$ determinizable $\leftrightarrow$ no rivals in $\mathcal{A}$

[Mohri]

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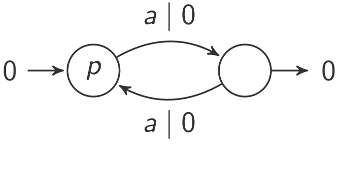

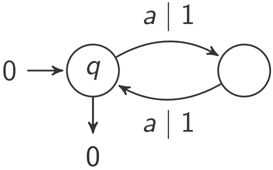

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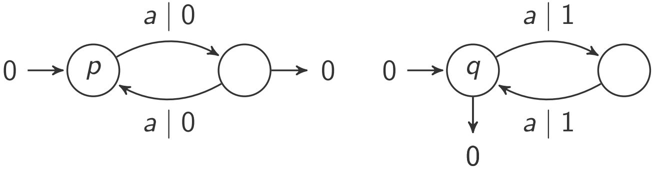

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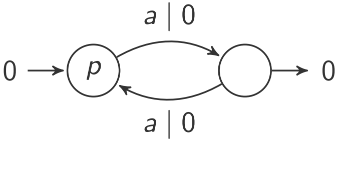

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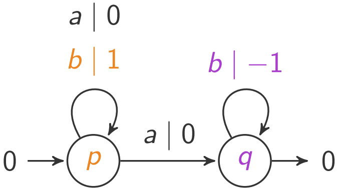

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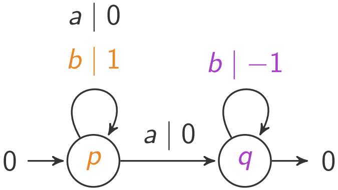

b's before last a

$\Updownarrow$

b's after last a

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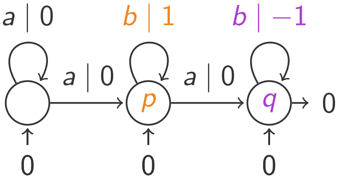

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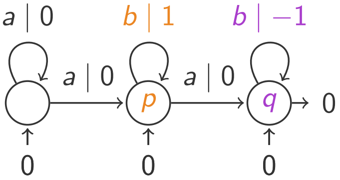

2 deterministic automata:

$a^* b^+ a b^*$ $a^* b^*$

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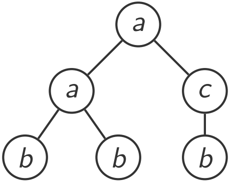
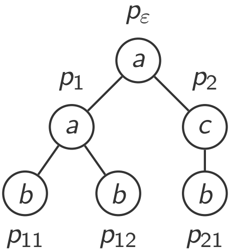
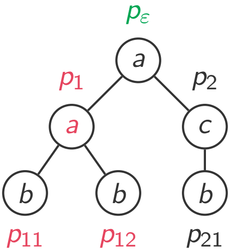

a
a
c
b
b
b


p_ε
a
p_1
p_2
a
c
b
b
b
p_{11}
p_{12}
p_{21}

weight of run =

transition weights + final weight

(p_{11}, p_{12}, a, p_1)



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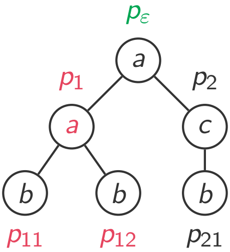
determinism: bottom-up



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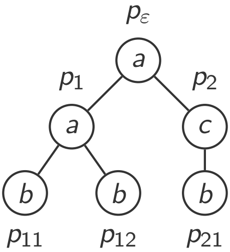

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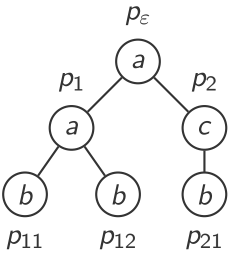

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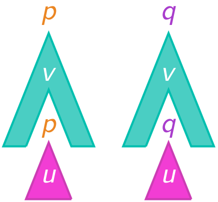

$x \neq y$

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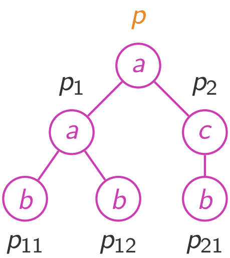

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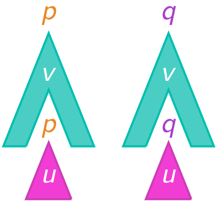

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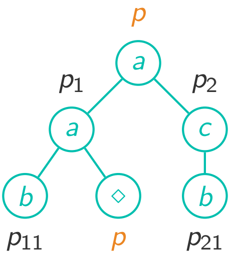

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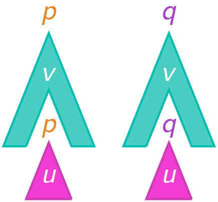

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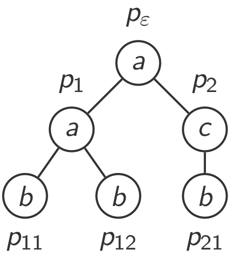

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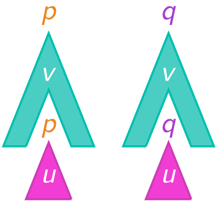

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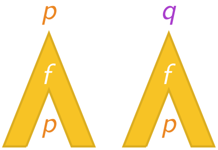
fork



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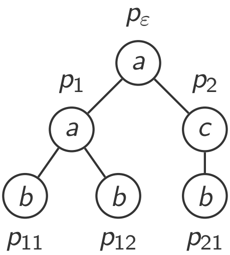

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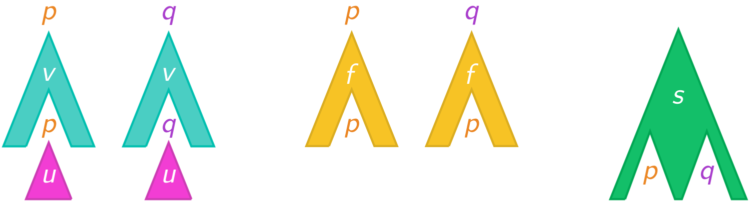

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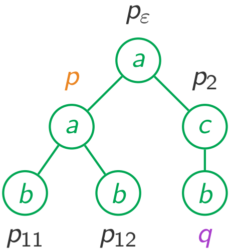

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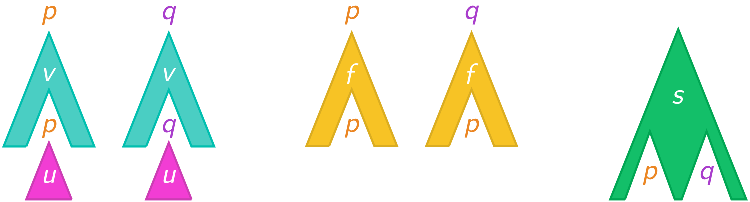

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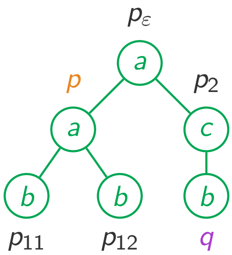

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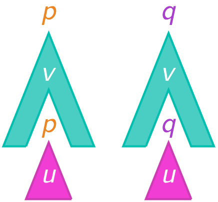

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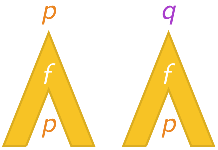
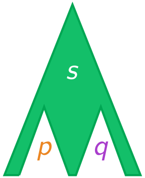


split



NEW THM

$\mathcal{A}$ unamb $\Rightarrow [\![\mathcal{A}]\!]$ fin seq $\leftrightarrow$ no forks $\wedge$ no splits