

# The DIAMOND System for Argumentation

## Preliminary Report

Stefan Ellmauthaler   Hannes Strass

Computer Science Institute  
Leipzig University  
Germany

ASPOCP 2013  
August 25<sup>th</sup> 2013

UNIVERSITÄT LEIPZIG

- 1 Motivation
- 2 Background
  - Abstract Argumentation Frameworks
  - Abstract Dialectical Frameworks
- 3 DIAMOND
- 4 Conclusion & Future Work

## 1 Motivation

## 2 Background

- Abstract Argumentation Frameworks
- Abstract Dialectical Frameworks

## 3 DIAMOND

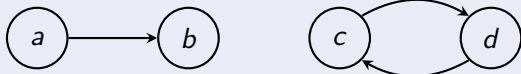
## 4 Conclusion & Future Work

# Motivation: AFs

State of the art in abstract argumentation

## Abstract Argumentation Frameworks (AFs)

- syntactically: directed graphs



- conceptually: nodes are arguments, edges denote attacks between arguments
- semantically: *extensions* are sets of “acceptable” arguments
- immensely popular in the argumentation community
- drawback: can only express attack

# Motivation: ADFs

Recent improvements

## Abstract Dialectical Frameworks (ADFs)

- generalise AFs, arguments are now called *statements*
- can also (although less directly) be visualised as graphs
- edges express that there is some relationship between the two statements
- relationship need not be “attack”, precise nature specified by *acceptance condition* for each statement
- acceptance condition specifies status of node given status of direct predecessors

# Motivation: Argumentation in practice

## Recent Projects and Ideas

- lawsuit analysis with argumentation<sup>1</sup>
- argumentation for discussion analysis on social media (e.g. facebook)<sup>2</sup>
- argumentation for blog-discussions<sup>3</sup>

---

<sup>1</sup>Marcello Ceci and Thomas Gordon. “Browsing Case-law: an Application of the Carneades Argumentation System”. In: *Proceedings of the RuleML2012@ECAI Challenge, at the 6th International Symposium on Rules, Montpellier, France, August 27th-29th, 2012*. Edited by Hassan Aït-Kaci, Yuh-Jong Hu, Grzegorz J. Nalepa, Monica Palmirani, and Dumitru Roman. Volume 874. CEUR Workshop Proceedings. CEUR-WS.org, 2012.

<sup>2</sup>Francesca Toni and Paolo Torroni. “Bottom-Up Argumentation”. In: *TAFIA*. edited by Sanjay Modgil, Nir Oren, and Francesca Toni. Volume 7132. Lecture Notes in Computer Science. Springer, 2011, pages 249–262. ISBN: 978-3-642-29183-8.

<sup>3</sup>Mark Snaith, Floris Bex, John Lawrence, and Chris Reed. “Implementing ArguBlogging”. In: *COMMA*. edited by Bart Verheij, Stefan Szeider, and Stefan Woltran. Volume 245. Frontiers in Artificial Intelligence and Applications. IOS Press, 2012, pages 511–512. ISBN: 978-1-61499-110-6.

## 1 Motivation

## 2 Background

- Abstract Argumentation Frameworks
- Abstract Dialectical Frameworks

## 3 DIAMOND

## 4 Conclusion & Future Work

# Abstract Argumentation

is for determining acceptance of abstract arguments

- argumentation framework<sup>4</sup>  $F = (A, R)$
- $A$  ... set of arguments
- $R \subseteq A \times A$  ... attack relation
- argument  $a \in A$  is *defended* by a set  $S \subseteq A$  if all attackers of  $a$  are attacked by  $S$ :

$$(\forall b \in A)(bRa \implies (\exists c \in S)cRb)$$

---

<sup>4</sup>Phan Minh Dung. "On the Acceptability of Arguments and its Fundamental Role in Nonmonotonic Reasoning, Logic Programming and n-Person Games". In: *Artificial Intelligence* 77 (1995), pages 321–358.



# AF Semantics

are defined via extensions

for AF  $F = (A, R)$ , a set  $S \subseteq A$  of arguments is

- *conflict-free* iff for all  $a, b \in S$ ,  $(a, b) \notin R$
- a conflict-free set  $S$  is
  - ▶ *admissible* iff it defends all arguments it contains
  - ▶ *preferred* iff it is  $\subseteq$ -maximal admissible
  - ▶ *complete* iff it contains exactly the arguments it defends
  - ▶ *grounded* iff it is  $\subseteq$ -minimal complete
  - ▶ *stable* iff it attacks all arguments not in  $S$

# AF Semantics

an example framework

- AF  $F = (A, R)$  with  $A = \{a, b, c, d\}$  and  $R = \{(a, b), (c, d), (d, c)\}$ :



- grounded extension:  $G = \{a\}$
- stable extensions:  $E_1 = \{a, c\}$  and  $E_2 = \{a, d\}$
- preferred extensions:  $E_1, E_2$
- complete extensions:  $G, E_1, E_2$

# Abstract Dialectical Frameworks<sup>5</sup>

## Syntax

### Definition: Abstract Dialectical Framework

An abstract dialectical framework (ADF) is a triple  $D = (S, L, C)$ ,

- $S$  ... set of statements (correspond to AF arguments)
- $L \subseteq S \times S$  ... links  $(par(s) = L^{-1}(s))$
- $C = \{C_s\}_{s \in S}$  ... acceptance conditions

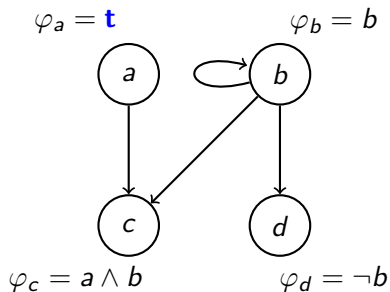
- links denote some kind of dependency relation
- acceptance condition: Boolean function  $C_s : 2^{par(s)} \rightarrow \{\mathbf{t}, \mathbf{f}\}$
- here:  $C_s$  often specified by propositional formula  $\varphi_s$

---

<sup>5</sup>Gerhard Brewka, Stefan Ellmauthaler, Hannes Strass, Johannes Peter Wallner, and Stefan Woltran. "Abstract Dialectical Frameworks Revisited". In: *IJCAI*. to appear. Beijing, China: AAAI Press, Aug. 2013.

# Abstract Dialectical Frameworks

## Example



# Abstract Dialectical Frameworks

## Semantics

### Truth values, interpretations

- truth values: true **t**, false **f**, unknown **u**
- interpretation:  $v : S \rightarrow \{\mathbf{t}, \mathbf{f}, \mathbf{u}\}$
- interpretations can be represented as consistent sets of literals

### Information ordering

- $\mathbf{u} <_i \mathbf{t}$  and  $\mathbf{u} <_i \mathbf{f}$  (as usual  $x \leq_i y$  iff  $x <_i y$  or  $x = y$ )
- *consensus*  $\sqcap$  is greatest lower bound w.r.t.  $\leq_i$ :  
 $\mathbf{t} \sqcap \mathbf{t} = \mathbf{t}$  and  $\mathbf{f} \sqcap \mathbf{f} = \mathbf{f}$ , otherwise  $x \sqcap y = \mathbf{u}$
- information ordering generalised to interpretations:  
 $v_1 \leq_i v_2$  iff  $v_1(s) \leq_i v_2(s)$  for all  $s \in S$

# Abstract Dialectical Frameworks

## Semantics

### Characteristic Operator

- for a valuation  $v$ , we define  $[v]_2 = \{v \leq_i w \mid w \text{ two-valued}\}$
- for ADF  $D$ , we define an operator  $\Gamma_D$  on interpretations
- for interpretation  $v : S \rightarrow \{\mathbf{t}, \mathbf{f}, \mathbf{u}\}$ , the operator yields a new interpretation (the consensus over  $[v]_2$ )

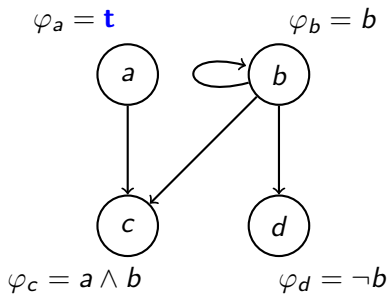
$$\Gamma_D(v) : S \rightarrow \{\mathbf{t}, \mathbf{f}, \mathbf{u}\} \quad s \mapsto \bigsqcap \{w(\varphi_s) \mid w \in [v]_2\}$$

### Semantics

- two-valued  $v$  is a model of  $D$  iff  $v(s) = v(\varphi_s)$  for all  $s \in S$
- $v$  is the grounded model of  $D$  iff it is the least fixpoint of  $\Gamma_D$

# Abstract Dialectical Frameworks

## Semantics: Example



- models:

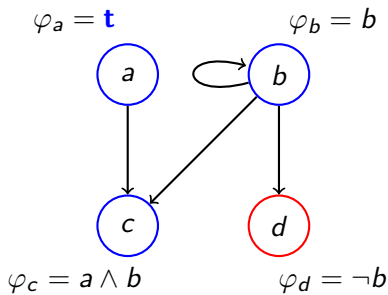
- $v_1 = \{a \mapsto \mathbf{t}, b \mapsto \mathbf{t}, c \mapsto \mathbf{t}, d \mapsto \mathbf{f}\} \hat{=} \{a, b, c, \neg d\}$

- $v_2 = \{a \mapsto \mathbf{t}, b \mapsto \mathbf{f}, c \mapsto \mathbf{f}, d \mapsto \mathbf{t}\} \hat{=} \{a, \neg b, \neg c, d\}$

- grounded model:  $v_3 = \{a \mapsto \mathbf{t}, b \mapsto \mathbf{u}, c \mapsto \mathbf{u}, d \mapsto \mathbf{u}\} \hat{=} \{a\}$

# Abstract Dialectical Frameworks

## Semantics: Example



- models:

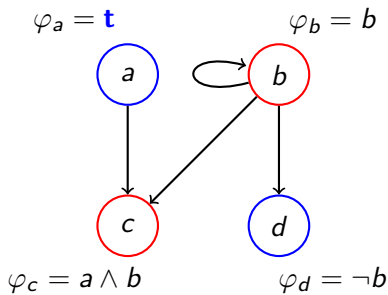
- $\triangleright v_1 = \{a \mapsto \mathbf{t}, b \mapsto \mathbf{t}, c \mapsto \mathbf{t}, d \mapsto \mathbf{f}\} \hat{=} \{a, b, c, \neg d\}$
- $\triangleright v_2 = \{a \mapsto \mathbf{t}, b \mapsto \mathbf{f}, c \mapsto \mathbf{f}, d \mapsto \mathbf{t}\} \hat{=} \{a, \neg b, \neg c, d\}$

- grounded model:  $v_3 = \{a \mapsto \mathbf{t}, b \mapsto \mathbf{u}, c \mapsto \mathbf{u}, d \mapsto \mathbf{u}\} \hat{=} \{a\}$



# Abstract Dialectical Frameworks

## Semantics: Example



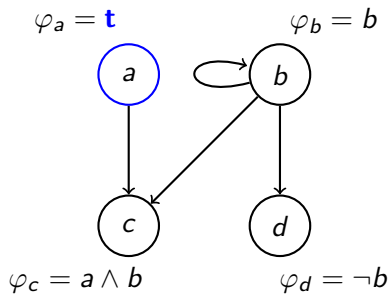
- models:

- $\triangleright v_1 = \{a \mapsto \mathbf{t}, b \mapsto \mathbf{t}, c \mapsto \mathbf{t}, d \mapsto \mathbf{f}\} \hat{=} \{a, b, c, \neg d\}$
- $\triangleright v_2 = \{a \mapsto \mathbf{t}, b \mapsto \mathbf{f}, c \mapsto \mathbf{f}, d \mapsto \mathbf{t}\} \hat{=} \{a, \neg b, \neg c, d\}$

- grounded model:  $v_3 = \{a \mapsto \mathbf{t}, b \mapsto \mathbf{u}, c \mapsto \mathbf{u}, d \mapsto \mathbf{u}\} \hat{=} \{a\}$

# Abstract Dialectical Frameworks

## Semantics: Example



- models:

- $\triangleright v_1 = \{a \mapsto \mathbf{t}, b \mapsto \mathbf{t}, c \mapsto \mathbf{t}, d \mapsto \mathbf{f}\} \hat{=} \{a, b, c, \neg d\}$
- $\triangleright v_2 = \{a \mapsto \mathbf{t}, b \mapsto \mathbf{f}, c \mapsto \mathbf{f}, d \mapsto \mathbf{t}\} \hat{=} \{a, \neg b, \neg c, d\}$

- grounded model:  $v_3 = \{a \mapsto \mathbf{t}, b \mapsto \mathbf{u}, c \mapsto \mathbf{u}, d \mapsto \mathbf{u}\} \hat{=} \{a\}$

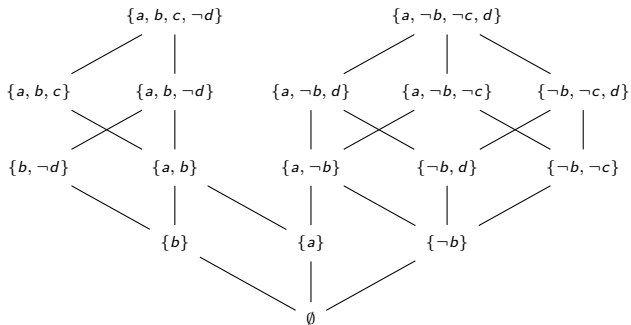
# Abstract Dialectical Frameworks

## Admissible Semantics

### Definition: Admissible

Interpretation  $v$  is *admissible* for ADF  $D$  iff  $v \leq_i \Gamma_D(v)$ .

- intuitively: does not contain too much information
- example ADF has 16 admissible interpretations:



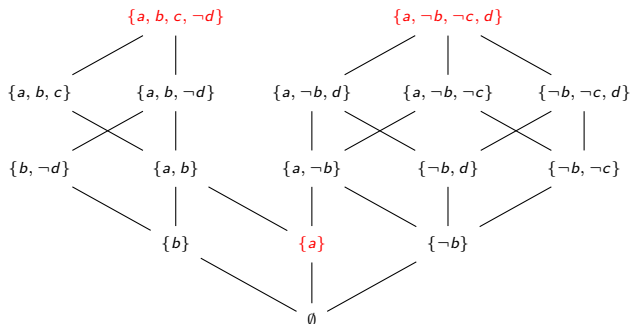
# Abstract Dialectical Frameworks

## Complete Semantics

### Definition: Complete

Interpretation  $v$  is *complete* for ADF  $D$  iff  $v = \Gamma_D(v)$ .

- complete interpretations are stationary w.r.t. revision operator



- 1 Motivation
- 2 Background
  - Abstract Argumentation Frameworks
  - Abstract Dialectical Frameworks
- 3 **DIAMOND**
- 4 Conclusion & Future Work

# DIAMOND

DIAl ectical MOdels eNcoDing

- Set of encodings utilizing the Potsdam Answer Set Solving Collection<sup>6</sup>
- Python script for command-line-interface
- ECL<sup>i</sup>PS<sup>e</sup>-Prolog program for fast input-format conversions

---

<sup>6</sup>M. Gebser, R. Kaminski, B. Kaufmann, M. Ostrowski, T. Schaub, and M. Schneider. "Potassco: The Potsdam Answer Set Solving Collection". In: *AI Communications* 24.2 (2011), pages 105–124.

# DIAMOND

## Alternative Input Instance Representations

### Propositional Formula Representation

- Statements are defined via the unary predicate `statement(a)`.
- Their acceptance condition is a propositional formula, represented by `ac(s,  $\phi$ )`, where  $\phi$  is a prop. formula in prefix notation
- links between statements are implicitly defined by variable occurrence in  $\phi$
- allowed constants in  $\phi$  are `c(v)`, `c(f)`
- allowed operators are `neg`, `and`, `or`, `imp`, and `iff`

### Example

```
statement(a). statement(b). statement(c). statement(d).  
ac(a,c(v)).      ac(b,b).  
ac(c,and(a,b)).  ac(d,neg(b)).
```

# DIAMOND

## Instance Representation

### Predicates

- unary predicate  $s(a)$  . to represent statements
- binary predicate  $l(b,a)$  . to represent links
- unary and ternary predicates  $ci$  and  $co$  to model the Boolean acceptance function

### Example

```
s(a). s(b). s(c). s(d).  
l(a,c). l(b,b). l(b,c). l(b,d).  
ci(a).  
co(b). ci(b,1,b).  
co(c). co(c,1,a). co(c,2,b). ci(c,3,a). ci(c,3,b).  
ci(d). co(d,1,b).
```



# DIAMOND

## Instance Representation

### Predicates

- unary predicate  $s(a)$  . to represent statements
- binary predicate  $l(b,a)$  . to represent links
- unary and ternary predicates  $ci$  and  $co$  to model the Boolean acceptance function

### Example

```
s(a). s(b). s(c). s(d).  
l(a,c). l(b,b). l(b,c). l(b,d).  
ci(a).  
co(b). ci(b,1,b).  
co(c). co(c,1,a). co(c,2,b). ci(c,3,a). ci(c,3,b).  
ci(d). co(d,1,b).
```

# DIAMOND

## Implementation of $\Gamma_D$

- semantics are based upon fixpoint computation of  $\Gamma_D$
- binary predicates  $\text{in}(X, I)$  and  $\text{out}(X, I)$  represent an interpretation at  $\text{step}(I)$
- operator applied if  $\text{step}(I)$  exists

# DIAMOND

Implementation of  $\Gamma_D$

## “Interesting” links at step I

```
ciui(S,J,I) :- lin(X,S,I), not ci(S,J,X), ci(S,J).  
ciui(S,J,I) :- lout(X,S,I), ci(S,J,X).  
cii(S,J,I) :- not ciui(S,J,I), ci(S,J), step(I).
```

## Validity check

```
pmodel(S,I) :- cii(S,J,I). pmodel(S,I) :- verum(S), step(I).  
pmodel(S,I) :- not lin(S,I), ci(S), step(I).  
valid(S,I) :- pmodel(S,I), not imodel(S,I).
```

## Fixpoint check

```
nofp(I) :- in(X,I), not valid(X,I), step(I).  
nofp(I) :- valid(X,I), not in(X,I), step(I).  
nofp(I) :- out(X,I), not unsat(X,I), step(I).  
nofp(I) :- unsat(X,I), not out(X,I), step(I).  
fp(I) :- not nofp(I), step(I).
```

# DIAMOND

Implementation of  $\Gamma_D$

## “Interesting” links at step I

```
ciui(S,J,I) :- lin(X,S,I), not ci(S,J,X), ci(S,J).  
ciui(S,J,I) :- lout(X,S,I), ci(S,J,X).  
cii(S,J,I) :- not ciui(S,J,I), ci(S,J), step(I).
```

## Validity check

```
pmodel(S,I) :- cii(S,J,I). pmodel(S,I) :- verum(S), step(I).  
pmodel(S,I) :- not lin(S,I), ci(S), step(I).  
valid(S,I) :- pmodel(S,I), not imodel(S,I).
```

## Fixpoint check

```
nofp(I) :- in(X,I), not valid(X,I), step(I).  
nofp(I) :- valid(X,I), not in(X,I), step(I).  
nofp(I) :- out(X,I), not unsat(X,I), step(I).  
nofp(I) :- unsat(X,I), not out(X,I), step(I).  
fp(I) :- not nofp(I), step(I).
```

# DIAMOND

Implementation of  $\Gamma_D$

## “Interesting” links at step I

```
ciui(S,J,I) :- lin(X,S,I), not ci(S,J,X), ci(S,J).  
ciui(S,J,I) :- lout(X,S,I), ci(S,J,X).  
cii(S,J,I) :- not ciui(S,J,I), ci(S,J), step(I).
```

## Validity check

```
pmodel(S,I) :- cii(S,J,I). pmodel(S,I) :- verum(S), step(I).  
pmodel(S,I) :- not lin(S,I), ci(S), step(I).  
valid(S,I) :- pmodel(S,I), not imodel(S,I).
```

## Fixpoint check

```
nofp(I) :- in(X,I), not valid(X,I), step(I).  
nofp(I) :- valid(X,I), not in(X,I), step(I).  
nofp(I) :- out(X,I), not unsat(X,I), step(I).  
nofp(I) :- unsat(X,I), not out(X,I), step(I).  
fp(I) :- not nofp(I), step(I).
```

# DIAMOND

## Semantics

### Admissible/Complete

```
step(0).  
in(S,0):s(S).  
out(S,0):s(S).  
:- in(S,0), out(S,0).  
:- in(S), not valid(S,0).  
:- out(S), not unsat(S,0).  
  
:- nofp(0).
```

### Grounded

```
maxit(I) :- I:=s(S). step(0).  
in(S,I+1) :- valid(S,I). out(S,I+1) :- unsat(S,I).  
step(I+1) :- step(I), not maxit(I).  
in(S) :- fp(I), in(S,I).  
out(S) :- fp(I), out(S,I).  
udec(S) :- fp(I), s(S), not in(S), not out(S).
```

# DIAMOND

## Semantics

### Admissible/Complete

```
step(0).  
in(S,0):s(S).  
out(S,0):s(S).  
:- in(S,0), out(S,0).  
:- in(S), not valid(S,0).  
:- out(S), not unsat(S,0).  
  
:- nofp(0).
```

### Grounded

```
maxit(I) :- I:=s(S). step(0).  
in(S,I+1) :- valid(S,I). out(S,I+1) :- unsat(S,I).  
step(I+1) :- step(I), not maxit(I).  
in(S) :- fp(I), in(S,I).  
out(S) :- fp(I), out(S,I).  
udec(S) :- fp(I), s(S), not in(S), not out(S).
```

- 1 Motivation
- 2 Background
  - Abstract Argumentation Frameworks
  - Abstract Dialectical Frameworks
- 3 DIAMOND
- 4 Conclusion & Future Work



# Conclusion

- DIAMOND utilizes ASP for implementing abstract argumentation (like ASPARTIX<sup>7</sup> does for AFs)
- All currently known representations for ADFs are understood by DIAMOND
- More complex semantics (e.g. preferred) can be implemented with Meta-ASP<sup>8</sup>
- ADFsys<sup>9</sup> is a similar system, but it
  - ▶ uses different semantics, and
  - ▶ it already needs disjunctive programs for the computation of  $\Gamma_D$

---

<sup>7</sup>Uwe Egly, Sarah Alice Gaggl, and Stefan Woltran. “Answer-set programming encodings for argumentation frameworks”. In: *Argument and Computation 1.2* (2010), pages 147–177.

<sup>8</sup>M. Gebser, R. Kaminski, and T. Schaub. “Complex Optimization in Answer Set Programming”. In: *Theory and Practice of Logic Programming 11.4–5* (2011), pages 821–839.

<sup>9</sup>Stefan Ellmauthaler and Johannes Peter Wallner. “Evaluating Abstract Dialectical Frameworks with ASP”. In: *COMMA*, edited by Bart Verheij, Stefan Szeider, and Stefan Woltran. Volume 245. *Frontiers in Artificial Intelligence and Applications*. IOS Press, 2012, pages 505–506. ISBN: 978-1-61499-110-6.

- Implementation of further semantics for ADFs
- Add more usability to DIAMOND
- Use ADFs and DIAMOND for discussion analysis in the web
- ...

**Thank you!**

# AF Semantics: Alternative Formulation

via operators

## Definition

For AF  $F = (A, R)$ , define two operators

- $U_F(S) = \{a \in A \mid S \text{ does not attack } a\}$  (Pollock)
  - $V_F(S) = \{a \in A \mid S \text{ defends } a\}$  (Dung)
- 
- $U_F$  is  $\subseteq$ -antimonotone ( $S_1 \subseteq S_2$  implies  $U_F(S_2) \subseteq U_F(S_1)$ )
  - $V_F$  is  $\subseteq$ -monotone ( $S_1 \subseteq S_2$  implies  $V_F(S_1) \subseteq V_F(S_2)$ )

## Lemma (Dung)

For any AF  $F$ , we have  $U_F^2 = V_F$ .

# AF Semantics: Alternative Formulation

are defined via extensions

for AF  $F = (A, R)$ , a set  $S \subseteq A$  of arguments is

- *conflict-free* iff  $S \subseteq U_F(S)$
- *admissible* iff  $S \subseteq U_F(S)$  and  $S \subseteq V_F(S)$
- *preferred* iff it is  $\subseteq$ -maximal admissible
- *complete* iff  $S \subseteq U_F(S)$  and  $S = V_F(S)$
- *grounded* iff it is the  $\subseteq$ -least fixpoint of  $V_F$
- *stable* iff  $S = U_F(S)$

# Stable Models

for ADFs

## Definition: Associated Extension

- For an interpretation  $v$ , the set  $E_v = \{s \in S \mid v(s) = \mathbf{t}\}$  defines the unique *extension* associated with  $v$ .

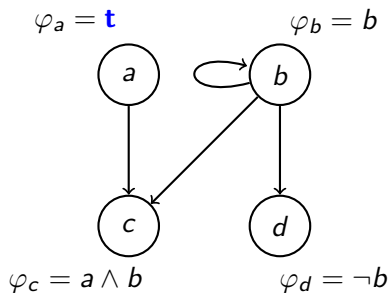
## Definition: Stable Model

Let  $D = (S, L, C)$  be an ADF with  $C = \{\varphi_s\}_{s \in S}$ . A two-valued model  $v$  of  $D$  is a *stable model* of  $D$  iff  $E_v$  equals the grounded extension of the reduced ADF  $D^v = (E_v, L^v, C^v)$ , where

- $L^v = L \cap (E_v \times E_v)$  and
- for  $s \in E_v$  we set  $\varphi_s^v = \varphi_s[b/\mathbf{f} : v(b) = \mathbf{f}]$ .

# Abstract Dialectical Frameworks

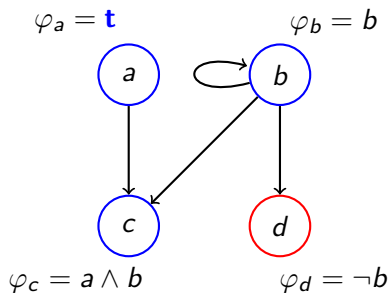
## Semantics: Example



- $v_1 \hat{=} \{a, b, c, \neg d\}$ : reduct  $D^{v_1}$  with grounded extension  $\{a\}$ ,  $\{a\} \neq E_{v_1}$ , thus  $v_1$  not stable (statements  $b$  and  $c$  unjustified)
- $v_2 \hat{=} \{a, \neg b, \neg c, d\}$ : reduct  $D^{v_2}$  with grounded extension  $\{a, d\} = E_{v_2}$ , thus  $v_2$  stable

# Abstract Dialectical Frameworks

## Semantics: Example

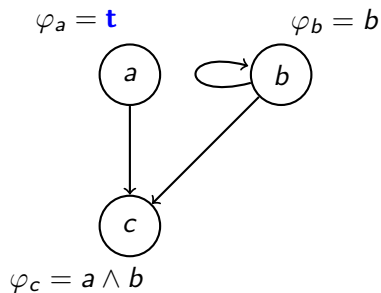


- $v_1 \hat{=} \{a, b, c, \neg d\}$ : reduct  $D^{v_1}$  with grounded extension  $\{a\}$ ,  $\{a\} \neq E_{v_1}$ , thus  $v_1$  not stable (statements  $b$  and  $c$  unjustified)
- $v_2 \hat{=} \{a, \neg b, \neg c, d\}$ : reduct  $D^{v_2}$  with grounded extension  $\{a, d\} = E_{v_2}$ , thus  $v_2$  stable



# Abstract Dialectical Frameworks

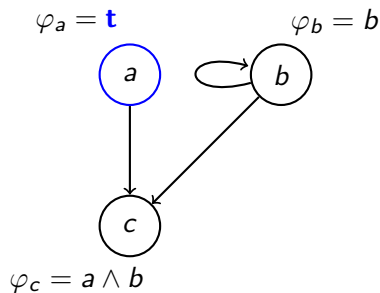
## Semantics: Example



- $v_1 \hat{=} \{a, b, c, \neg d\}$ : reduct  $D^{v_1}$  with grounded extension  $\{a\}$ ,  $\{a\} \neq E_{v_1}$ , thus  $v_1$  not stable (statements  $b$  and  $c$  unjustified)
- $v_2 \hat{=} \{a, \neg b, \neg c, d\}$ : reduct  $D^{v_2}$  with grounded extension  $\{a, d\} = E_{v_2}$ , thus  $v_2$  stable

# Abstract Dialectical Frameworks

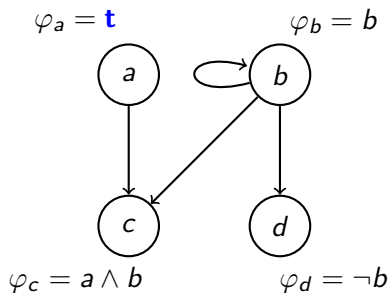
## Semantics: Example



- $v_1 \hat{=} \{a, b, c, \neg d\}$ : reduct  $D^{v_1}$  with grounded extension  $\{a\}$ ,  $\{a\} \neq E_{v_1}$ , thus  $v_1$  not stable (statements  $b$  and  $c$  unjustified)
- $v_2 \hat{=} \{a, \neg b, \neg c, d\}$ : reduct  $D^{v_2}$  with grounded extension  $\{a, d\} = E_{v_2}$ , thus  $v_2$  stable

# Abstract Dialectical Frameworks

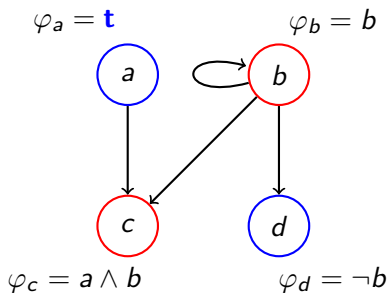
## Semantics: Example



- $v_1 \hat{=} \{a, b, c, \neg d\}$ : reduct  $D^{v_1}$  with grounded extension  $\{a\}$ ,  $\{a\} \neq E_{v_1}$ , thus  $v_1$  not stable (statements  $b$  and  $c$  unjustified)
- $v_2 \hat{=} \{a, \neg b, \neg c, d\}$ : reduct  $D^{v_2}$  with grounded extension  $\{a, d\} = E_{v_2}$ , thus  $v_2$  stable

# Abstract Dialectical Frameworks

## Semantics: Example



- $v_1 \hat{=} \{a, b, c, \neg d\}$ : reduct  $D^{v_1}$  with grounded extension  $\{a\}$ ,  $\{a\} \neq E_{v_1}$ , thus  $v_1$  not stable (statements  $b$  and  $c$  unjustified)
- $v_2 \hat{=} \{a, \neg b, \neg c, d\}$ : reduct  $D^{v_2}$  with grounded extension  $\{a, d\} = E_{v_2}$ , thus  $v_2$  stable

# Abstract Dialectical Frameworks

## Semantics: Example

$$\varphi_a = \mathbf{t}$$



$$\varphi_d = \neg \mathbf{f}$$

- $v_1 \hat{=} \{a, b, c, \neg d\}$ : reduct  $D^{v_1}$  with grounded extension  $\{a\}$ ,  $\{a\} \neq E_{v_1}$ , thus  $v_1$  not stable (statements  $b$  and  $c$  unjustified)
- $v_2 \hat{=} \{a, \neg b, \neg c, d\}$ : reduct  $D^{v_2}$  with grounded extension  $\{a, d\} = E_{v_2}$ , thus  $v_2$  stable

# Abstract Dialectical Frameworks

## Semantics: Example

$$\varphi_a = \mathbf{t}$$

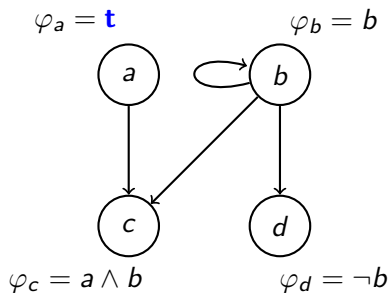


$$\varphi_d = \neg \mathbf{f}$$

- $v_1 \hat{=} \{a, b, c, \neg d\}$ : reduct  $D^{v_1}$  with grounded extension  $\{a\}$ ,  $\{a\} \neq E_{v_1}$ , thus  $v_1$  not stable (statements  $b$  and  $c$  unjustified)
- $v_2 \hat{=} \{a, \neg b, \neg c, d\}$ : reduct  $D^{v_2}$  with grounded extension  $\{a, d\} = E_{v_2}$ , thus  $v_2$  stable

# Abstract Dialectical Frameworks

## Semantics: Example



- $v_1 \hat{=} \{a, b, c, \neg d\}$ : reduct  $D^{v_1}$  with grounded extension  $\{a\}$ ,  $\{a\} \neq E_{v_1}$ , thus  $v_1$  not stable (statements  $b$  and  $c$  unjustified)
- $v_2 \hat{=} \{a, \neg b, \neg c, d\}$ : reduct  $D^{v_2}$  with grounded extension  $\{a, d\} = E_{v_2}$ , thus  $v_2$  stable

**Thank you!**