Magisterarbeit

Semantical Investigations in the Logic of Actions and Norms

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1 Introduction

Deontic logic is concerned with the normative concepts such as obligation, permission, or prohibition. These concepts can be applied to actions or to states of affairs. In ordinary language the first use seems to prevail. Thus, deontic logic should be primarily designed to accommodate norms about action. In other words, we need a logic of the concept of "Ought-to-do" (Tun-Sollen.) However, contemporary deontic logic cannot capture the notion of Tun-Sollen adequately. It can at most formalize the concept of "Ought-to-be" (Sein-Sollen,) despite all efforts of von Wright ([43], [44], [45], [47]) and others to develop logics of norms about action. The present paper is an attempt to fill this gap. I try to develop a system of deontic logic which possesses the resources to formalize adequately action and change. I shall show that my system is not afflicted with the well-known paradoxes of the standard modal approach to deontic logic.

My principal working hypothesis is as follows. Contemporary deontic logic is dominated by the standard modal approach, but this approach can at most formalize the concept of Ought-to-be. The attemts to interpret standard deontic logic as a logic the practical concept of Ought or the concept of Ought-to-do leads to several paradoxical results. Most of the so-called paradoxes of deontic logic, for example Ross' paradox and the Good Samaritan paradox, lose their paradoxical character if they are interpreted properly, viz., as sentences of the logic of ought-to-be. This shows that the notion of Ought-to-do cannot be captured formally in the standard modal approach. To formalize the practical concept of Ought we need another logic. I shall develop a deontic logic based on a logic of action. The logic of action is in turn an extension of propositional dynamic logic.

My paper shall focus on problems of semantics. Syntactical issues such as questions of axiomatization will play only a secondary role. The main objective of the essay is to find a plausible semantics for normative statements. The formal semantics of normative concepts must be derived from their informal semantics in natural language. The meaning and truth conditions of the normative statements in the formal semantics to be found must agree with those in practical discourse, otherwise the logic would be useless and does not deserve the name "deontic logic." In order to test the suitability of the logic we must show that it allows us to reconstruct concrete normative circumstances in a way that does not violate our natural language intuitions. In particular, the well-known paradoxes of deontic logic must be avoided.

The terminus "semantics for normative statements" is still ambiguous: it can mean semantics for norms or semantics for norm-propositions, because normative sentences can be used in two different ways: prescriptively or descriptively. For example, a sentence like "You may go to the cinema now" can be used to give a permission. It functions as a prescription, and is neither true nor false. It is quite controversial if a logic of prescriptions is possible. I shall not here discuss the issue, but adopt the descriptive view of deontic sentences. According to this view, deontic sentences (represented by the formulae of deontic logic) describe what is regarded as permitted, obligatory, forbidden, etc., in some unspecified normative system. For example, the same sentence "You may go to the cinema now" can be used as a proposition to describe that such a permission exists, according to some underlying norm-system. When I use the sentence in this sense I do not give an permission, but describe that someone has given the permission, and my hearer can do things according to the norm. My investigations will concentrate only on questions of the semantics for descriptive normative sentences. Those sentences have, in contrast to the presciptive ones, a definite truth-value, true or false. The logic we get will be then a logic of norm-propositions.

What is regarded as permitted, obligatory or forbidden can be states of affairs or actions, or more precisely, names of generic acts. Let x be a variable which stands for an act-name. It can be a simple or a complex act, comprising many simpler acts. The next considerations apply for norms about actions, but they can also be transferred easily to the case of norms about states of affairs. If we use the operator "P" for permission, then "Px" would be a deontic sentence. It is usually read "x is permitted", but according to our descriptive interpretation, its proper reading should be: "Norm-system N permits the performance of x", where N is the normative system under consideration. However, the formula "Px" carries too little information. It does not say, for example, who may do x, if all agents may do this action or only certain agents. Moreover, it does not say when the action x is permitted. Thus, many relevant factors which are needed for a meaningful normative argumentation would be not available.

To cope with the first difficulty we can introduce a parameter for the agent. Then we can form agent-specific deontic sentences such as $P(i, x)$ with the intended meaning "the agent i may do x". A general deontic sentences such as Px are often used in the sense that all agents may perform the action x, in particular, the agent under consideration is allowed to do x. For the remaining of the paper I assume that we have only one agent, so that the distinction between agent-dependent and agent-independent norms disappears. Thus, the index i can be omitted. The generalization to the case of multiple agents is straightforward. The term "agent" should not be understood too narrowly: it can denote a single person, but it can also be used to denote a group of people or even robots.

Let us turn to the temporal dimension of normative sentences. Sometimes normative statements contain an explicit temporal reference, from which we can conclude when a certain action is permitted or forbidden. However, it is very often the case that normative sentences do not refer to any time. In such cases we can only decide from the context what is most probably meant. A number of those sentences say that the norm exists *now*. For example, the obligation given by the imperative "Shut the door!" belongs to this class. Such sentences can be formalized as atomic deontic sentences. In this case, following the approach in temporal logic we assume tacitly that a time point is fixed ("the immediate situation", "now") where these sentences are valuated, so that we do not need to introduce a temporal parameter to the language. However, there are a number of normative sentences which cannot be formalized in this way. These sentences express that the corresponding norm exists at all time point. They could be termed "basic norms" or "universal norms." For example, the norm "it is forbidden to kill" should be treated as a universal one, which always exists. It should not be formalized as "Fy", where "F" is the operator of prohibition ("forbidden") and "y" is the act "kill". The logical form of this sentence is rather "LFy", where "L" is the temporal operator "at any time" or "always", i.e., a sentence "LA" is true if and only if the sentence "A" is true at all possible times, or "always true", for short. Standard deontic logic does not pay enough attention to this distinction between time-dependent and time-independent norms, so that a number of paradoxes arise, for example Chisholm's contrary-to-duty paradox. I shall show later that such paradoxical results can be avoided easily if the mentioned distinction is considered.

The plan of the paper is as follows. In the next section I shall review briefly the standard modal approach to deontic logic and examine the motivations for developing a semantics of deontic logic. This examination is important because the formal semantics must be based on intuitions of the practical discourse. I shall show that there are two plausible starting points. On the one hand one can imagine that there are possible states of affairs ("possible worlds") where all obligations of our actual world are fulfilled and investigate what must be the case in such states of affairs. Thus, we can try to derive the truth-conditions of the normative sentences with recourse to imaginary worlds which are in some sense deontic alternatives of our world. This is the way initiated by J. Hintikka, S. Kanger, and others. On the other hand we can try to derive the semantics of the normative sentences from the semantics of other, for us more familiar notions. A. R. Anderson has suggested a way to reduce deontic logic to modal logic. He proposed a schema how to define the normative concept in terms of the concept of necessity and that of a sanction. Thus, if we have a semantics of these two concepts then we can derive a semantics of the normative concepts. I shall examine the relationship between the two semantical approaches.

In the next step I discuss briefly some paradoxes of deontic logic. I shall show that many of them only arise because one tries to interpret formulae of a logic of "Ought-to-be" as theorems of a logic of the concept of Ought-to-do. However, I show that Chisholm's paradox of contrary-to-duty imperatives cannot be solved in this way. This paradox arises because the standard approach does not take into account the temporal dimension of normative sentences. In order to solve the paradox I shall suggest a simple system of temporal deontic logic. But even this logic lacks the resources to formalize adequately the concepts of action and change. To solve the problem I define a rudimentary logic of action based on propositional dynamic logic. On the basis of this logic of action I develop a system of dynamic deontic logic and show that this logic is suited to formalize the concept of "Ought-to-do." The semantics of this dynamic deontic logic is based on similar intuitions as the semantics for standard deontic logic, but the formal treatment of the intuitive ideas is quite different.

2 Standard deontic logic

2.1 Syntax and Proof Theory

By "standard deontic logic" one often means the system SDL, first formulated by B. Hansson ([19]), which goes back essentially to G. H. von Wright ([42]). I shall represent it briefly in a slightly modified form, insofar as the restriction on the syntax of the logic is removed.

Definition 1 *A set of atomic formulae is assumed. The set of SDL-formulae is the least set containing all atomic formulae and is closed under the following rule: if A, B are formulae then $\neg A$, $A \wedge B$, OA are formulae.*

OA is read "A is obligatory" where A is any formula. It may describe an action, a state of affairs, a norm, or some mixture of all that. Originally, von Wright used the "objective formulae" (formed from the atomic formulae using the Boolean connectives) as names to denote actions. Higher-order norms (such as OOA) and mixed formulae (such as $A \supset OA$) are not allowed ([42].) However, this reading causes difficulties with the interpretation of the connectives. This leads to a re-interpretation of the formulae as sentences describing states of affairs, and not as act-names. Most authors have adopted this view. (cf. [19], [15].) However, I shall show later that this interpretation cannot capture the characteristic features of actions, so that it makes it impossible to apply deontic logic to analyze norms about actions. Many authors have suggested the reading of "Op" as "it is obligatory to see to it that p" or "it is obligatory to bring about that p" (see, e.g., [43], [28].) At first sight this reading seems to accommodate also action sentences. However, it does not offer a genuine logic of action, because it is unable of doing justice to the action connectives "not", "and", "or", as we shall see when discussing the paradoxes.

Permission and prohibition are introduced by the following definition:

$$PA =_{def} \neg O \neg A$$

$$FA =_{def} O \neg A$$

The other truth-functional connectives (conjunction, implication, biconditional,) as well as the propositional constants $true$ and $false$ are defined as follows:

$$(A \vee B) =_{def} \neg(\neg A \wedge \neg B)$$

$$(A \supset B) =_{def} (\neg A \vee B)$$

$$(A \leftrightarrow B) =_{def} ((A \supset B) \wedge (B \supset A))$$

$$true =_{def} (A \vee \neg A)$$

$$false =_{def} (A \wedge \neg A)$$

The usual conventions on parenthesis are adopted. The system SDL is axiomatized by the well-known modal axioms K and D (in the language with the operator O.) This system is also known in the literature on modal logic under the name KD.

Definition 2 *The system SDL is determined by the following axiom schemata and rules of inference:*

A0. All axiom schemata of the propositional calculus

A1. $O(A \supset B) \supset (OA \supset OB)$

A2. $OA \supset PA$

R1. Modus ponens: from A and $A \supset B$, infer B

R2. O-Generalization: if A is atheorem then so is OA

The notions of a proof, a provable formula and a set of consistent formulae are defined as usual.

2.2 The Standard Semantics: Deontically Perfect Worlds

The intuitive idea behind this semantics is as follows: in our actual world the norms may or may not be satisfied. But we can imagine possible worlds where all obligations (or O-norms) of our world are satisfied. Such possible worlds are called deontically perfect or deontic alternatives of our world. (In every deontic alternative of our world there exist also certain norms which may or may not be identical with the norms of our world. Whether or not these norms are satisfied in this deontic alternative is another question.) In our actual world something is obligatory if and only if it is the case in all deontically perfect worlds. (If it were not realized then the deontic alternatives would not be "deontically perfect".) Analogously, something is permitted if and only if it is the case in at least one deontically perfect world, and something is forbidden if and only if it is not realized in any deontic alternative.

Definition 3 *A deontic model is a structure $M = (W, R, V)$ where W is a non-empty set, R is a binary relation on W, and V is a function which assigns to every atomic formula p a set $V(p) \subseteq W$.*

The set W is often called the set of possible worlds. For later comparison we note the following. As all sentences of SDL are atemporal, a possible world from W should be interpreted as the total history of a world through time. Later we shall encounter another interpretation where the worlds are interpreted at "snapshots" of the world at different time points. The relation R is called "deontic alternativeness relation." wRv should express that v is a deontic alternative of w in the sense that all duties of w are fulfilled in v. The set $V(p)$ is interpreted as the set of all possible worlds where p is true. A model $M = (W, R, V)$ is called serial if the relation R is serial, i.e., if for all $w \in W$ there is some $v \in W$ such that wRv.

Definition 4 *Let $M = (W, R, V)$ be a deontic model, $w \in W$ and A a formula of SDL. The relation $M, w \models A$ (read: A is true at w in M) is defined recursively as follows:*

1. *$M, w \models p$ iff $w \in V(p)$ for all $p \in At$*
2. *$M, w \models \neg A$ iff $M, w \not\models A$*
3. *$M, w \models A \wedge B$ iff $M, w \models A$ and $M, w \models B$*
4. *$M, w \models OA$ iff $M, v \models A$ for all $v \in W$ with wRv.*

If it is understood which model M is meant we shall write $w \models A$ for $M, w \models A$. From the definition we can derive the truth condition for the operators P and F as follows:

- $M, w \models PA$ iff $M, v \models A$ for some $v \in W$ with wRv.
- $M, w \models FA$ iff $M, v \not\models A$ for all $v \in W$ with wRv.

Definition 5 *A formula A is said to be valid in a model M (in symbol: $M \models A$) if and only if it is true in all possible worlds in M, that is, $M, w \models A$ for all $w \in W$. Let $\mathcal{K}$ be a class of deontic models. A is called valid in $\mathcal{K}$, in symbol $\mathcal{K} \models \mathcal{A}$ if it is valid in all models $M \in \mathcal{K}$. The formula A is called satisfiable (relatively to $\mathcal{K}$) if and only if it has a model in $\mathcal{K}$, that is, if there is a model M and a world w such that $M, w \models A$.*

The following well-known theorem states that SDL axiomatizes completely the class of serial deontic models.

Theorem 6 (Soundness and Completeness of SDL) *A formula A is a theorem of SDL if and only if it is valid in all serial models.*

The proof is quite standard and is omitted here.

2.3 Deriving Semantics for Deontic Logic from the Semantics for Modal Logic

A. R. Anderson and S. Kanger have observed that standard deontic logic can be reduced to modal logic in the following sense: if one adds a propositional constant S into the language of modal logic and then introduces the deontic operator O by the definition $OA =_{def} \Box(\neg A \supset S)$, where $\Box$ is the necessity operator, then the axioms of SDL can be proved from the axioms of modal logic, plus the schema: $\neg\Box S$. The intuition behind the famous "reduction schema" is as follows: normative concepts are closely related to the concept of punishment, or sanction. It would be pointless if a norm exists, but any people who violate the norm are immune to punishment (or sanction): in such a case it is not quite reasonable to speak of a norm. Thus, we can introduce the propositional constant S describing a state of affairs which counts as a sanction or punishment. The interpretation of S as the liability (as opposed to immunity) to sanction, is more plausible, cf. [15], p. 20. I shall adopt this more liberal interpretation. An action is obligatory if and only if one cannot omit it without becoming liable to punishment. The permission to do something means that one can do it without sanction, and prohibition means that the performance of a forbidden action necessarily leads to sanction.

The propositional constant S can be treated as semantically primitive, so that its semantical treatment falls within the scope of the semantics for propositional logic. The semantics of modal logic is also well-developed. Thus, via Anderson's reduction schema we can derive the semantics for deontic logic from the semantics of modal logic. Assuming the language $\mathcal{L}_M$ which extends the standard language of modal logic (with the necessity operator $\Box$) by the propositional constant S, we can define the semantics for deontic logic in terms of modal semantics. The deontic operators are introduced into the language by the following definitions:

$$OA =_{def} \Box(\neg A \supset S)$$

$$PA =_{def} \neg O \neg A$$

$$FA =_{def} O \neg A$$

By the deontic fragment of the modal language $\mathcal{L}_M$ we mean the least set of all formulae which contains all propositional letters and is closed under the Boolean connectives and the operator O defined before. We can identify this fragment with the language of SDL.

Definition 7 *A modal-deontic model is a structure $M = (W, R, V)$ where W is a non-empty set, R is a binary relation on W, and V is a function $V : At \cup \{S\} \mapsto 2^W$, that is, V assigns to every propositional letter and to the propositional constant S a subset of W.*

Definition 8 *Let $M = (W, R, V)$ be a modal-deontic model, $w \in W$ and A a formula of $\mathcal{L}_M$. The relation $M, w \models A$ (read: A is true at w in M) is defined recursively as follows:*

1. $M, w \models p$ *iff* $w \in V(p)$ *for all* $p \in At \cup \{S\}$
2. $M, w \models \neg A$ *iff* $M, w \not\models A$
3. $M, w \models A \wedge B$ *iff* $M, w \models A$ *and* $M, w \models B$
4. $M, w \models \Box A$ *iff* $M, v \models A$ *for all* $v \in W$ *with* wRv.

From the definition we can derive the truth condition for the deontic operators O, P and F as follows:

- $M, w \models OA$ iff for all $v \in W$ such that wRv, if $M, v \not\models A$ then $M, v \models S$.
- $M, w \models PA$ iff there exists some $v \in W$ such that wRv, $M, v \models A$ and $M, v \not\models S$.
- $M, w \models FA$ iff for all $v \in W$ with wRv, if $M, v \models A$ then $M, v \not\models S$.

The notions of validity, satisfiability ect. are defined as in the previous section. Every modal logic L in the language $\mathcal{L}_M$ generates a corresponding deontic logic via Anderson's reduction schema. It turns out that SDL coincides with the set of all formulae of the deontic fragment of $\mathcal{L}_M$ which are valid in a certain class of modal-deontic models, namely the class of all opt-serial models in the following sense:

Definition 9 *Let $M = (W, R, V)$ be a modal-deontic model. We say that R is opt-serial if and only if for all w there is some v such that wRv and $v \notin V(S)$.*

Theorem 10 *A SDL-formula is a theorem of SDL if and only if it is valid in all opt-serial modal-deontic models.*

Proof. In order to prove this theorem it suffices to show that for any SDL-formula B, B is satisfiable in a serial deontic model if and only if it is satisfiable in an opt-serial modal-deontic model. Both directions are quite simple.

First, assume that B is true at w in a serial deontic model $M = (W, R, V)$. Let $M' = (W', R', V')$ be the following modal-deontic model: $W' = W$, $R' = R$, $V'(p) = V(p)$ for all propositional letters p, and $v \in V'(S)$ if and only if there is no $u \in W$ such that uRv. Let u be any element in $W'(= W)$. By seriality of R there is some $v \in W$ such that uRv. Then $uR'v$ (by definition of R) and $v \notin V'(S)$ (by definition of V'), hence R' is opt-serial. A simple induction on the formation of the formulae will show that for all $v \in W$ and all SDL-formulae A, $M, v \models A$ if and only if $M', v \models A$. It follows that B is true at a point (viz. w) in an opt-serial modal-deontic model (viz. M').

Conversely, assume that B is true at a point w in an opt-serial modal-deontic model $M = (W, R, V)$. We construct a deontic model $M' = (W', R', V')$ as follows: $W' = W$, $uR'v$ if and only if uRv and $v \notin V(S)$, and $V'(p) = V(p)$ for all propositional letters p. That R' is serial follows from the fact that R is opt-serial: for all $u \in W$ there is a $v \in W$ such that uRv and $v \notin V(S)$, i.e., $uR'v$. A simple induction shows that for all points $v \in W$ and all SDL-formulae A, $M, v \models A$ if and only if $M', v \models A$. It follows that $M', w \models B$, i.e., B is true at some point in some serial deontic model. □

The interesting point in the previous proof is that it shows that there is a close relationship between the two semantical approaches, although they start at different points, with different underlying intuitions. As the construction above shows, one can identify the set of "deontically perfect" worlds of the one, direct approach with the set of possible worlds in which the proposition S is not true in the other, indirect approach. The intuition on which the construction is based is the following: a possible world is "deontically perfect" if no sanction can be applied to it, otherwise we have a world which is not so perfect, i.e., where some norms are dissatisfied. Such "deontically imperfect" world cannot bear the relation of deontic alternativeness to our actual world, although they may be alternatives of our world in some other sense, for example in logical, physical or metaphysical sense.

2.4 Paradoxes of Standard Deontic Logic

2.4.1 A Taxonomy of the Paradoxes

Although SDL is very poor in expressive power and also quite weak, it contains a number of paradoxical theorems. In fact, for almost every theorem of SDL we can find a natural language interpretation of the theorem which sound paradoxical. Such examples can be found very often in the literature on deontic logic. Extensive discussions of the paradoxes can be found in [18], [4], or [15]. It seems, however, that deontic logicians do not have a systematical way to find paradoxes and to argue that they are really paradoxes. They pick out by chance any arbitrary theorem and look for its possible natural language interpretations. If an interpretation sounds paradoxical then the theorem is rejected. Such an approach must rely heavily on intuitions, which are too often not formulated precisely. Thus, there exist contrary opinions among deontic logicians what formulae are to be regarded as paradoxes. This lack of methodology makes it very difficult to discuss the paradoxes.

I shall propose a criterion to classify paradoxes and to discuss the intuitions on which the paradoxical character of the formulae is based. In my opinion, there are three main groups of paradoxes. Some paradoxes arise when one interprets the disjunction within the scope of the deontic operators of SDL as the "or" in action sentences. I shall call them paradoxes of disjunctive norms. The second group, the group of paradoxes of conjunctive norms, contains those paradoxes which arise when one interprets the conjunction of SDL as the "and" in action sentences. The last group are paradoxes of conditional norms, which arise because SDL does not have the resources to formalize conditional norms adequately. The paradoxes of the first two groups arise from the fact that the two types of deontic logic are confused, viz. logics of the concept of Sein-Sollen and logics of the concept of Tun-Sollen. In contrast, the paradoxes of the third group arise because conditional norms are not treated properly. It should be clear that these paradoxes require a different solution than that to the former.

2.4.2 Paradoxes of Disjunctive Norms

In standard deontic logic (SDL) disjunctive norms are formalized by means of the truth-functional disjunction. This leads to several paradoxical results. They are known as Ross' paradox and the paradox of free-choice permission.

It is easily checked that the following formula is a theorem of SDL:

(1) $Op \supset O(p \vee q)$

Let "p" stand for "You post the letter" and "q" for "You burn the letter", then the formula seems to say that if you ought to post the letter then you ought to post it or to burn it. It sounds very awkward. The paradox involved in it is named after Alf Ross. Closely related to this paradox is the SDL-theorem

(2) $Pp \supset P(p \vee q)$

which seems to say, for example, that if it is permitted to smoke then it is also permitted to smoke or to kill. Similar interpretations can also be found for the following related SDL-theorems (cf. [18]):

(3) $Op \vee Oq \supset O(p \vee q)$

(4) $Op \vee Oq \supset P(p \vee q)$

(5) $Pp \vee Pq \supset P(p \vee q)$

(6) $Op \supset P(p \vee q)$

Why are these formulae paradoxes? Because in natural language the word "or", when used to combine two actions, expresses the idea of free choice. The formula "$P(p \vee q)$" should be properly read: you may choose to do p or to do q. The mentioned paradoxes show that the idea of free choice cannot be formalized by means of the disjunction within the scope of the deontic operators.

On the other hand, if one pays enough attention to the proper interpretation of SDL then the above formulae have nothing paradoxical. Standard deontic logic should be interpreted as a logic of the notion of "Ought-to-be", and the connective "or" should be interpreted as the usual disjunction, and not as the free-choice "or".

Consider Ross' paradox. In terms of the semantics presented in the previous sections, formula (1) can be interpreted as saying that if p is true in all deontically perfect worlds then $p \vee q$ is also true in all deontically perfect worlds. It can also be interpreted as saying that if it is obligatory to bring about a state of affairs where p is true then one must also bring about a state of affairs where $p \vee q$ is true. These interpretations are no more paradoxical than the fact that $p \vee q$ is a logical consequence of p. For this reason many authors think that there is nothing paradoxical with Ross' example and related formulae.

To cope with the difficulty caused by the notion of free-choice permission one may introduce an additional, stronger notion of permission and postulate an axiom to the effect that it is permitted to choose between two actions if and only if both actions are permitted, i.e., the axiom:

$$P^*(p \vee q) \leftrightarrow P^*p \wedge P^*q$$

This axiom seems very natural and plausible. However, it is also quite ad hoc and does not have a sound semantical foundation in the possible worlds semantics. I shall offer another solution later.

2.4.3 Paradoxes of conjunctive norms

In SDL the following equivalent formulae are provable:

(7) $Fp \supset F(p \wedge q)$

(8) $\neg Pp \supset \neg P(p \wedge q)$

These formulae are a variant of the so-called "Good Samaritan paradox." It suggests that if it is forbidden (not permitted) to offend someone, then it is forbidden (not permitted) to offend him and then to apologize. So, if I have offended someone, then I would violate one more norm if I apologize. This sounds very unintuitive.

Note that if the mentioned SDL-theorems are interpreted in terms of the standard semantics of deontically perfect worlds then they are not paradoxical at all. Formula (7), for example, says that if p is false in all deontically perfect worlds then $p \wedge q$ is also false in all deontically perfect worlds. One can also interpret this formula in the following way: if bringing about a state of affairs where p holds inevitably leads to sanction, then bringing about a state of affairs where $p \wedge q$ holds also leads to sanction. This is nothing paradoxical because one cannot bring about a world in which $p \wedge q$ holds without bringing about a world in which p is true, simply because p is a logical consequence of $p \wedge q$. Thus, the so-called Good Samaritan Paradox loses its paradoxical character as soon as we interpret it properly.

The following theorems of SDL seem at first sight very innocuous. However, paradoxical interpretations can be found for them, too.

(9) $P(p \wedge q) \supset (Pp \wedge Pq)$

(10) $P(p \wedge q) \supset P(p \vee q)$

(11) $O(p \wedge q) \supset (Op \wedge Oq)$

(12) $(Op \wedge Oq) \supset O(p \wedge q)$

Counterexamples for these theorems are discussed in ([18], [46]). All such counterexamples are based on the following intuition: the deontic status of a conjunctive norm and the deontic status of its components are independent. Some natural language readings of the theorems involving the conjunction within the scope of the deontic operators sound very paradoxical because the effects of two different actions, when they are performed in isolation, are generally different from the effects when the actions are done in conjunction. One can read "conjunctive action" as "parallel actions" (doing many actions concurrently) or as "consecutive actions" (doing an action, followed by others.) In standard deontic logic we have only one way to formalize these two types of conjunctive actions, namely by means of the truth-functional conjunction. In any case it can happen that doing one of the two actions may have desirable consequences, but doing them concurrently or consecutively may lead to very undesirable effects, and vice versa. A rather radical example of this sort is the following. If a doctor administers two different medicaments A and B to his patient, then the patient will recover quickly from his illness. Then it is permitted, and perhaps obligatory to administer the two medicaments. But can we infer that the doctor may (or shall) administer the medicament A? The answer seems to be negative. The medicament A alone may well cause the patient's death. Thus, it is clear why the mentioned formulae cannot be accepted as theorems of a logic of Ought-to-do. As I already noted, the reading of them as theorems about the concept of Ought-to-be removes their paradoxical character. We simply need another logic to formalize the concept of Ought-to-do.

2.4.4 Paradoxes of Conditional Norms

Conditional norms play an important role in normative argumentation. A large number, if not the majority of norms have a conditional form: "if ... then it is obligatory (permitted, forbidden) that ...". An important class among the conditional obligations are the so-called commitments, that is, sentences of the form "the performance of p commits one person to perform q". Among conditional (or hypothetical) norms the so-called conditional obligations have attracted most attention. The concept of conditional obligation has caused deontic logicians much trouble. Many proposals were made how to formalize the notion properly, but none of them has proved satisfactory. The dilemma can be summarized as follows: it seems necessary to have a form of detachment rule so that one can proceed from conditional to unconditional norms, if the condition is satisfied. On the other hand, if the rule of detachment is allowed, then in many cases one can deduce too much: from intuitively consistent premisses one can deduce contradictory consequences. The best-known example for such counter-intuitive results is Chisholm's contrary-to-duty paradox. I shall show that the dilemma can be solved in a very simple way. In the following I shall concentrate on conditional obligation only. The other conditional norms can be treated in a similar way.

In monadic deontic logic, there are two natural candidates for formalizing the notion of conditional obligation. Some authors suggest the formula "$O(p \supset q)$", others the formula "$p \supset Oq$" as the formalization of the sentence: "q ought to be under the condition p". The first formula is favored, e.g., by von Wright ([42], [48]), and the second by, e.g. Chisholm ([14]) and Weinberger ([40].) Both suggestions have their own drawbacks. It is well-known that the following formulae are valid in SDL:

(13) $Fp \supset O(p \supset q)$

(14) $Oq \supset O(p \supset q)$

Thus, if "$O(p \supset q)$" is to be read as "p commits you to do q" then formula (13) suggests that doing something forbidden commits one to do anything, and formula (14) that anything commits one to do one's duty. One may try otherwise and use "$p \supset Oq$" to formalize the notion of commitment. However, similar problems arise: the following formulae are valid for SDL:

(15) $\neg p \supset (p \supset Oq)$

(16) $Oq \supset (p \supset Oq)$

Formula (15) seems to say that whatever is not done (or is not the case) commits one to do anything, and formula (16) says that anything commits one to do one's duty. The above formulae (13–16) are quite unobjectionable as such. More serious problems arise when one try to formalize concrete normative circumstances adequately. The following paradox was detected by R. Chisholm ([14]) and has been discussed extensively in the literature. I shall discuss it in the form presented by Åqvist ([4]). Let us consider the following four sentences:

(I) It ought to be that John does not impregnate Suzy Mae.

(II) Not-impregnating Suzy Mae commits John to not marrying her.

(III) Impregnating Suzy Mae commits John to marry her.

(IV) John impregnates Suzy Mae.

Let "p" stand to the sentence "John impregnates Suzy Mae" and "q" for the sentence "John marries Suzy Mae". The first and the fourth sentence seem to have a canonical formalization. They do not cause any difficulty: (I) and (IV) can be formalized as follows:

(Ia) $O\neg p$

(IVa) p

The sentences (II) and (III) are conditional obligations. In the language of SDL, (II) can be formalized by either "$\neg p \supset O\neg q$" or "$O(\neg p \supset \neg q)$. The former formula is a logical consequence of (IVa), therefore it is not a suitable formalization of (II), because intuitively (II) is not a consequence of (IV). Analogously, from (Ia) one can prove (in SDL) the formula "$O(p \supset q)$", therefore the latter formula is not a suitable formalization of (III), because (III) is not a consequence of (I). Thus, the only suitable candidates for (II) and (III) are:

(IIa) $O(\neg p \supset \neg q)$

(IIIa) $p \supset Oq$

In standard deontic logic the following distribution principle is valid:

(17) $O(A \supset B) \supset (OA \supset OB)$

Thus, from (Ia) and (IIa) we can deduce "$O\neg q$". From (IIIa) and (IVa) we can deduce Oq, by detachment. From "$O\neg q$" one can infer "$\neg Oq$". Thus, one can deduce (in standard deontic logic) a contradiction from (Ia)–(IVa), although (I)–(IV) are perfectly consistent intuitively. Note that if the principle $O\neg A \supset \neg OA$ is not valid then we cannot deduce a formal contradiction from (Ia)–(IVa). But in this case we can infer $O\neg q$ and Oq, which is also paradoxical, and (I)–(IV) do not seem to suggest that both q and $\neg q$ are obligatory.

The discussed paradox suggests that it is not possible to formalize adequately the notion of commitment in standard monadic deontic logic. Some authors have proposed a new primitive dyadic operator $O(q/p)$ with the intended meaning "q is obligatory in circumstances p". Unconditional obligation can be defined in the following way:

$$Op =_{def} O(p/true)$$

With this new operator, the sentences (I)–(IV) above can be formalized as follows:

(Ib) $O\neg p$

(IIb) $O(\neg q/\neg p)$

(IIIb) $O(q/p)$

(IVb) p

Åqvist ([4]) has shown that (Ib)–(IVb) are consistent in his strongest dyadic system. However, the price one has to pay for this consistency is too high: there is no rule of detachment available for conditional obligation. From $O(q/p)$ and p one cannot deduce Oq. This lack of detachment imposes a heavy restriction on the application of deontic logic: how can conditional obligations play their important role in normative argumentation if they cannot, by way of detachment, lead to unconditional obligations?

The difficulties with Chisholm's paradox leads Åqvist to the question: "We seem to feel that detachment should be possible after all. But we cannot have things both ways, can we? This is the dilemma on commitment and detachment." ([4], p. 658.)

My claim is that we can have things both ways, commitment and detachment. I shall argue that both formalizations above fail because they do not pay attention to the temporal (or situational) aspect of the sentences (I)–(IV). I shall suggest a simple temporal deontic logic and show that the discussed paradox can be solved in a simple way. In this system TDL it is possible to formalize these sentences correctly, and the intuitively valid inferences are also TDL-valid.

2.5 Temporal Deontic Logic and Chisholm's Paradox

Let us take a closer look at the sentences (I)–(IV). We consider what can be inferred from them by common-sense. We want to see which of these intuitively correct inferences are also formally correct in SDL and in dyadic deontic logic, and which of the formally correct inferences are unintuitive from the standpoint of common-sense. Intuitively, from (III) and (IV) it follows "John ought to marry Suzy Mae", that is, Oq. This conclusion can be drawn from $(IIIa)$ and (IVa) in SDL. However, it is not possible to infer Oq from $(IIIb)$ and (IVb) in Åqvist dyadic systems. On the other hand, the set $\{(I), (II), (III), (IV)\}$ is perfectly consistent from an intuitive point of view. Thus, the sentence "John ought not to marry Suzy Mae" ($O\neg q$) should not be deducible from this set, because in case it is deducible we would have a contradiction. Clearly, it seems very odd if this inference should be valid: we would have the norm that John ought not to marry Suzy Mae, although he has impregnated her, and at the same time we have another norm saying that impregnating Suzy Mae commits John to marrying her. We saw above that $O\neg q$ is a consequence from $\{(Ia), \ldots, (IVa)\}$, making the set inconsistent. This inference is intuitively unacceptable. Thus, a solution to the paradox must make the inference based on the distribution principle (17) invalid. In the formalization in dyadic system (formulae (Ib)–(IVb),) however, $O\neg q$ is derivable from (Ib) and (IIb), in conflict with our intuitions. Thus, this formalization avoids inconsistency, but at the price that an intuitively valid inference is formally not valid, and an intuitively invalid inference is formally valid. So it is not a satisfactory solution either.

How does common-sense work to infer Oq from (III) and (IV)? If we examine (III) and (IV) carefully, we find that they have very different structures: (IV) refers to a single time point (or situation.) Its structure is: at time t it is the case that John impregnates Suzy Mae. On the other hand, (III) involves a temporal (or situational) quantification: (III) should be read as follows:

(IIIt) At any time, impregnating Suzy Mae commits John to marrying her.

From (IIIt) it follows: if John impregnates Suzy Mae at time t, then he ought to marry her (at some time later than t.) Because the premise "John impregnates Suzy Mae at t" is satisfied, one can deduce "(After t,) John ought to marry Suzy Mae".

For dealing with this temporal quantification formally, an easy way is to introduce a modal operator. Let 'L' stand for 'for all time points'. Then (IIIt) can be formalized by the formula '$LO\neg p$', for example. Our temporal logic will be very simple: we do not care about the structure of time and we do not need the usual temporal operators of past and future. But it should be sufficient for our purposes. Formally, we add the operator "L" into the language of deontic logic. The definition of formulae is modified accordingly. We assume that the logic of "ought" is KD and the logic of (temporal) necessity is $S5$. Thus, we need to extend SDL by the axioms and inference rules of $S5$ (for the operator L.)

(L1) $L(A \supset B) \supset (LA \supset LB)$

(L2) $LA \supset A$

(L3) $LA \supset LLA$

(L4) $\neg LA \supset L\neg LA$

(LR) L-rule: if A is a theorem, so is LA

As we know from multi-modal logic, this logic $S5/KD$ is characterized by the class of models with two binary relations: an equivalence relation R_L (corresponding to L) and a serial relation R_O (corresponding to O). Note that a world in such a model should be interpreted now as a total state of a world at a given time.

Remark: Some words about the truth-condition of formulae involving the operator L. The logic of temporal necessity (in the sense of "eternity": truth at all past, present, and future time points) is $S5$. If necessity is used in the above sense, then it is often reasonable to require R_L to be the universal relation, that is, $R_L = W \times W$. However, in many cases we are not interested in all possible time points, but only in those which are in some sense compatible with the immediate time point. Thus, we use LA in the sense "A is true at all relevant time points". To achieve the distinction between relevant and irrelevant time points we require L_R to be an equivalent relation. To illustrate this point consider the following example. If we want to formalize the norm "It is forbidden to kill" then we shall use the formula "LFq" (where q describes the action "kill"). In this case we would be tempted to require R_L to be the universal relation, because all possible time points are relevant. On the other hand, the situation is different if we want to formalize the sentence "John ought not to impregnate Suzy Mae." (John and Suzy Mae are perhaps the most famous couple in the community of deontic logicians!) We could formalize the sentence by means of the formula "$LO\neg p$", where "p" stands for "John impregnates Suzy Mae". The temporal operator is necessity, because the norm is clearly not intended to say that John ought not to impregnate Suzy Mae *now.* But it also sounds odd to say, for example, that John ought not to impregnate Suzy Mae after he has married her. Thus, the relevant situations for this norm do not exhaust all possible time points. It includes, for example, only those time points where John can impregnate Suzy Mae, where they are still unmarried, et cetera.

With the formal apparatus developed so far, let us reconsider the above example. Let us fix a time point t for evaluating the sentences (I)–(IV). The sentence (IV) can be formalized by a propositional letter p as before. Sentences like (I) are ambiguous: (I) can mean that John ought not to impregnate Suzy Mae at time t, or it can mean that John ought not to impregnate Suzy Mae at any (relevant) time. In this context, the second reading is more plausible. The two sentences (II) and (III) express conditional obligations: whenever something is the case, then some action ought to be done. This 'whenever' must be formalized by the operator L. Thus, we can formalize (I)-(IV) as follows:

(Ic) $LO\neg p$

(IIc) $L(\neg p \supset O\neg q)$

(IIIc) $L(p \supset Oq)$

(IVc) p

We easily see that the set $\{(Ic), (IIc), (IIIc), (IVc)\}$ is consistent. The construction of a $S5/KD$-model for this set of formulae is an easy exercise. Moreover, from $(IIIc)$ and (IVc) we can infer Oq, as desired. We can also find models of the above set in which $O\neg q$ is false, that is, $O\neg q$ is not a consequence from (Ic)–(IVc), in accordance with our informal considerations above.

The formalization of conditional obligation "if p then it shall be that q" by means of the formula $L(p \supset Oq)$ provides not only a simple solution to the dilemma of conditional obligation and detachment. It also solves the paradoxes of commitments. One can check easily that all criteria of adequacy originated by Anderson ([2], also listed in Åqvist's paper ([4])) are satisfied.

3 Interim Conclusions

We now arrive at a point where we can look back and draw some interim conclusions. Standard deontic logic is quite poor in expressive power. It possesses an adequate possible worlds semantics and is deductively complete with respect to this semantics. But standard deontic logic can at best formalize the concept of Ought-to-be. In this interpretation, the paradoxes of deontic logic which arise from the improper reading of the connectives "and" and "or" disappear. If in addition we pay sufficient attention to the temporal dimension of normative statements and introduce a temporal operator into deontic logic, then we can also solve the paradoxes of conditional norms easily.

However, the paradoxes also show that standard deontic logic is too poor to be able to capture the notion of Ought-to-do, or the practical concept of Ought. It cannot represent action and change adequately. This is pointed out by, for example, Castañeda ([10], p. 675.) But a satisfactory solution to the problem has still not been found. I shall not discuss in details the attempts in the literature (e.g., Castañeda ([10]), Hilpinen ([21]), Segerberg ([32]), Buck ([6]), and others) to develop a semantics for the Ought-to-Do. I shall rather present my own semantical approach.

Three main intuitions underlie my approach. First, I agree with Hector-Neri Castañeda that there is a fundamental semantic distinction between practitions and propositions, between actions practically (or deontically) considered and the circumstances of action, including actions considered as circumstances or conditions of other actions. Because of this distinction, practitions and propositions must be treated in different ways. This requirement has been overlooked by the standard modal approach. In my approach, actions (practitions) will be treated dynamically, and propositions will be treated statically (in the sense that will be made more precise later.) Thus the logic we get will be a form of dynamic logic.

Second, it is true that the connectives such as "not", "and", "or" can also be used to form complex actions (practitions) from simpler ones. However, their logical behavior in this context and the behavior of the same words when they are used to form complex propositions from simpler ones are very different. In the latter case they function as truth-functional connectives, but not in the former. Therefore, these two uses should not be confused. Many paradoxes of deontic logic arise from the confusion of these two different uses and can be avoided if the mentioned connectives are treated in two different ways.

Third, normative concepts are closely related to the concept of punishment, or sanction. It would be pointless if a norm exists, but any people who violate the norm are immune to punishment (or sanction): in such a case it is not quite reasonable to speak of a norm. Based on similar intuitions A. R. Anderson has tried to reduce deontic logic to modal logic. Although Anderson's attempt was not satisfactory, it remains a valuable stimulus for my own work. Our intuition and that of Anderson agree in the following point: an action is forbidden just in case its performance (necessarily, inevitably) leads to the liability to punishment. An action is obligatory if one cannot omit it without becoming liable to punishment, and the permission to do something means that one can do it without sanction. However, Anderson's formalization of this idea using strict implication leads to several difficulties. We shall choose another formalization.

In the next section I shall discuss the intuitions about action and norms more thoroughly. Then I define a system of dynamic deontic logic formally. Next I shall reconsider some paradoxes of deontic logic. Finally I discuss the possibilities to develop some variations of DDL and some open problems.

4 A Basic Logic of Actions

4.1 Intuitions about Actions

Let us assume that a certain agent is in the world. She performs certain actions and changes the world through her actions. Actions can be described at different levels. I shall not here discuss the question if there are reliable criteria to identify actions, and if yes, what are such criteria. I do not discuss the issue what constitutes an action, or when are two actions identical. I simply assume that we have such criteria, so that at every situation we can (in principle) decide what actions, if any, the agent is performing. It seems that a reasonable concept of action should satisfy the following minimal condition: different course of actions have different effects. In the paper I shall adopt this condition. An immediate consequence of this requirement is that the effect of the performance of an action and that of its omission must be different.

We shall describe actions in terms of change in the world. Assume that s is the total state of the world at a certain time point. If the agent performs an action x at the state (or situation) s, then the world will be at some state t after x is performed. The outcome is in general not determined: it depends on other factors such as external events which take place at s and other agents' action. Thus, the effect of an action is described by a relation, rather than a function, on the set of possible total states of the world. We consider only the case of one agent, so that other agents' action can be treated as external events. They form the circumstances where the agent acts, and the effects of her action depend strongly on those events.

The world picture just drawed resembles that of dynamic logic, a logic that was developed for reasoning about the behavior of computer programs. Now we can interpret human actions as programs and try to use dynamic logic to describe actions. In contrast to traditional, "static" logic, we have now a logic which describes changes in the world explicitly. Thus it is worth trying to develop deontic logic to a logic of action.

4.2 Syntax and Semantics of Propositional Dynamic Logic

The formal language of Propositional Dynamic Logic (PDL) is built up from two sets of primitive symbols, a countable set At of atomic formulae and a countable set Prg of atomic programs. The set of formulae is the least set containing At and is closed under the rules: if A and B are formulae then $\neg A$ and $A \wedge B$ are formulae; if A is a formula and x is a program then $< x > A$ is a formula. The set of programs is the least set containing all atomic programs and closed under the rule: if x and y are programs then $x; y$, $x \cup y$ and x^* are programs.

The dual operator to $< x >$ is defined as follows:

$$[x]A =_{def} \neg < x > \neg A$$

The intuitive reading of the program connectives and the formulae are as follows:

- "$x; y$": "perform x and then y" (composition)
- "$x \cup y$": "do either x or y" (non-deterministic choice)
- "x^*": "perform x finitely many times" (iteration)
- "$[x]A$": "A always holds after x is performed"
- "$< x > A$": "sometimes after x is performed, A holds".)

A PDL-model is a structure $M = (W, R, V)$, where W is a non-empty set, R is a function which assigns to every program x a binary relation $R(x)$ on W, and V is a valuation function which assigns to every atomic formula pa subset $V(p)$ of W.

To preserve the intuitive reading of the program connectives the following formal requirements must be made:

$$R(x; y) = R(x); R(y)$$

$$R(x \cup y) = R(x) \cup R(y)$$

$$R(x^*) = (R(x))^*$$

In these conditions, $R; S$ is the composition of the two relations, i.e., $(s, t) \in R; S$ iff there exists a u such that $(s, u) \in R$ and $(u, t) \in S$, and R^* is the reflexive, transitive closure of R.

The satisfaction relation "A is true (holds) at point s in model M", denoted $M, s \models A$, is defined inductively on the formation of A as follows:

1. $M, s \models P$ iff $s \in V(P)$ for any atomic formula P
2. $M, s \models \neg A$ iff $M, s \not\models A$
3. $M, s \models A \wedge B$ iff $M, s \models A$ and $M, s \models B$
4. $M, s \models < x > A$ iff there is some $t \in W$ such that $(s, t) \in R(x)$ and $M, s \models A$

A is true in model M, denoted $M \models A$, if it is true at all points in M, i.e., if $M, s \models A$ for all $s \in W$. A is valid in PDL, denoted $\models A$ if it is true in all PDL-models.

We identify the logic PDL with the set of all PDL-formulae which are valid in all PDL-models. Well-known results say that PDL is decidable and finitely axiomatizable. (cf. [20])

4.3 PDL as a logic of action

To interpret PDL as a logic of action we interpret the programs of PDL as human action. The states of a PDL-models are interpreted as total states of the world at different time points. By performing an action the world will be changed. This change is recorded by the relation associated with the action under consideration. The transition relations corresponding to the actions generate implicitly a discrete temporal order over the universe of total states. If an action x cannot be performed at a certain state s the we can express it in terms of the semantics for PDL by the condition: there is no state t such that $(s,t) \in R(x)$.

The sketched idea of interpretation is not without difficulty. The first obvious drawback of PDL is that many of its model structures can hardly be interpreted reasonably as models of action. If we interpret the states of a model as possible total states of the worlds at different time points and if the temporal relation is defined implicitly through the transition relation associated with the programs then we would have no constraint on the temporal relation; it would be possible, for example, to "bring about the past": there are states s, t such that if one performs some course of actions at t one arrives at t, and at t one can produce the past state t by some course of actions. This idea is not logically inconsistent, but we shall better deal with more plausible time structures. When we describe actions it is much more intuitive to assume a tree-like structure. We can imagine the following picture of the world. Up to now the world has a linear history: the total states of the world at different past time points can be ordered linearly. The past cannot be changed through our action. At the present time point we can do different things, which influence the future course of history. According to what we do at present (and what else happens: which extraneous events take place, which actions do other agents perform, etc.) the world history can be continued in different directions. If we choose to do something now, then the development up to the next moment is determined, there is one unique transition step to the next moment (which is determined by our action and other events,) and the history up to that moment is again linear, determined and cannot be changed when we are already at the next moment. At that moment we have again different options, so that the future relative to the next moment is again a tree.

The intuitive picture just drawed can be captured by the concept of a tree-like PDL-model. We can view a PDL-model $M = (W, R, V)$ as a labeled directed multigraph with the node set W, the arc set $\bigcup\{R(x) : x \in \text{Prg}\}$, and the labeling function l generated by V assigns to each state s the set $l(s)$ of all atomic formulae p such that $s \in V(p)$. A model M is called tree-like if it is acyclic, each node has at most one predecessor, and there is no multiple arc, that is, for any $s, t \in W$ there is at most one atomic program x such that $(s,t) \in R(x)$.

Curiously, the class of all PDL-models and the class of tree-like PDL-models determine the same logic.

Theorem 11 *A PDL-formula is satisfiable in the class of all PDL-models if and only if it is satisfiable in the class of all tree-like models.*

Proof. The "only if"-clause is trivial. To prove the "if"-clause we assume that a formula is satisfiable at a point s in a dynamic model $M = (W, R, V)$. Let N be the set of natural numbers and X be a fixed atomic program. We define then the structure $M' = (W', R', V')$, which is called the structure obtained by unwinding M starting at state $s \in W$, where W', R', are respectively the least subset of $W \times Prg \times N$, $W' \times W'$ such that:

1. $(s, X, 0) \in W'$
2. if $(s, Y, n) \in W'$ and $(s, t) \in R(Z)$ then $(t, Z, n+1) \in W'$, where Y and Z are any atomic programs.
3. if $(s, t) \in R(Z)$ then $((s, Y, n), (t, Z, n+1)) \in R'(Z)$, for any atomic programs Y, Z.

It is easily verified that M' is tree-like. (In fact, it is a tree.) First, we observe the following. For any program x, if $(s, t) \in R(x)$ then for any $(s, Y, n) \in W'$ there is some $(t, Z, m) \in W'$ such that $((s, Y, n), (y, Z, m)) \in R'(x)$. Converserly, if $((s, Y, n), (y, Z, m)) \in R'(x)$ then $(s, t) \in R(x)$.

The valuation V' is defined in the obvious way: for any propositional letter P, $V'(P) = W' \cap (V(P) \times Prg \times N)$, that is, for all $(s, Y, n) \in W'$, $(s, Y, n) \in V'(P)$ if and only if $s \in V(P)$. We show by induction that a PDL-formula A holds at s in M if and only if it holds at all points (s, Y, n) in M' which are generated from s.

If A is a propositional letter P then $M, s \models P$, i.e., $s \in V(P)$ implies $(s, Y, n) \in V'(P)$ for all points (s, Y, n), hence $M, (s, Y, n) \models P$. If $M, s \not\models P$ then $(s, Y, n) \notin V'(P)$, hence $M, (s, Y, n) \not\models P$. If A is a negation or a conjunction then the proof is trivial. Now assume that A is the formula $< x > B$. Suppose that $M, s \models < x > B$. It follows that $M, t \models B$ for some t such that $(s, t) \in R(x)$. By induction hypothesis, $M, t \models B$ if and only if $M, (t, Z, m) \models B$ for all (t, Z, m). Moreover, $(s, t) \in R(x)$ implies $((s, Y, n), (t, Z, m)) \in R'(x)$ for some (t, Z, m), hence $M, (s, Y, n) \models < x > B$. Conversely, assume that $M, (s, Y, n) \models < x > B$. By definition, there is some $(t, Z, m) \in W'$ such that $((s, Y, n), (y, Z, m)) \in R'(x)$ and $M, (y, Z, m) \models B$. It follows that $(s, t) \in R(x)$ (by the above observation) and $M, y \models B$ (by induction hypothesis), hence $M, s \models < x > B$. This completes our induction.

We have supposed that the formula A holds at a state s in M. Then it holds at the state $(s, X, 0)$ in the tree-like model M'. □

Having this result we can restrict our investigations on tree-like models of PDL. In order to get a logic of action we must extend PDL to accommodate parallel actions. As we see from the semantics of PDL, this logic allows reasoning about a single action at a time point. In order to cope with the difficulty we can introduce a new program connective $\|$ into the language with the meaning: do x and y simultaneously (cf. [26], [17]). I do not choose this solution because of the following reasons. First, it makes the basic logic too complicated and therefore unhandy. Second, to define the semantics for the new connective one must make the assumption that the atomic programs are totally independent and the effects of a concurrent action can be determined from the effects of its components. This assumption seems to be too strong. Third, I think that we have simpler ways to solve the problem more easily and more intuitively.

Let us consider the parallel performance of two basic actions. The effect when two actions are performed in parallel is often totally unrelated to the effect when one of the two actions is performed in isolation. It seems that there is no reasonable way to describe the effect of "$X \| Y$" (that is, of doing X and Y in parallel) in terms of X and Y, in contrast to the action connectives of composition, choice, omission, and iteration. One might be tempting to require that doing X and Y in parallel should lead to a state which can also be brought about by doing either X or Y. However, this requirement is unreasonable. A rather radical example is the following. An agent sits in a room in front of two buttons. If she pushes the left button, the door and the left window will open. If she pushes the right window then the door and the right window will open. But if she pushes both buttons at the same time then neither the door nor the windows will open. The outcomes of the actions X, Y and $X \| Y$ are totally unrelated. Therefore it is convenient to regard the totality of all atomic actions which the agent performs at a certain state s as a quasi-atomic action. Thus, at any state s the agent does only one action, although this action can involve the parallel performance of (finitely) many atomic actions, or of one single action, or even of no action at all. The latter case is when the agent does nothing, i.e., when **skip** $(= \emptyset)$ is performed ("the agent lets things happen.") The possibility to describe the change in the world when the agent does nothing is another advantage of this approach. It is clear that the world history "does not stop" when the agent does nothing, because other agents may do other things, and also external events may happen, independent on the will of the agent.

To summarize, we can define our basic logic of action as follows:

Definition 12 *Let Prg be a countable set of atomic actions and At be a countable set of propositional variables (atomic formulae.) Every finite set X of atomic actions is a quasi-atomic action. The set of all actions is the least set which contains all quasi-atomic actions and satisfies the following condition: If x and y are actions, then $(x; y)$, $(x \cup y)$ and x^* are actions. The set Fml of formulae is the least set which contains all propositional letters and satisfies the following condition: If A, B are formulae and x is an action, then $\neg A$, $(A \wedge B)$ and $[x]A$ are formulae.*

The intended reading of a quasi-atomic action "X" is that all atomic actions from X are performed in parallel. We assume that an agent can execute only a finite number of atomic actions at a time point. (It follows that the set of quasi-atomic actions is also countable.) Note that $\emptyset$ is also a quasi-atomic action. Its intended reading is: "the agent does nothing". This 'action' is also denoted by the name "**skip**" or "**donothing**". The quasi-atomic actions will be treated like the atomic programs of PDL.

Notational Convention: If X is the set $\{X_1, \ldots, X_n\}$, $(n \geq 1)$, then we also write X in the form $X = X_1 \| \ldots \| X_n$. In particular, in case of $n = 1$ we write X instead of $\{X\}$.

Definition 13 *An action model is a tree-like structure $M = (W, R, V)$, where W is a non-empty set, R is a function which assigns to every action x a binary relation $R(x)$ on W, and V is a valuation function which assigns to every atomic formula p a subset $V(p)$ of W. The function R satisfies the following conditions:*

$$R(x; y) = R(x); R(y)$$

$$R(x \cup y) = R(x) \cup R(y)$$

$$R(x^*) = (R(x))^*$$

Our logic of action is no more than a basic logic. It is very poor in expressive power. We can only say what can be and what must be the case after a certain action is performed. We cannot say, for example, that a certain action is performed at a certain state. Neither can we express what is the case when certain actions are not performed. Furthermore, some more extensions of the basic logic seem meaningful. We can require, for example, that the world history has no beginning and no end, that is, every state must have a predecessor and a successor. Such extensions can be defined when necessary. A pleasant feature of the basic action logic is that it is possible to say in our framework when a certain action can be executed. The agent can do x at s if and only if there is some t such that $(s, t) \in R(x)$. This is equivalent to $M, s \models < x > true$. Thus, we can define an operator Can which applies to act-names to form formulae with the intended reading that the agent can perform the action under consideration. We have the following definition:

$$Can(x) =_{def} < x > true$$

On the basis of the basic action logic we shall develop a deontic logic which deals explicitly with norms about action. For reasoning about norms about action it is necessary not only to speak about what the agent does, but it is also necessary to speak about what she omits. Thus, we have to extend the language by a new operator, the operator of omission. This will be done in the next section.

5 Propositional Dynamic Deontic Logic (DDL)

5.1 Truth Conditions for Normative Statements

Let us now examine the truth-conditions of the norm-propositions. When do we say that an action is permitted, forbidden, or obligatory? Assume that an action x is permitted. Assume further that the agent performs x. Does she violate the norm? The answer is "no", of course. Therefore, the agent is not liable to punishment. Thus, it seems reasonable to adopt the following principle: according to a certain code, an action is permitted if and only if after performing the action the agent is not liable (i.e., immune) to punishment. If we write "$[x]A$" for "always after performing x, A holds" and "S" for "the agent is liable to sanction", then Px (that is, the formalization of "x is permitted") can be defined as $[x]\neg S$.

It can be argued that this definition of the notion of permission is too strong. It says that no performance of the permitted thing leads to punishment. The critic of this definition could argue as follows: in addition to the permitted action the agent can also perform another action at the same time. If a person performs a permitted act x, then surely she should not be punished for x, but she may at the same time perform another act y which is forbidden, and thus implies S. Performance of a permitted act cannot guarantee an escape from S, but it preserves the possibility of such an escape. This criticism is beside the point, however: our act-names refer to the totality of all (atomic) actions that the agent performs at a certain time, and not to the components of such actions. Thus, in our example, we shall not say that the agent performs x or that she performs y, but we must say that she perform the (quasi-atomic) act $x \| y$.

Next, consider prohibitions. We assume that an action y is forbidden. If the agent performs y, then she violates the norm, therefore she is liable to punishment. Thus, an action is forbidden just in case its performance leads to sanction (or the liability of sanction.)

What about obligation? Assume that it is obligatory that the agent carry out a certain action, say z . Let us assume that the agent does not do z. In this case, the agent violates the norm, therefore, according to the norm, she is liable to punishment. Thus we can formulate the condition for obligation as follows: an action is obligatory if and only if omitting the action leads to punishment.

The above notions of permission, prohibition and obligation seem to be the most common in ordinary language. However, we also find other uses of these normative concepts. For example, sometimes a permission is regarded simply as the absence of a prohibition, or a prohibition as the absence of the corresponding permission. Obligation is often used in the sense that whatever is obligatory is also permitted. Such normative concepts are stronger or weaker than ours. They can also be defined in my framework without any difficulty.

5.2 Syntax of DDL

To define dynamic deontic logic we extend the language of our basic logic of action by a new action operator, the omission operator. Furthermore, as in section 2.3 we add a propositional constant S into the language and then define the deontic operators according to our informal discussion in the last section.

Definition 14 *Let Prg a countable set of atomic actions and let At be a countable set of propositional variables. Moreover, let S be a propositional constant. The constant S and the propositional variables are called atomic formulae. Every finite set X of atomic actions is a quasi-atomic action.*

The intended reading of the propositional constant "S" is: "the agent is liable to sanction". The intended reading of a quasi-atomic action "X" is as before. The set of quasi-atomic actions generated by the set Prg of atomic actions is denoted by QA.

Definition 15

1. *The set UA of all actions is the least set which contains all quasi-atomic actions and satisfies the following condition: If x and y are actions, then $(x; y)$, $(x \cup y)$, $-x$ and x^* are actions.*
2. *The set Fml of formulae is the least set which contains all atomic formulae and satisfies the following condition: If A, B are formulae and x is an action, then $\neg A$, $(A \wedge B)$, LA and $< x > A$ are formulae.*

UA stands for "universe of actions." The Boolean connectives, the operator [] and the dual operator M of L are defined as usual. The intuitive reading of the action "$-x$" is "the agent omits x". The interpretation of LA is "A is true at all relevant time points".

Now we go on define the deontic concepts. As I have remarked before, our deontic operators are prefixed to act-names. The result is a proposition to the effect that a certain action is permitted, forbidden, or obligatory.

Definition 16 *The deontic operators are introduced by the following definitions:*

$$Ox =_{def} [-x]S$$

$$Px =_{def} [x]\neg S$$

$$Fa =_{def} [x]S$$

The three concepts O, P and F are called obligation, permission, and prohibition, respectively. Besides these three concepts we can define a number of stronger and weaker notions of obligation, permission, and prohibition in our framework. We shall discuss the deontic concepts and their relations to each other after we have defined a precise semantics for our logic. The deontic operators defined above shall be called standard, or explicit ones, when we need to distinguish them from other deontic concepts.

5.3 Semantics of DDL

The semantics of DDL extends the semantics for our basic logic of action by the conditions for the omission operator. The propositional constant S does not require any special treatment. Following von Wright ([44]) we assume two minimal principles for the omission operator. First, the effects when an action is performed and the effects when it is omitted must be different. Second, the omission to omit an action and the performance of the action are the same. In our approach these principles can be expressed by the conditions that the transition relations associated with x and $-x$ are disjoint for any action x, and those of x and $--x$ are identical.

In ([44]) von Wright also mentioned the following principle of acting and omitting: if someone can perform an action then she can also omit it, and if she can omit an action then she can perform it. Most action theoreticians would accept this principle as valid. It makes little sense to say that someone can do an action but cannot omit it, because in case of her doing it we rather say that she does not perform the action at all. It seems that in von Wright's system we cannot capture the principle formally. In our system we can, viz. by means of the formula $Can(x) \leftrightarrow Can(-x)$, that is, $< x > true \leftrightarrow < -x > true$. In terms of the semantics it is expressed by the following condition on DDL-models:

$$\forall s(\exists t : (s,t) \in R(x) \iff \exists u : (s,u) \in R(-x))$$

These observations give rise for the following definition of the semantics for propositional dynamic deontic logic.

Definition 17 *A model for DDL is a structure $M = (W, R_L, R, V)$ where W is a non-empty set, R_L is an equivalence relation on W, R is a function which assigns to every action x a binary relation $R(x)$ on W, and V is a valuation function which assigns to each atomic formula P a subset $V(P)$ of W. The function R satisfies the following conditions: for all $x, y \in UA$:*

$$R(x;y) = R(x);R(y)$$

$$R(x \cup y) = R(x) \cup R(y)$$

$$R(-x) \cap R(x) = \emptyset$$

$$R(--x) = R(x)$$

$$\forall s(\exists t : (s,t) \in R(x) \iff \exists u : (s,u) \in R(-x))$$

$$R(x^*) = R(x)^*$$

Moreover, $(W, \bigcup\{R(X) : X \in QA\})$ is a tree-like graph.

Remark. The intended meaning of W, R and V are as in the basic logic of action: W is the set of all possible total states of the world. $R(x)$ is the transition relation associated with the action x, that is, $(s,t) \in R(x)$ means that if the action x is performed at s then the resulting state of the world can be t. In particular, if the action x cannot be performed at state s then there exists no state t such that $(s,t) \in R(x)$. $V(P)$ is meant to be the set of states where P is true.

Definition 18 *For all formulae A from Fml we define the relation $M, s \models A$ as follows:*

1. $M, s \models P$ *iff* $s \in V(P)$ *for any atomic formula* P
2. $M, s \models \neg A$ *iff* $M, s \not\models A$
3. $M, s \models A \vee B$ *iff* $M, s \models A$ *or* $M, s \models B$
4. $M, s \models LA$ *iff for all* $t \in W$, $(s,t) \in R_L$ *implies* $M, t \models A$
5. $M, s \models [x]A$ *iff for all* $t \in W$, $(s,t) \in R(x)$ *implies* $M, s \models A$

If it is clear from the context what model M is meant we also write $s \models A$ instead of $M, s \models A$. A formula is said to be true at a state s in the model M if and only if $M, s \models A$. It is said to be true in M (symbolized by $M \models A$) if it is true at all states in W. The formula A is valid (in symbol: $\models A$) if it is true in all DDL-models.

From the above definition we can derive the truth-conditions for our deontic sentences as follows:

- $M, s \models Px$ iff for all $t \in W$, $(s,t) \in R(x)$ implies $M, t \models \neg S$ (i.e., $M, t \not\models S$)
- $M, s \models Fx$ iff for all $t \in W$, $(s,t) \in R(x)$ implies $M, t \models S$
- $M, s \models Ox$ iff for all $t \in W$, $(s,t) \in R(-x)$ implies $M, t \models S$.

5.4 Some features of DDL

Obviously all theorems of propositional dynamic logic, including all theorems of the propositional calculus, are valid formulae of DDL. Moreover, all theorems of the modal logic $S5$ are valid for our operator L. Interesting theorems for us are those about the normative concepts.

In order to compare SDL and DDL we need a notion of corresponding formulae of the two languages. Because DDL does not allow iteration of deontic operators we consider only SDL-formulae which do not contain any iteration of deontic operators. Without loss of generality we can assume that the SDL-formulae do not contain any occurrence of the material implication and material equivalence. A SDL-formula can be translated to a DDL-formula by the rule: all occurrences of the propositional letters and the Boolean connectives outside the scope of the deontic operators are replaced by the propositional letters and the Boolean connectives of DDL. The propositional letters and the connectives "not", "and", "or" within the scope of the deontic operators are replaced by the action variables and the corresponding action connectives of DDL, respectively. The corresponding action connectives for negation and disjunction are the omission operator and the choice operator, respectively. We have two different ways to translate the "and" within the scope of the deontic operators, namely by means of the connectives ";" and "$\|$".

In standard deontic logic all deontic operators are interdefinable. In DDL it is not so: only obligation and prohibition are interdefinable. We have a theorem stating that an obligation to do some action is equivalent to the prohibition to omit the action:

(18) $Ox \leftrightarrow F(-x)$

(19) $Fx \leftrightarrow O(-x)$

The next result shows that our logic is able to describe "open" normative systems: it is not the case that any action is either permitted or forbidden, that is, $\not\models Px \vee Fx$. The rationale of this principle is that we are concerned with the concept of explicit permission. If we identify permission (in the weak sense) simply with the absence of a prohibition, as in standard deontic logic, then the normative systems will be trivially closed. But then many phenomena cannot be captured. For example, we can ask what sense does it make to write into the constitution that every citizen has the right to vote, if the constitution does not explicitly forbid anyone to vote.

What about the inconsistency of norm-systems? Following Bulygin and Anchourrón ([8]) I shall call two norms inconsistent if the satisfaction of the one precludes the satisfaction of the other. An example for such inconsistencies is the norm saying that one shall do and omit the same action at the same time. In standard deontic logic we can express this norm by the formula $Op \wedge O\neg p$. This formula is not satisfiable in SDL, therefore all norm systems described by SDL are consistent. In our approach the corresponding formula $Ox \wedge O - x$ is satisfiable. Thus, we are able to describe inconsistent normative systems. In moral or legal philosophy such situations would be called "conflicts of duty." In such situations one cannot fulfill all duties at the same time, so that one must rely on other criteria to choose what to do in the situation.

From the consistency of $Ox \wedge O - x$ in DDL we can conclude that $O - x$ does not imply $\neg Ox$ in DDL, that is, $\not\models O - x \supset \neg Ox$. This results agrees with the fact that there are normative conflicts in the world which cannot be eliminated by logical reasons.

Besides the discussed type of inconsistency we sometimes find in the literature a discussion of another type of normative conflict, viz. that between a prohibition and a permission (cf. [8], [50]). An example of this type in standard deontic logic is the formula $Pp \wedge Fp$. As Bulygin and Anchourrón ([8]) remarked, conflicts of this type are not genuine conflicts at all. A norm-system consisting of the permission to do something and the prohibition to do the same thing is satisfiable, namely when one does not perform the action under consideration. Thus, there is no conflict at all. The reason is that a permission cannot be satisfied or dissatisfied: one just makes use or one does not make use of the permission. Why such norms are sometimes regarded inconsistent is perhaps derived from the following fact: in SDL, the formula $Pp \wedge Fp$ is unsatisfiable, because it is equivalent to $Pp \wedge \neg Pp$. This shows that SDL is not able to formalize permission and prohibition of actions adequately: consistent circumstances are formalized by inconsistent formulae. In contrast to SDL we have in dynamic deontic logic: $\not\models \neg(Px \wedge Fx)$, that is, the formula $Px \wedge Fx$ is perfectly consistent.

Another pleasant feature of DDL is that it does not require any action to be permitted, forbidden or obligatory from logical ground alone. As we know the following SDL-formulae $\neg O(p \wedge \neg p)$ and $P(p \vee \neg p)$ are SDL-valid. The first says that it is not obligatory to do the impossible, and the second says that it is permitted (not forbidden) to do any logical truth. In DDL the corresponding formulae are not valid: neither $O(X; (-X))$ nor $O(X \| -X)$ nor $P(X \cup -X)$ is valid for DDL. In general we can show that for every action x, each of the formulae Ox, Px, Fx is satisfiable. This seems to fit better to our intuitions: there is no logical reason why one shall do certain things and omit others.

We shall discuss the so-called "distribution principles" such as the formulae (11), (12) or (17) in the next section when we discuss the paradoxes.

5.5 Paradoxes of Deontic Logic Revisited

5.5.1 Paradoxes of Disjunctive Norms

In our approach we have the possibility to capture formally the intuition about the choice between two actions, namely by means of the connective "$\cup$". Then the paradoxes are avoided. It is easily shown that the following DDL-formulae (which correspond to the SDL-formulae above) are not valid in DDL:

(20) $Ox \supset O(x \cup y)$

(21) $Px \supset P(p \cup q)$

(22) $Ox \vee Oy \supset O(x \cup y)$

(23) $Ox \vee Oy \supset P(x \cup y)$

(24) $Px \vee Py \supset P(x \cup y)$

(25) $Ox \supset P(x \cup y)$

For the free-choice interpretation of "or", a permission should satisfy the condition: one is permitted to choose between two actions if and only if both actions are permitted. In DDL the formula corresponding to this idea is valid, viz. the following formula:

(26) $P(x \cup y) \leftrightarrow Px \wedge Py$

This validity makes disjunctive permission a genuine permission. If we have only

(27) $P(p \vee q) \supset (Pp \vee Pq)$

as in SDL, then "$P(p \vee q)$" is in reality no permission at all: it says that "p" or "q" is permitted, but does not say which of the two actions is permitted, so that the agent still does not know what she may do without becoming liable to punishment (cf. [45]).

5.5.2 Paradoxes of Conjunctive Norms

In our system we can distinguish between parallel and consecutive performance of two actions. The following theorem shows that the source of all paradoxes of conjunctive norms is removed.

Theorem 19 *Let D be any deontic operator (O, P or F). Then the four sentences $D(X;Y)$, $D(X\|Y)$, DX and DY are logically independent for all deontic operators D. In particular, the following "principles of distribution over conjunction" (corresponding to the formulae (11) and (12) of section 2.4.3) are not valid:*

(28) $O(X\|Y) \supset (OX \wedge OY)$

(29) $(OX \wedge OY) \supset O(X\|Y)$

(30) $O(X;Y) \supset (OX \wedge OY)$

(31) $(OX \wedge OY) \supset O(X;Y)$

Proof. We can construct easily DDL-models in which three of the four formulae $D(X;Y)$, $D(X\|Y)$, DX and DY are ture but the fourth is false. We omit the details.
$\square$

The next theorem lists some more simple facts about DDL. Its proof is very simple and is omitted.

Theorem 20 *None of the formulae which correspond to the paradoxical formulae of section 2.4.3 is DDL-valid.*

In light of the previous theorems we see that in our system the discussed version of the Good Samaritan paradox can be solved easily. We can verify that none of the corresponding formulae to the paradox is valid in our system:

(32) $\neg Px \supset \neg P(x\|y)$

(33) $\neg Px \supset \neg P(x;y)$

Furthermore, we find a solution to a problem posed by H. N. Castañeda ([9]), viz. the problem of the relationship of Act-Utilitarianism to deontic logic. According to Castañeda, the main thesis of Act-Utilitarianism can be summarized as follows:

(U) The agent X ought to do A in situation C if and only if for all A', if A' is an alternative to A in C and A' is different from A then the consequences of X's doing A in C are are better than those of X's doing A' in C.

Now assume that X ought to do the conjunctive action "P and Q". In standard deontic logic we can express it by the formula $O(P \wedge Q)$. Moreover, assume that the three actions "$P \wedge Q$", "P", "Q" are open to the agent on an occasion C. One would agree that the three actions are different and P, Q are alternatives to "$P \wedge Q$" in C. According to (U), the consequences of doing $P \wedge Q$ are better than those of doing P, because the agent ought to do $P \wedge Q$, by our assumption. It is reasonable to assume the relation "better than" to be asymmetric. Thus, we can conclude that the consequences of doing P are not better than those of doing $P \wedge Q$. By standard deontic logic, from $O(P \wedge Q)$ we can infer OP. From the latter and (U) we can conclude that the consequences of doing P are better than those of doing $P \wedge Q$, which contradicts our earlier result.

From this example we see that Act-Utilitarianism and standard deontic logic seem not to go together. In our logic the problem is solved because the obligation to do a conjunctive action does not imply the obligation to do any of its components, that is, we cannot infer that P is obligatory from the assumption that "P and Q" is obligatory, and it is reasonable so.

5.5.3 Paradoxes of Conditional Norms

Now let us examine how our logic can solve the paradox of contrary-to-duty imperatives. Let "x" denote the action "impregnating Suzy Mae" and "y" denote the action "marrying Suzy Mae." We stay first at the model level. The first three sentences are normative ones, and they do not refer to any specific time point. They can be expressed in our terminology as follows:

(Id) For all $t \in W$, $t \models O - x$.

(IId) For all t, s, if $(t, s) \in R(-x)$ then $s \models O - y$.

(IIId) For all t, s, if $(t,s) \in R(x)$ then $s \models Oy$.

The sentence (IV) describes a state of affairs. It says that in the actual course of events ("actual history of the world",) John impregnates Suzy Mae at some time point u. In our terminology we can express it as follows:

(IVd) $(u,v) \in R(x)$.

Here u, v are two successive states of the world, where v is the state after John has impregnated Suzy Mae at u.

We could use the truth-conditions of Ox to analyze (Id)–(IVd) further. Sentence (I), for example, says that John ought not to impregnate Suzy Mae. In our terminology, it means that for all t, s such that $(t,s) \in R(x)$ (i.e., where John has done the action x at t,) John has violated an obligation, therefore he is liable to sanction at s. However, it is not really necessary for our purpose. It is obvious that the set (Id)–(IVd) is consistent. Moreover, from (IIId) and (IVd) we can deduce the following: $u \models O(y)$, that is, John ought to marry Suzy Mae after he has impregnated her, in accordance with our informal discussion above. Furthermore, there is no way to infer $u \models O-y$, as it was possible in Åqvist's dyadic system. This is again an intuitively correct result.

The discussion up to now took place at the model level. Now we go to the language level. We do not have the resource in the language to say directly that a certain action is performed, so that we cannot formalize the sentence (IV) directly. To cope with the difficulty we could add an operator "*Do*" into the language which indicates what action is performed at he immediate situation. Then, we can formalize (IV) as "$Do(x)$". Of course, we must add new conditions to ensure the intended meaning of "*Do*". The semantics of DDL must be modified accordingly. We have to define the notion of a path, that is, a sequence of successive states, and to valuate the formulae involving the operator *Do* along the paths. A designated path h is interpreted as the "real" history of the world, and a state s in the path as the immediate state of the world. The sentence "The agent performs the action x" is formalized by the formula $Do(x)$, which is true at s in h if and only if $(s,t) \in R(x)$ for some future state t in h. The truth conditions for $Do(x)$ cannot be defined at states alone. I shall not here pursue this way any further, but choose another way to approximate the meaning of the sentences (I)–(IV), utilizing only the resources of the language of DDL.

The sentence (I) is an unconditional obligation (or at least there is no explicitly stated condition.) It can be formalized as $LO-x$. The sentences (II) and (III) are commitments, that is, sentences about conditional obligations which obtain when certain actions are performed or omitted. In these sentences the performance and the omission of x play the role of the circumstance for other actions. Sentence (II) says that whenever x has ben omitted, the obligation to omit y exists. In DDL we can formalize it by means of the formulae $L([-x]O-y$. Similarly, the formalization of sentence (III) is $L([x]Oy)$.

The last of the four sentences describes an action which forms the circumstance of other actions. We can formalize it with the aid of a propositional constant p with the meaning that John has just impregnated Suzy Mae. In terms of semantics we say that p is true at a state s in a model M if and only if there is a relevant state t such that $(t, s) \in R_L$ and $(t, s) \in R(x)$, that is, s is the state after x has been done at the previous state t. To preserve the intended meaning of "p" we need to add to our formalization the set of all formulae which are true in all models of p. Formally, let x be a quasi-atomic action and let p be a propositional constant. We define Γ to be the set of all DDL-formulae which are is true in all DDL-models $M = (W, R_L, R, V)$ such that for all $s \in W$, $M, s \models p$ if and only if there is some $t \in W$ such that $(t, s) \in R_L$ and $(t, s) \in R(x)$.

Thus, the formalization of (I)–(IV) in the language of DDL is as follows:

(Ie) $LO - x$

(IIe) $L([-x]O - y$

(IIIe) $L([x]Oy)$

(IVe) p

(IVe*) Γ

Our formalization would be much nicer if we could replace the theory Γ by a single formula which still preserves the intended meaning of p. This could be a formula saying that p is true whenever x has been done, that p is false whenever x has been omitted, and that anything that always holds after x has been done must also hold where p holds. However, it seems that there is no formula which exhausts the whole meaning of Γ. My conjecture is that we cannot find a DDL-formula which has the same set of models as Γ. Fortunately, we do not need the whole set Γ in our reasoning, but only a part of it. Our next lemma shows that the formula $L([x]p \wedge [-x]\neg p)$ belongs to Γ, and so do all instances of the schema $L[x]A \supset L(p \supset A)$. The former formula says that p always holds after x is performed, and it negation always holds after x is omitted. The latter schema says that if a formula A always holds after x is performed, then A must hold whenever p holds, because p indicates that x has been done.

Lemma 21 *Let $M = (W, R_L, R, V)$ be a DDL-model, let x be a quasi-atomic action and let p be a DDL-formula such that for all $s \in W$, $M, s \models p$ if and only if there is some $t \in W$ such that $(t, s) \in R_L$ and $(t, s) \in R(x)$. Then $M \models L([x]p \wedge [-x]\neg p)$ and $M \models L[x]A \supset L(p \supset A)$.*

We see that the translation (Ie)–(IVe*) is consistent. We can construct a model for (Ie)–(IVe) which also satisfies Γ, that is, we can construct a model M of the translation (Ie)-(IVe) such that for all states t of M, p is true at s in M if and only if $(t,s) \in R_L \cap R(x)$ for some state t. Furthermore, we can infer Oy from Γ and (Ie)–(IVe), but we cannot infer $O-y$, as a satisfactory solution to the problem of Chisholm should be.

To conclude this section a short remark on the so-called principle of amalgation, which is valid for standard deontic logic (cf. [13]). This principle says that if an action p commits one to do q then any action $p \wedge r$ also commits one to do q. This principle seems to be paradoxical to some deontic logicians: it seems to say that if impregnating Suzy Mae commits John to marry her, then John ought to marry her after John has impregnated and then killed her. In dynamic deontic logic the principle is not valid in general: neither $[x;z]Oy$ nor $[x\|z]Oy$ is a consequence of $[x]Oy$. The performance of $(x;z)$ or $(x\|z)$ ("impregnating and killing") brings about another situation than the performance of x alone, therefore the duties associated with x do not hold for the other situation.

6 Variations of DDL

6.1 Additional deontic concepts

In standard deontic logic one can start with one of the three concepts and then define the other ones using negation. As I noted before, our concepts cannot be defined in this way. However, we can also start with one of our concepts and define additional deontic operators with the aid of the two sorts of negation: propositional negation (i.e., the usual truth-functional negation which applies to formulae) and act-negation (i.e., the omission operator.) The defined concepts are in general weaker then the corresponding standard ones. We define weak (or implicit) permission as the absence of an explicit prohibition, and weak or implicit prohibition as the absence of an explicit permission. Weak obligation is defined as the weak prohibition to omit the action under consideration:

$$P^w x =_{def} \neg F x$$

$$F^w x =_{def} \neg P x$$

$$O^w x =_{def} F^w(-x)$$

From these definitions and the definition of the standard concepts we have the following equivalences:

(34) $P^w x \leftrightarrow < x > \neg S$

(35) $F^w x \leftrightarrow < x > S$

(36) $O^w x \leftrightarrow < -x > S$

The intuitive idea behind the weak concept of permission, for instance, is that an action is permitted if one can find a way to do it without leading to a sanction. As I noted in the informal discussion, the strong deontic concepts seem to capture our intuitions about norms better than the weak ones. The notion of weak permission, for example, has some undesirable properties of the permission operator in standard deontic logic, for example one can prove the validity of the paradoxical formula $P^w x \supset P^w(x \cup y)$. In normative discourse, one usually refers to the normative concepts in the strong sense. However, the weak ones have some pleasant features of normative notions where the strong ones do not, namely, the famous Kantian paradigm of "Ought implies Can". We can prove that if an action is obligatory (or forbidden, or permitted) in the weak sense, then the same action can be performed:

(37) $P^w x \supset Can(x)$

(38) $F^w x \supset Can(x)$

(39) $O^w x \supset Can(x)$

To prove the last formula we assume that $O^w x$ is true at some state s. Then there is some t such that $(s,t) \in R(-x)$. It follows that there is some u such that $(s,u) \in R(x)$, that is, $Can(x)$ is true at s. The other implications are even simpler to prove.

The corresponding formulae are, however, not valid for the standard concepts. This leads us to the definition of the following strong concepts:

$$P^s x =_{def} Px \wedge Can(x)$$

$$F^s x =_{def} Fx \wedge Can(x)$$

$$O^s x =_{def} Ox \wedge Can(x)$$

It is then obvious that the strong concepts imply the corresponding standard ones. Moreover, they satisfy the principle "Ought implies Can." Whereas the standard concepts do not in general imply the weak ones, the strong concepts do: the following formulae are DDL-valid:

(40) $P^s x \supset P^w(x)$

(41) $F^s x \supset F^w(x)$

(42) $O^s x \supset O^w(x)$

To prove the first of the three formulae, for example, let us assume that at the state s the action x is permitted in the strong sense. It follows that there is some t

such that $(s,t) \in R(x)$ and $\neg S$ is true at all u such that $(s,u) \in R(x)$, hence $\neg S$ is true at t, and so $P^w x$ is true at s.

In contrast to the strong deontic concepts the standard ones do not imply the weak concepts. It can happen that the action cannot be performed at some state. In this case the standard permission holds, for example, but not the weak permission. Thus, the standard concepts imply the weak ones just in case the action under consideration can be performed.

The principle "Ought implies Can" has another reading. We can interpret it as saying that whatever is obligatory is also permitted. In DDL we can distinguish between the two readings, which was not possible in SDL. (To be fair, the SDL-formula $OA \supset PA$ expresses the idea that obligation implies permission, and the formula $\neg O(false)$ could be construed as saying that it is not obligatory to bring about the impossible, a weaker version of the Ought-implies-Can principle. However, the idea that some action can be performed cannot be formalized directly in SDL. Moreover, the mentioned formulae are equivalent given the other axioms and rules of SDL, so that they are logically indistinguishable.) In SDL the principle that obligation implies permission, expressed by the formula $OA \supset PA$, is valid. In DDL the corresponding principles do not hold: none of the obligation concepts implies any of the permission concepts. However, we can define further notions of obligation which possess the desired feature. For example, a very strong concept of obligation can be defined as the conjunction of strong obligation and strong permission. It is clear that this obligation implies Ought, Can and May in all the senses we have considered.

6.2 Dynamic Deontic Logic without the Constant S

With the aid of the concept of sanction we have been able to define the semantics of the normative concepts in a very elegant way. The natural question arises if the meaning of the latter concepts depends crucially on the meaning of the former. In other words, we can ask if it is possible to define the semantics for the deontic concepts without explicit reference to the notion of sanction. We can investigate the fragment of the language of DDL without the propositional constant S and ask how can we define a semantics for the operators O, F and P. (These notions are now regarded as primitive and not as defined ones.) The answer is that we can do it quite easily. The intuitive idea is that the possible world states where there is no sanction can be regarded as deontic alternatives of the current state, that is, states where all duties of our current state are fulfilled and no prohibition is violated. Thus, we can eliminate the constant S and replace it by a set *Opt* of states that are somehow "optimal". (This is basically the same idea we used to show the close relationship between SDL and modal-deontic logic.) The relationship between DDL and its S-free fragment is analogous that between modal-deontic logic and its deontic fragment, which can be identified with SDL.

Formally, the S-free fragment of DDL comprises all formulae that are built up from the propositional letters such that for all formulae A, B and for all actions x the following are formulae: $\neg A$, $A \wedge B$, LA, $< x > A$, Ox, Fx and Px. A model for S-free DDL is a tupel $M = (W, R_L, R, Opt, V)$, where W, R_L, R are as in the case of DDL-models, Opt is a subset of W, and V is a valuation that assigns to every propositional letter a subset of W. Intuitively, Opt is the set of states where the agent is not liable to sanction. The truth conditions for the propositional letters, the Boolean combinations and the operators L, $< x >$ remain unchanged. The truth-conditions for the deontic operators now become:

- $M, s \models Px$ iff for all $t \in W$, $(s,t) \in R(x)$ implies $t \in Opt$
- $M, s \models Fx$ iff for all $t \in W$, $(s,t) \in R(x)$ implies $t \notin Opt$
- $M, s \models Ox$ iff for all $t \in W$, $(s,t) \in R(-x)$ implies $t \notin Opt$

That this semantics preserves the desirable features of DDL can be seen as follows. Starting with a DDL-model we can construct a model for S-free DDL with the same set of states such that the two models verify the same formulae (not involving the constant S) at every state. On the other hand, every model for S-free DDL induces a DDL-model. The construction is obvious. Let $M = (W, R_L, R, V)$ be a DDL-model. Let $M' = (W', R'_L, R', Opt, V')$ be defined as follows: $W' = W$, $R'_L = R_L$, $R' = R$, $s \in Opt$ iff $s \notin V(S)$, and $V'(P) = V(P)$ for all propositional letters P. Conversely, let the model M' be given, we can construct M by defining $V(S) = W \setminus Opt$. (The other components of the model remain unchanged.) It is easily checked that for all formulae A of S-free DDL and for all states s, $M, s \models A$ if and only if $M', s \models A$. In particular, all DDL-valid formulae of the S-free fragment remain valid.

7 Conclusions and Open Problems

7.1 Related Works

The idea to base deontic logic on a logic of action in order to avoid the deontic paradoxes has been developed in a number of works by von Wright (c.f. [43], [44], [45], [46], [47].) My paper has been inspired partly by his works. There are, however, a number of important differences between his approach and mine. The most obvious difference is that my approach is semantic, whereas von Wright's approach is syntactic. I try to capture the intuitions about the meaning and truth of normative sentences and then derive the laws of action and deontic logic from the semantics. In contrast, von Wright tries to extract the axioms of the logic of action and norms from the standard usage of the normative statements in ordinary language. His logics do not possess a formal semantics. Without a formal semantics it is very difficult to see why one should accept certain axioms as valid and other as not valid. Moreover, it seems to me that the different meanings of the connectives "not", "and", "or" in different contexts, namely as propositional connectives on the one hand and as action connectives on the other hand, have not been recognized clearly and treated accordingly.

My approach to action logic is most similar to that of Segerberg ([33], [35].) In contrast to my approach, Segerberg studies the logic of action based on dynamic logic for its own sake and does not relate it to deontic logic. In an earlier paper ([32]) Segerberg has developed a system of deontic logic of action. However, this system has a very restricted language, so that it is not suitable for formalizing normative statements. For example, one cannot formalize conditional norms in this system. Due to its poor expressive power it is hardly possible to compare it with our system DDL.

As far as I know, the first attempt to found deontic logic on dynamic logic — interpreted as a logic of action — is B. Buck's dissertation ([6].) However, her work contains too little philosophical reflection about the problems of deontic logic, so that many problems are overlooked. Although her technical apparatus is much more complicated than mine, her system does not solve many problems of deontic logic. Among the valid formulae of her system one can find several paradoxes, for instance, Ross's paradox $Ox \supset O(x \cup y)$ or the paradoxes of conjunctive norms such as $O(x;y) \supset Ox$ or $P(x;y) \supset Px$. One of the main reasons for this failure is that the notion of omission has not been paid enough attention. Moreover, the temporal aspect of normative sentences has not been recognized clearly, so that certain normative statements cannot be formalized properly in Buck's system. For these reasons, this attempt must be regarded unsuccessful.

7.2 Open Problems

I have not been able to solve the problem whether or not the system DDL is decidable. We know that propositional deontic logic is decidable (c.f. [20],) but our logic is somewhat richer. If it is decidable, then how is the exact complexity of the decision problem? Provided that a logic is decidable, then we have already a recursive axiomatization of it: we simply choose the set of all valid formulae as the set of axioms. But we are normally interested in "nice" axiomatizations. If DDL were decidable, then we are still interested in finding a nice axiom system for it. However, it is even not clear if DDL or its extensions can be axiomatized adequately at all. In other words: is the set of all valid formulae of DDL recursively enumerable?

I have left out a number of important philosophical questions concerning deontic logic. Our systems developed so far are concerned with the norm-propositions. Can we have logics of norms as well? If yes, how is the structure of such a logic? Another question is based on an important distinction made by the moral philosopher W. D. Ross, namely the distinction between prima facie obligations and all-things-considered obligations (c.f. [29], [30].) A prima facie obligation exists if there are (moral, legal or other normative) reasons to do something. But prima facie obligations may be, and often are defeated by other, weightier obligations, either alone or in concert. An undefeated prima facie obligation is called all-things-considered obligation. The concept of obligation of deontic logic can be best construed as all-things-considered obligation. Our systems are no exception. But in ordinary language, an important class of normative statements refer to prima facie norms. An interesting question which arises in this context is how to accommodate the notion of prima facie obligations in our systems.

A third group of open problems arise when we try to extend our logics to accommodate other aspects of the concepts of action and of norm and to discuss the case of many agents. I have stressed before that our action logic is no more than a basic one: we have only one agent, and the actions are considered from a single perspective, namely as state-changes in the world. But we know that actions have many other aspects such as the agent's intention or knowledge, so that a comprehensive treatment of the logic of action must discuss these issues as well. As soon as we consider the multi-agent case, additional problems arise. In our context we can discuss the case that different agents play their different roles, for example, some agent is the norm-authority, who may give norms, and others are subjects of the norms. Some norms hold for certain agents, but not for others. The are not only actions of the autonomous agents but also interactions between the agents. The communication among the agents will play an important role. The behavior of an agent depends not only on her intention, her knowledge and her obligations, but also on her knowledge of other agents' actions, intentions, duties, belief etc. The discussion of such issues would exceed the scope of the paper, so that I shall not discuss them here and hope to return to the topic in future work.

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Erklärung

Hiermit erkläre ich, daß ich die Arbeit selbständig verfaßt und andere als die hier angegebenen Quellen nicht benutzt habe.

Leipzig, den 05.01.1995

Ho Ngoc Duc